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Impedance Calculation Algorithm For Microprocessor Based Digital Protective Relay

Neel Pawar [1], Saniya Vichare[2], Shikha Singh [3], Vedant Bhatkar[4], Rajeev Valunjkar [5]
[1](Electrical Engineering, Atharva College of Engineering/Mumbai University, India)
[2](Electrical Engineering, Atharva College of Engineering/Mumbai University, India)
[3](Electrical Engineering, Atharva College of Engineering/Mumbai University, India)
[4](Electrical Engineering, Atharva College of Engineering/Mumbai University, India)
[5](Electrical Engineering, Atharva College of Engineering/Mumbai University, India)

Abstract: *Earlier, line relaying algorithms represented most of the activity in computer relaying. This paper presents an algorithm to calculate the impedance of a transmission line fault. The impedance is calculated using differential equation algorithm (DEA) from sampled data. DEA, which is a parameter estimation method, is model-based and is independent of any form of input signal (voltage/current). The intended algorithm requires 3 samples at successive intervals of time. In this, examination of differential equation algorithm is done and attempts are made to draw a conclusion about the characteristics of this algorithm. DEA is an alternative method to Fourier Transform and similar algorithms and it shows that it can be used effectively in digital relays to find the distance of a fault from the relaying point in a transmission line.*
Key words: *Differential equation algorithm, Digital protection, Impedance relay.*

I. Introduction

Electromechanical and static relays were used in the beginning which had several drawbacks such as high burden on instrument transformers, long operating time, contact problems, inflexibility, inadaptability to changing system conditions, complexity and cost etc. Microcontroller based relays, which avoid most of these disadvantages are rapidly replacing these relays. Programmable equipment can respond fast and maybe used to implement complex threshold characteristics at low cost. They can also be self-checking in nature thereby requiring less maintenance and providing greater reliability. Implementation of digital relaying was first proposed by G.D. Rockefeller in 1969. Recent literature on digital protection of transmission lines is dedicated to developing suitable algorithms. A number of algorithms can be used as impedance calculations in which the fundamental frequency components of both voltages and currents are obtained from the samples. Amongst the various available algorithms, the differential equation algorithm which is based on a series R-L model of a transmission line is reviewed in the literature to follow.

II. Impedance calculations

The transmission line is represented as a series lumped R–L circuit as shown in the Fig. (i), for a single phase line.

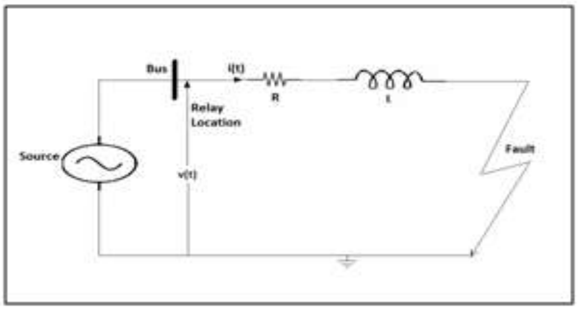

Fig. (i)

Here, the shunt capacitance of the line is ignored and it is assumed that fault resistance is zero. v(t) and i(t) are the voltage and current available at the relaying point. The sampling instants and sampled values of voltage and current are shown in Fig. (ii)[1]

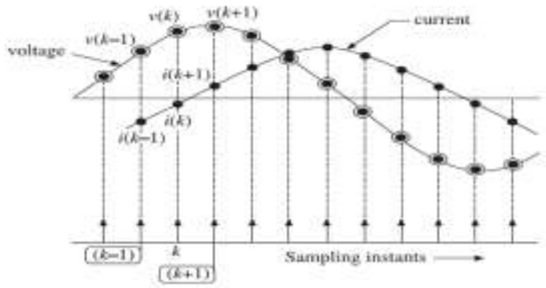

Fig. (ii)

Applying KVL around the loop formed because of fault, at instant t_k

$$v(t_k) = Ri(t_k) + L\frac{di(t_k)}{dt} \quad (1)$$

It can be seen that in the above differential equation, the known quantities are $v(t_k)$ and $i(t_k)$, and the unknowns are R and L, i.e. the resistance and the inductance of the transmission line from relay location up to the point of fault. In order to solve for the two unknowns, two equations are needed. The second equation can be generated by applying KVL at another instant of time say the $(k+1)^{th}$ instant as shown below:

$$v(t_{k+1}) = Ri(t_{k+1}) + L\frac{di(t_{k+1})}{dt} \quad (2)$$

Representing di(t)/dt by i'(t) and rewriting equations (1) and (2) as:

$$v(k) = Ri(k) + Li'(k)$$
$$v(k+1) = Ri(k+1) + Li'(k+1)$$

The matrix form of the above two equations is:

$$\begin{bmatrix} v(k) \\ v(k+1) \end{bmatrix} = \begin{bmatrix} i(k) & i'(k) \\ i(k+1) & i'(k+1) \end{bmatrix} \begin{bmatrix} R \\ L \end{bmatrix}$$

Thus,

$$\begin{bmatrix} R \\ L \end{bmatrix} = \begin{bmatrix} i(k) & i'(k) \\ i(k+1) & i'(k+1) \end{bmatrix}^{-1} \begin{bmatrix} v(k) \\ v(k+1) \end{bmatrix}$$

$$\begin{bmatrix} R \\ L \end{bmatrix} = \frac{1}{i(k)i'(k+1) - i'(k)i(k+1)} \begin{bmatrix} i'(k+1) & -i'(k) \\ -i(k+1) & i(k) \end{bmatrix} \begin{bmatrix} v(k) \\ v(k+1) \end{bmatrix}$$

Finally 'R' and 'L' can be expressed as a function of various samples of voltage & current & computed derivatives of current

$$R = \frac{i'(k+1)v(k) - i'(k)v(k+1)}{i(k)i'(k+1) - i'(k)i(k+1)}$$

$$L = \frac{i(k)v(k+1) - i(k+1)v(k)}{i(k)i'(k+1) - i'(k)i(k+1)}$$

The numerical derivatives can be calculated using the central difference formula as:

$$v'(k) = \frac{v(k+1) - v(k-1)}{2\Delta t}$$
$$i'(k) = \frac{i(k+1) - v(k-1)}{2\Delta t}$$

Where,

Δt is the sampling time period given by $1/f_{sampling}$.

Therefore,

$$R = \frac{[v(k)+v(k-1)][i(k+1)-i(k)]-[v(k+1)+v(k)][i(k)-i(k-1)]}{[i(k)+i(k-1)][i(k+1)-i(k)]-[i(k+1)+i(k)][i(k)-i(k-1)]} \quad (3)$$

$$L = \frac{2\Delta t[v(k)+v(k+1)][i(k-1)+i(k)]-[v(k-1)+v(k)][i(k)+i(k+1)]}{[i(k)+i(k-1)][i(k+1)-i(k)]-[i(k+1)+i(k)][i(k)-i(k-1)]} \quad (4)$$

Then impedance is thus, calculated as

$$Z = \sqrt{R^2 + X^2}$$

Where,

$$X = \omega L/2$$

The block-diagram showing the input to the algorithm and the block required for the implementation of specific distance relay characteristics is shown in Fig. (iii).

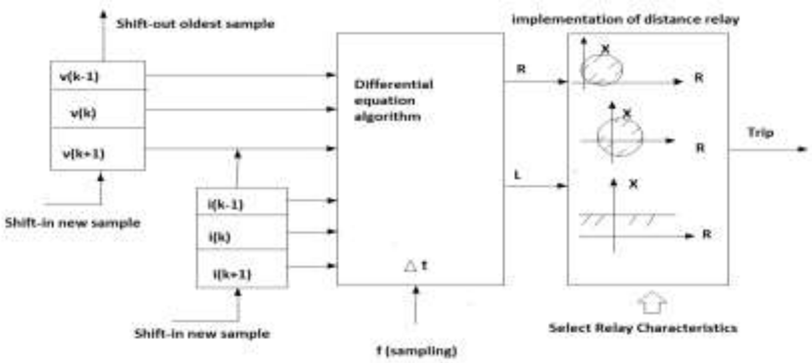

Fig. (iii)

III. Extension Of Differential Equation Algorithm To A Three Phase Line

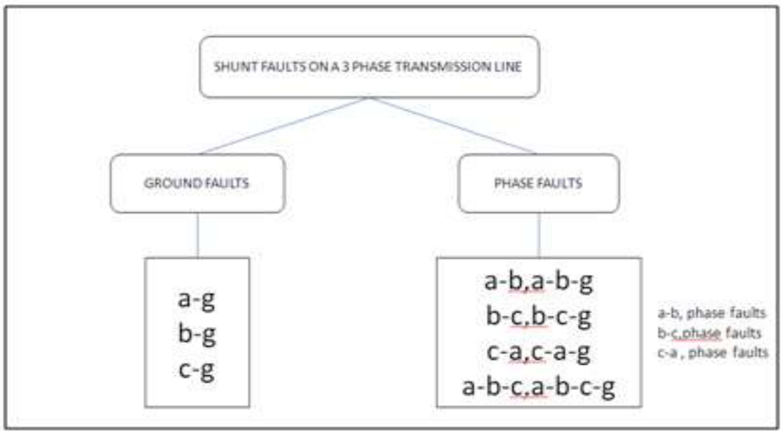

Fig. (iv)

Differential equation algorithm when applied to three-phase line needs careful consideration. This is because there are a total of 11 shunt faults that can take place on the three-phase line. Further, whatever happens in one-phase of a three-phase line depends upon what is happening in other two-phase because of the inductive and capacitive coupling between the phases. Therefore, appropriate voltage and current signals need to be selected from the three voltage signals (v_a, v_b, v_c) and three current signals (i_a, i_b, i_c) available at the relaying location. Thus, six numbers of measuring units are required for implementing protection against all 11 shunt faults that can occur on a three-phase line as shown below Fig. (iv).

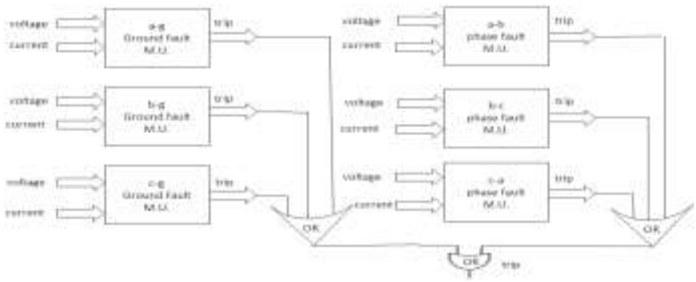

Fig. (v)

A simple logic diagram for the organization of distance protection is shown in Fig. (v).
As seen, the multitude of faults occurring on a three phase line can be divided into two categories:

1. Ground Fault Protection of Three-Phase Line

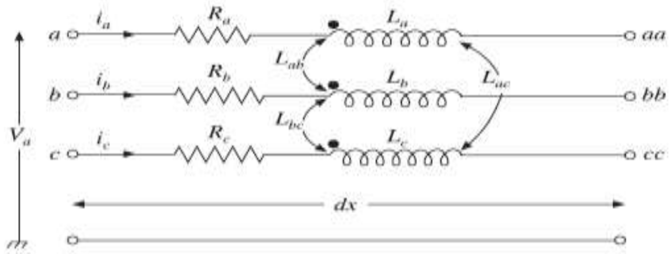

Fig. (vi)

The model shown in Fig. (vi) is used for the sake of developing differential equation algorithm for three-phase lines. It can be seen that the shunt capacitance of the line is neglected. Initially, equations based on the series distributed parameter model are developed and then the parameters are lumped.
Voltage drop across a small length 'dx' of phase 'a' of the line, denoted by $dV_{aa'}$ can be written as:

$$dV_{aa'} = (R_a dx)I_a + L_a dx\left(\frac{dI_a}{dt}\right) + L_{ab}\,dx\left(\frac{dI_b}{dt}\right) + L_{ac}\,dx\left(\frac{dI_c}{dt}\right)$$

Assuming that there is a single line to ground bolted fault at length 'x' from the sending end, the voltage across length 'x' can then be written as:

$$V_{aa'} = (R_a x)I_a + L_a x\left(\frac{dI_a}{dt}\right) + L_{ab}\,x\left(\frac{dI_b}{dt}\right) + L_{ac}\,x\left(\frac{dI_c}{dt}\right)$$

$$V_{aa'} = x(R_a)I_a + xL_a\left(\frac{dI_a}{dt}\right) + xL_{ab}\left(\frac{dI_b}{dt}\right) + xL_{ac}\left(\frac{dI_c}{dt}\right)$$

$$V_{aa'} = x(R_a)I_a + xL_a\frac{d}{dt}\left(I_a + \left(\frac{L_{ab}}{L_a}\right)I_b + \left(\frac{L_{ac}}{L_a}\right)I_c\right)$$

Note that $V_{aa'} = V_a$ for line to ground fault on phase 'a'. Hence,

$$V_a = x(R_a)I_p + xL_a\frac{dI_q}{dt} \qquad (5)$$

Where $I_p=I_a$ and $I_q = I_a + \left(\frac{L_{ab}}{L_a}\right)I_b + \left(\frac{L_{ac}}{L_a}\right)I_c$

Similarly,

$$V_b = x(R_b)I_p + xL_b\frac{dI_q}{dt} \qquad (6)$$

Where $I_p=I_b$ and $I_q = I_b + \left(\frac{L_{ab}}{L_b}\right)I_a + \left(\frac{L_{bc}}{L_b}\right)I_c$.

$$V_c = x(R_c)I_p + xL_c\frac{dI_q}{dt} \qquad (7)$$

Where $I_p=I_c$ and $I_q = I_c + \left(\frac{L_{ca}}{L_c}\right)I_a + \left(\frac{L_{cb}}{L_c}\right)I_b$

In each of these equations, there are two unknowns (xR_a) and (xL_a) and thus, the equation is in the form of $v = R\, i(t) + L\, di(t)/dt$; albeit the current terms i(t) and di(t)/dt are different. Thus, the same methodologies that were used for the single phase line can be followed henceforth.

2. Phase Fault Protection of Three-Phase Line

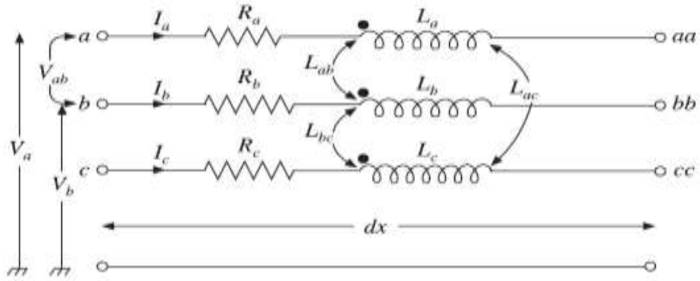

Fig. (vii)

From Fig. (vii)[1], the equations are,

$$v_{a=}x(R_a)I_a + x(L_a)\frac{dI_a}{dt} + xL_{ab}\frac{dI_b}{dt} + xL_{ac}\frac{dI_c}{dt}$$
$$v_b = x(R_b)I_b + x(L_{ab})\frac{dI_a}{dt} + xL_b\frac{dI_b}{dt} + xL_{bc}\frac{dI_c}{dt}$$
$$v_a - v_b = x(R_a)I_a - x(R_b)I_b + \left(x(L_a)\frac{dI_a}{dt} - x(L_{ab})\frac{dI_a}{dt}\right) + \left(x(L_{ab})\frac{dI_b}{dt} - x(L_b)\frac{dI_b}{dt}\right) + \left(x(L_{ac})\frac{dI_c}{dt} - x(L_{bc})\frac{dI_c}{dt}\right)$$
$$v_a - v_b = xR_a\left(I_a - \frac{R_b}{R_a}I_b\right) + x(L_a - L_{ab})\frac{dI_a}{dt} + x(L_{ab} - L_b)\frac{dI_b}{dt} + x(L_{ac} - L_{bc})\frac{dI_c}{dt}$$

For a fully transposed line it can be assumed that $L_{ac} = L_{bc}$. Hence, the last term in the previous expression vanishes, giving:

$$v_a - v_b = xR_a\left(I_a - \frac{R_b}{R_a}I_b\right) + x(L_a - L_{ab})\left(\frac{dI_a}{dt} + \left(\frac{L_{ab} - L_b}{L_a - L_{ab}}\right)\frac{dI_b}{dt}\right)$$
$$v_a - v_b = xR_a\left(I_a - \frac{R_b}{R_a}I_b\right) + x(L_a - L_{ab})\frac{d}{dt}\left(I_a - \left(\frac{L_b - L_{ab}}{L_a - L_{ab}}\right)I_b\right)$$

For a symmetric line, L_b - L_{ab}/L_a - L_{ab} and R_b/R_a will be equal to '1' and the above equation reduces to:

$$V_a - V_b = xR_a(i_a - i_b) + x(L_a - L_{ab})\frac{d(i_a - i_b)}{dt} \qquad (8)$$

Here also, there are two unknowns ***(xR$_a$)*** and ***x(L$_a$-L$_{ab}$)*** . The term ***x(L$_a$-L$_{ab}$)*** arises as the fault has been assumed to have occurred between phases 'a' and 'b'. Equations similar to equation (8) will be formed for fault between other phases. Thus, again the equations are in the form, ***v= R i(t)+ L di(t)/dt .***

IV. Conclusion

The advantages of differential equation algorithm are – higher accuracy and efficiency at low voltages, absence of filters makes the circuit compact, rather than using a single frequency model, this approach has the advantage of allowing all signals that satisfy the differential equation to be used in estimating the R and L of the model. This simplifies the design of a differential equation based numerical relay.

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