

Optimization of Gear Pairs Using Genetic Algorithm

¹Y.K.Mogal , ²D.D.Palande, ³V.D.Wakchaure

Department of Mechanical Engineering, MCERC ,Nasik
Department of Mechanical Engineering, AVCOE,Sangamner

Abstract: - Up to now many optimization techniques have been developed & used for optimization of engineering problems to find optimum design. Solving engineering problems can be complex & time consuming when there are large number of design variables & constraints. A Gear design require the designer to compromise many design variables; i.e. continuous, discrete & integer variables in order to determine best performance of gear set. Therefore a conventional optimization technique has difficulty in solving those kinds of problem. In this paper Genetic algorithm is introduced for,

- 1] Minimization of power loss of worm gear mechanism with respect to specified set of constraints.
- 2] Minimization of volume of two-stage gear train.

Key words: - Optimization, Genetic Algorithm and Gear

1. INTRODUCTION

Design optimization for many engineering applications is multiobjective in nature. Due to conflicts among objectives, it is impossible to obtain a single design that corresponds to optima of all the objectives. A multiobjective optimization problem can be solved using single-objective optimization methods, if all the objectives can be combined into a single objective function by assigning each with a weight coefficient or if all but one objective can be converted into constraints [1]. Solving engineering problems can be complex and a time consuming process when there are large numbers of design variables and constraints. Hence, there is a need for more efficient and reliable algorithms that solve such problems. The improvement of faster computer has given chance for more robust and efficient optimization methods. Genetic algorithm is one of these methods.[3]

Originally developed by Holland, a genetic algorithm (GA) is a robust technique, based on the natural selection and genetic production mechanism. The genetic algorithm works with a group of possible solutions within a search space instead of working with a single solution as is seen in gradient optimization methods. [2]

The flow chart of genetic algorithm is shown in Fig.1. An initial population is chosen randomly at the beginning, and fitness of initial population individuals are evaluated. Then an iterative process starts until the termination criteria have been run across. After the evaluation of individual fitness in the population, the genetic operators, selection, crossover and mutation are applied to breed a new generation. The newly created individuals replace the existing generation and reevaluation is started for fitness of new individuals. The loop is repeated until acceptable solution is found.[3]

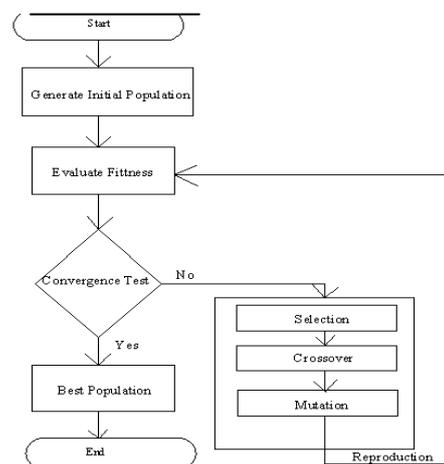


Fig.1.Flow chart for GA

The design of gear train is a kind of mixed problems which have to determine various types of design variables; i.e.,continuous, discrete, and integer variables.. In this study, the Genetic Algorithm is introduced for geometrical volume (size) minimization problem of the two-stage gear train and the simple planetary gear train to show that genetic algorithm is better than the conventional algorithms for solving the problems that have [4]

Finite element analysis (FEA) has proved to be a useful method to analyze the strength of worm gears; however, it is usually time consuming and difficult for parametric design, and, hence, it is difficult to apply FEA for design optimization. To overcome the problem, the authors developed a method, which combines ANN and GA to optimize worm gear design based on FEA results. [6]

The reduction of vibration and noise is nowadays a very important issue in the design of gear power transmissions. Many authors underlined

the effect of modifications on the dynamics of gears: Kahraman and Blankenship experimentally measured the effect of the involutes tip relief on the dynamic response, while Bonori et al. simulated the effect of different profile modifications and manufacturing errors on gear noise. A key point on the design process of spur gears is the choice of the best profile modification; it is not trivial difficult to define design guidelines for profile modifications.[7]

2. APPLICATION OF GA FOR POWER LOSS OPTIMIZATION

In this study, a non-conventional algorithm namely genetic algorithm is presented for minimization of power-loss of worm gear mechanism with respect to specified set of constraints. Number of gear tooth, friction coefficient, and helix (thread) angle of worm are used as design variables and linear pressure, bending strength of tooth, and deformation of worm are set as constraints. The results for minimization of power-loss of worm gear mechanism are presented to provide a comparison with analytical method

In the design of gear set, the shafts angle is 90°. Transmission rate is 1/15; and three Thread worm. In the worm gear system because of friction, missing power is turned to heat. The objective function is the power loss with respect to constraints such as linear pressure worm gear tooth, bending stress of gear tooth for acceptable deflection, acceptable deflection of worm shaft.[3]

Power loss, ΔP, formulated as:

$$\Delta P = P_i - P_o$$

Where, P_i: input power,

P_o: output power

$$P_o = F_n (\cos \alpha_n \cos \gamma_n - \mu \sin \gamma_n) \frac{m a z_g w_w}{2 * i}$$

Where,

- F_n: Normal force
- α_n: Pressure angle
- γ_n: Helix angle
- μ: Friction coefficient
- m_a: Module
- z_g: Number of gear tooth
- w_w: Angular velocity of worm
- i: Transmission rate.

$$F_n = \frac{F_{t1}}{\cos \alpha_n \cos \gamma_n + \mu \sin \gamma_n}$$

$$F_{t1} = \frac{2\pi n}{60} * d_{ow}$$

Where,

- F_{t1}: Tangential force
- n: Number of revolution of worm
- d_w: worm diameter

2.1 Objective Function

It is desired to obtain the lowest power loss of worm gear mechanism subject to linear Pressure worm gear, bending stress of gear tooth and deflection of worm shaft under the Load. The objective function is formulated as:

$$F_{obj} = F(z_g, \gamma_n, \mu) = P_i - F_n (\cos \alpha_n \cos \gamma_n - \mu \sin \gamma_n) \frac{m a z_g}{2 * i}$$

2.2 Design Variables and Parameters

- Gear tooth number 21 ≤ Z_g ≤ 80
- Friction coefficient 0.03 ≤ μ ≤ 0.05
- Helix angle 15° ≤ γ_n ≤ 25°

Table 1: Coefficients and input values for sample design practice

Definition	Symbol	Unit	Values
Input power	P _o	KW	11
Number of revolution tour of worm	n	Rpm	720
Transmission rate	i	-	15 +-0.4
Center distance of worm gear pair	a	mm	200
Distance between of worm shaft bearings	L	mm	330
Module	m _a	mm	7
Number of worm teeth	z _w	-	3
Worm diameter	d _{ow}	mm	71
Bottom of teeth diameter of worm	d _{f_w}	mm	55
Pressure angle	α _n	Degree	22.5
Elasticity module	E	N/mm ²	21.10 ⁴
Inertia	I	mm ⁴	449000

2.3 Constraints

Constraints are conditions that must be met in the optimum design and include restrictions on the design variables. These constraints define the boundaries of the feasible and infeasible design space domain. The constraints considered for the optimum design of the power loss of worm gear are the following

$$g_j = (Z_g, \gamma_n, \mu) \leq 0$$

j= 1,..... n No. of constraints

$$g_1(x) = \frac{F_{t2}}{b_{og} * m_a * z_g} * 2.5 - 3.6 \leq 0$$

$$g_2(x) = \frac{F_{t2}}{\pi * m_a * b_{og}} - 30 \leq 0$$

Number of Individual	Randomized binary string
1	1 1 0 1 0 0 0 0 0 0 1 0 0 0 0 1 1 1 0 0 1 0 1 0 1 1 1 1
2	0 1 0 1 1 0 0 1 1 1 0 1 0 0 0 1 1 0 0 0 1 0 0 1 1 1 1 0
...
85	0 0 0 1 1 0 0 0 0 0 0 1 0 1 0 0 1 0 0 0 1 0 0 1 1 1 1 1 0

$$g_3(x) = \frac{df_w}{1000} - \frac{F_t R_1 + L_3}{48 EI} \leq 0$$

Where,

- g_1x : Linear pressure worm gear tooth
- g_2x : Bending stress of gear tooth
- g_3x : Acceptable deflection of worm shaft
- $F_{t2} = F_n (\cos \alpha_n \cos \gamma_n - \mu \sin \gamma_n)$
- $b_{og} = 0.45(d_{o1} + 6m_a)$

$$F_{r1} = \sqrt{F_t^2 + F_r^2}$$

Where,

- F_{r1} : Radial force represented with Equation
- $F_r = F_n \sin \alpha_n$
- Fitness Function = $F - [F(Z_g, \gamma_n, \mu) + PF]$

$$PF = \sum_{j=1}^n r_j [\max(0, g_j)]^2$$

Table 2. Coding of binary design variable vectors into binary digits

Design variables vector	Lower limit	Upper limit	Precision	String length
Z_g	21	80	1	9
γ_n	0.03	0.05	0.0001	9
μ	15	25	0.1	9

To start the algorithm, an initial population set is randomly assigned. This set of initialized population is potential solution to the problem. The binary string representation for the design variables (Z_g, γ_n, μ) in Table 3 gives an example of a chromosome that represents design variables accordingly. This design string is composed of 27 ones and zeros.

Table 3: The binary string representation of the

Design Variables		
Gear tooth number (Z_g)	Friction coefficient (μ)	Helix angle (γ_n)
1 1 0 1 0 0 0 0 0	0 1 0 0 0 0 1 1 1	0 0 1 0 1 0 1 1 1

Concatenated variables head-to-tail
1 1 0 1 0 0 0 0 0 0 1 0 0 0 0 1 1 1 0 0 1 0 1 0 1 1 1

Population Size = $165.2^{0.21 * 1}$

For a string length of 27bits, an optimal population size of 85 maybe used.

Table 4. A set of starting population

The setting parameters of genetic algorithm for this study are chosen as follows:

- Chromosome length = 27,
- Population size = 85,
- Number of generation = 100,
- Crossover Probability = 0.5,
- Mutation probability = 0.005,

2.4 Results & Discussion

Fig 2 shows the 3-D plots of design variables values during the working of GA. Design variables have been got different values and take on minimum value of objective function at 69-th generation shown in Fig 2. It has been shown that design variables take on values as: number of gear teeth is 44, friction coefficient is 0.0305, and helix angle is 5.246o. Fig 2 shows the plots of the helix angle and number of worm gear teeth in each generation as optimization proceed. The overall results show that the best design converge 69-th generation and refine the design over remaining generations,. The results compared with results of analytical method, shown in Table 5.

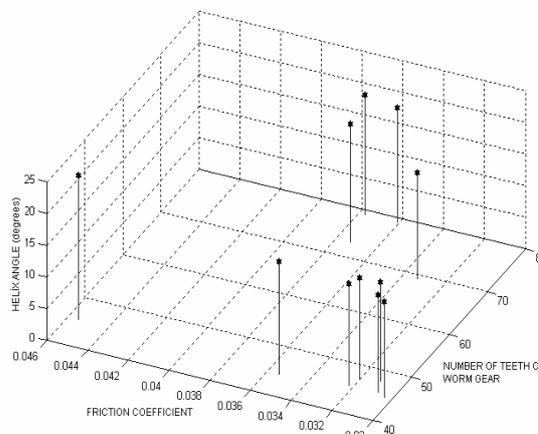


Fig.2. Variation of variables through generations

Table 5 Comparison of the results

Design Variables	Analytical method	Genetic algorithm
Number of teeth of worm gear (Z_g)	46	44
Friction coefficient (μ)	0.0390	0.0350
Helix angle (γ_n)	16.280°	15.246°
Minimum power loss	1.362 KW	0.881 KW

The starting point of analytical method $Z_g = 45$, $\mu=0.046$, $\gamma_n= 22.5^\circ$. The programme, developed in MATLAB 7.0 for analytical method has been run several time for different values of design variables. The results obtained are given in Table 5. As can be seen from the results, the genetic algorithm produced much better results than analytical method.

3. APPLICATION OF GA FOR GEOMETRICAL VOLUME & GEAR RATIO OPTIMIZATION

The objectives of the design are geometrical volume (size) minimization of 2-stage gear train and that of simple planetary gear train that has very valuable aspects in lightweight, small size and high strength. With regard to these design objects, we determine the number of teeth, module, face width, and helix angle, considering constraints such as strength (durability), interference, contact ratio and other factors based on AGMA standards so that these gear trains can perform the tasks required in design specification.[4]

3.1 Volume Minimization of 2-Stage Gear Train

The objective function and constraints can be written as,

$$F_{\text{objective}} = w_1 F_{\text{obj1}} + w_2 F_{\text{obj2}} + \sum_{j=1}^P \gamma_j (\text{Max}(G_j, 0))^2$$

$$F_{\text{obj1}} = \mu \frac{Z_2 s_1 + Z_2 s_2}{Z_1 s_1 + Z_1 s_2}$$

$$F_{\text{obj2}} = b_{s1} \left(\frac{m n s_1}{\cos \beta s_1} \right)^2 (z_{1s1}^2 + z_{2s1}^2) + b_{s2} \left(\frac{m n s_2}{\cos \beta s_2} \right)^2 (z_{1s2}^2 + z_{2s2}^2)$$

$$G_1 = 1.2 \sigma_{\text{HS1}} - \sigma_{\text{H lim}}$$

$$G_2 = 1.2 \sigma_{\text{HS2}} - \sigma_{\text{H lim}}$$

$$G_3 = 1.15 \sigma_{\text{Fs1}} - \sigma_{\text{F lim}}$$

$$G_4 = 1.15 \sigma_{\text{Fs1}} - \sigma_{\text{F lim}}$$

$$G_5 = v_{ts1} - v_{\text{tmax}}$$

$$G_6 = v_{ts2} - v_{\text{tmax}}$$

$$G_7 = 1 - \epsilon_{as1}$$

$$G_8 = \epsilon_{as1} - 2.5$$

$$G_9 = 1 - \epsilon_{as2}$$

$$G_{10} = \epsilon_{as2} - 2.5$$

$$G_{11} = 0.8 - \epsilon_{\beta s1}$$

$$G_{12} = \epsilon_{\beta s1} - 6$$

$$G_{13} = 0.8 - \epsilon_{\beta s2}$$

$$G_{14} = \epsilon_{\beta s2} - 6$$

$$G_{15} = b_{s1} \cos \beta_{s1} - 2 Z_{1s1} m_{ns1}$$

$$G_{16} = 0.5 Z_{1s1} m_{ns1} - b_{s1} \cos \beta_{s1}$$

$$G_{17} = b_{s2} \cos \beta_{s2} - 2 Z_{1s2} m_{ns2}$$

$$G_{18} = 0.5 Z_{1s2} m_{ns2} - b_{s2} \cos \beta_{s2}$$

$$G_{19} = Z_{1s1 \text{ lim}} - Z_{1s1}$$

$$G_{20} = Z_{1s2 \text{ lim}} - Z_{1s2}$$

$$G_{21} = \frac{Z_2 s_2}{Z_1 s_2} - \frac{Z_2 s_1}{Z_1 s_1}$$

$$G_{22} = \frac{Z_2 s_2 m n s_1}{\cos \beta s_1} - \frac{Z_2 s_2 m n s_2}{\cos \beta s_2}$$

where, F_{obj1} is the objective function for volume minimization, F_{obj2} is the objective function

for reduction gear ratio, w_1 and w_2 are weight factors for F_{obj1} and F_{obj2} respectively, γ_p is the penalty coefficient, and G_j is the violent value of constraints. $G_1 \sim G_4$ represent the constraints for the bending strength and pitting resistance on the 1st and 2nd stage gears considering factor of safety. $G_5 \sim G_6$ represent the constraints for pitch line velocity. $G_7 \sim G_{14}$ Represent the constraints for contact ratios. $G_{15} \sim G_{18}$ represent the constraints for aspect ratios of pinion of 1st and 2nd stage. $G_{19} \sim G_{20}$ represent the constraints for undercut. $G_{21} \sim G_{22}$ represent constraints for reduction gear ratio and pitch diameter of gear respectively. Fitness function is constructed by subtracting constructed single objective from Cmax, what is a adequate value which prevent the fitness value from being negative

$$F_{\text{fitness}} = C_{\text{max}} - F_{\text{objective}}$$

As for parameters used in genetic algorithm, number of individuals is 30, the probability of crossover is 0.8, the probability of mutation is 0.3, and the algorithm is set to terminate when the number of a shift in a generation reaches 10000.

Table 6. Bounds of design variables for 2-stage gear train for escalator

Design variable	Bounds
Z_{1s1}	14 - 77
Z_{2s1}	14 - 141
Z_{1s2}	14 - 141
Z_{2s2}	14 - 141
Normal module (mm)	0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.8, 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
b_{s1} (mm)	1 - 500
b_{s2} (mm)	1 - 500
β_{s1}	0 - 45
β_{s2}	0 - 45

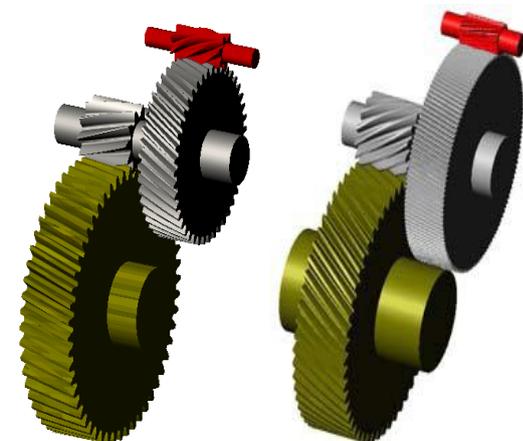
Table 7 Design specification of 2-stage gear train for escalator

Item	1-stage		2-stage	
	Pinion n	Gear r	Pinion n	Gear
Normal module [mm]	2.75		3.75	
Number of teeth	7	43	11	53
Total gear ratio	29.59			
Helix angle [°]	28		15	
Addendum modification coefficient	0.52	-.50	0.433	-0.483

Effective face width [mm]	37	50
Pressure angle [°]	20	
Rotational speed of input driver [rpm]	1750	
Transmitted power [kW]	7.5	
Grade (AGMA)	9	
Material	SCM415	
Heat treatment	Carburized & case hardened	
Surface hardness	HRC 60	
Volume (pitch circle) [mm ³]	2269249.613	

Table 8. Design result of optimization

Item	1-stage		2-stage	
	Pinion	Gear	Pinion	Gear
Normal module [mm]	1		2.5	
Number of teeth	18	134	14	54
Total gear ratio	28.71428			
Helix angle [°]	12.9		35.5	
Addendum modification coefficient	0	0	0	0
Effective face width [mm]	30		40.1	
Pressure angle [°]	20			
Rotational speed of input driver [rpm]	1750			
Contact stress [MPa]	1309		1330	
Allowable contact stress [MPa]	1531		1531	
Bending Stress [MPa]	351		338	
Allowable bending stress [MPa]	423		423	
Volume (pitch circle) [mm ³]	1377539.057			



(a) Old model (b) New model

Fig.3 Model of 2-stage gear trains of escalator

3.2 Result & Discussion

Table 8 shows design results that are obtained at 8664th generation, and from this, we can easily notice optimum values of integer and discrete variables are found adequately satisfying constraints. In this design results, the correction of solution according to the types of variables after design is not needed. Therefore genetic algorithm is the effective method in designing gear trains.

Fig.3 shows the comparison between the model of existing product and the model having dimensions found by optimization in this research. In this result, the volume of pitch diameter and face width is reduced about 40%, and the error of reduction gear ratio between objective and result are about 3%.

4. CONCLUSION

By studying above cases we concluded that, while optimizing the complex problems of mechanical system a Genetic Algorithm is important tool. So optimization of gear pair consisting of various parameters, objectives and constraints can be done easily using Non conventional optimization technique i.e. Genetic Algorithm as compared to conventional techniques.

5. FUTURE SCOPE

By using GA we can optimize gearbox by considering different objective functions and constraints.

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