IOSR Journal of Engineering (IOSRJEN) **www.iosrjen.org**
ISSN (e): 2250-3021, ISSN (p): 2278-8719
Vol. 10, Issue 3, March 2020, ||Series -II|| PP 32-40

Influence of the Aspect Ratio on the Thermal Performance of Axial Extended Surface in Electric Motor by the Generalized One-Dimensional Radial Fin Using the Frobenius Method

Élcio Nogueira
Received 05 March 2020; Accepted 19 March 2020

ABSTRACT: Analytical solution using the Frobenius Method for the generalized one-dimensional radial fin is applied to analyze the influence of the aspect ratio on the temperature distribution, heat transfer rate, efficiency, and effectiveness on the extended axial surface of an electric motor. Cast Iron and Aluminum materials were used for the fin. For high value for the aspect ratio and external heat transfer coefficient, the temperature, along the entire length of the fin, tends to the temperature of the medium. In contrast, the fin temperature tends to the surface temperature of the electric motor when the heat transfer coefficient decreases. The thermal performance does not increase indefinitely with the increase of the aspect ratio, and for relatively high values for aspect ratio, the heat transfer rate remains stagnant, with a tendency to decrease. The aspect ratio does not have much influence on the results in the efficiency and effectiveness, but the difference of the heat conduction coefficients between the used materials are very relevant.
KEY WORDS: Frobenius Method; Axial Extended Surface; One-dimensional Radial Fin; Electric Motor, Cast Iron; Aluminum.

I. INTRODUCTION

The serial expansion of the modified power, the Frobenius Method, is applied to a specific geometry, Figure 1, and converted to a special and particular geometry called one-dimensional radial fin in the specialized literature, Figure 2.

Extended surfaces are extensively used in engineering applications to increase heat transfer efficiency. The longitudinal fin of rectangular profile and the one-dimensional radial fin are the most commonly used Suárez[1], Soares and Nogueira[2], Santos[3], Freire et al.[4], Yuri et al.[5], Novaes et al.[6], Campo and Kundu[7], Cotta and Ramos[8], Sommers and Jacobi[9], Kraus[10], Aparecido and Cotta[11,12].

The applications in compact heat exchanger increase the interest in accessible and applicable models for fins systems Oktay[13].

Nogueira[14] demonstrated that the one-dimensional model, using Frobenius Method, is suitable for compact fins systems and that the smaller the value of the aspect ratio, the higher the range of Biot number in which the one-dimensional model works correctly.

Nogueira[15] presented a comparison between the Frobenius Method and the mathematical model already established and validated, called “Improved One-Dimensional Classic Radial Fin” in the literature Cotta and Mikhailov[16]. The equivalence of the one-dimensional models presented is evident, considering the Reynolds number used for analysis. For relatively low values of the Biot number, the efficiency and effectiveness justify the placement of the fins and use of one of the two one-dimensional models.

II. OBJECTIVES

To analyze the influence of the aspect ratio on the thermal performance of the radial fin using a generalized one-dimensional model by the Frobenius method.

III. THEORETICAL ANALYSIS

Generalized One-Dimensional Radial Fin by Frobenius Method

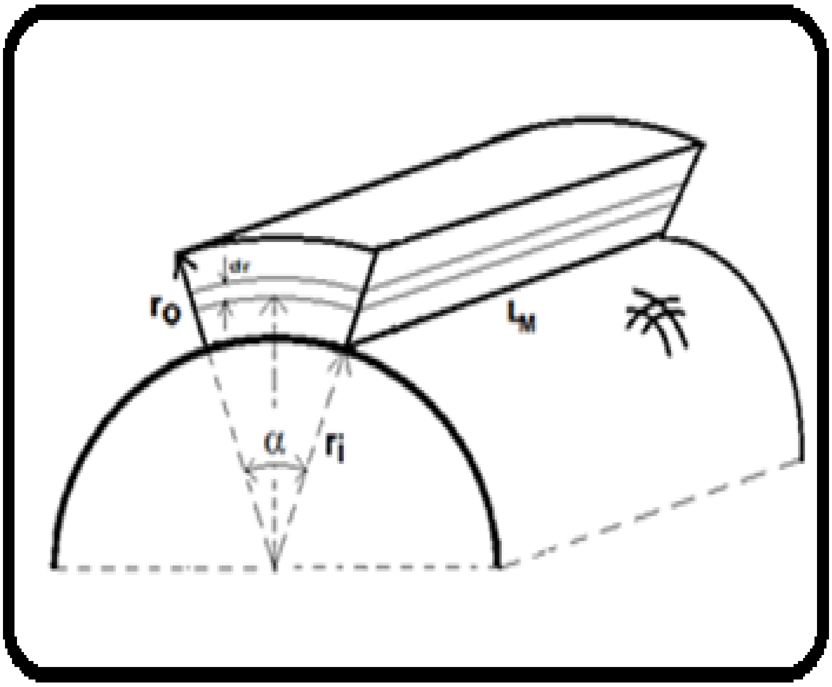

Figure 1 – Generalized Geometry for One-Dimensional Radial Fin Analysis

Table 1 – Data for typical finned Radial Electric Motor DC

Basic information for the fin system	
Engine width - L_M	130,13 mm
Width of fin base - *w*	5,84 mm
Internal radius – r_i	69,8 mm
External radius - r_o	86,8 mm
Fin height - L_o	17,00 mm
Conductivity - k	80W/(m.K)
Maximum base Temperature - T_b	98ºC
Maximum external Temperature - T_∞	40ºC

Consider steady-state, one-dimensional heat conduction through a radial fin, with constant conductivity, k, and subjected an ambient temperature, T_∞.

$$\frac{d}{dr}\left[r\frac{dT(r)}{dr}\right] = \frac{2h_2[\alpha r + L_M]}{k\alpha L_M}[T(r) - T_\infty] \quad (01)$$

By definition:

$$\theta(r) = \frac{T(r) - T_\infty}{T(r_i) - T_\infty} \quad (02)$$

Then, we have:

$$r^2\frac{d^2\theta(r)}{dr^2} + r\frac{d\theta(r)}{dr} = \frac{2h_2[\alpha r^2 + L_M r]}{k\alpha L_M}\theta(R) \quad (03)$$

$$R = \frac{r - r_i}{r_o - r_i} \quad \rightarrow \quad r = L_o R + r_i \; with \;\; L_o = r_o - r_i \quad (04)$$

$$[R + \frac{r_i}{L_o}]^2\frac{d^2\theta(R)}{dR^2} + \left(R + \frac{r_i}{L_o}\right)\frac{d\theta(R)}{dR} = \frac{2h_2[\alpha(L_o R + r_i)^2 + L_M(L_o R + r_i)]}{k\alpha L_M}\theta(R) \quad (05)$$

$$\mathbb{P}(R)\frac{d^2\theta(R)}{dR^2} + \mathbb{Q}(R)\frac{d\theta(R)}{dR} - \mathbb{W}(R)\theta(R) = 0 \quad (06)$$

At where

$$\mathbb{P}(R) = P_1 R^2 + P_2 R + P_3;\ \mathbb{Q}(R) = Q_1 R + Q_2;\ \mathbb{W}(R) = W_1 R^2 + W_2 R + W_3 \qquad (07)$$

and

$$P_1 = 1.0;\ P_2 = 2K_1; P_3 = {K_1}^2;\ Q_1 = 1.0;\ Q_2 = K_1 \qquad (08)$$

$$W_1 = \frac{2B_{i2}K^2}{K_2};\ W_2 = B_{i2}\left[\frac{4K^2K_1}{K_2} + \frac{2K}{\alpha}\right];\ W_3 = B_{i2}[\frac{2K^2K_1}{K_2} + \frac{2KK_1}{\alpha}] \qquad (09)$$

The dimensionless groups were defined for the problem in analysis:

$$K = \frac{L_o}{w/2}; K_1 = \frac{r_i}{L_o}\ ; K_2 = \frac{L_M}{w/2};\ B_{i1} = \frac{h_2(\frac{w}{2})}{k}; B_{i2} = \frac{h_2(\frac{w}{2})}{k} \qquad (10)$$

For the situation under analysis, an electric motor with axial fins, radial profile, and prescribed temperature in the base, Figure 2, the following simplifications are necessary:

$$B_{i1} \to \infty;\ r_i = 0 \to\ K_1 = 0\ and\ \alpha \to \infty \qquad (11)$$

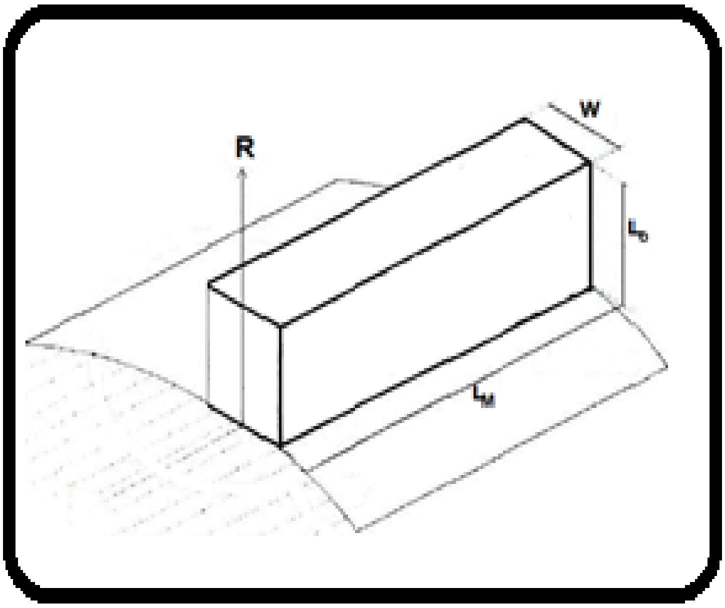

Figure 2 – Geometric Representation of Classical One-dimensional Radial Fin

Then,

$$P_2 = 0;\ P_3 = 0;\ Q_2 = 0;\ W_2 = 0\ and\ W_3 = 0 \qquad (12)$$

And

$$R^2\frac{d^2\theta(R)}{dR^2} + R\frac{d\theta(R)}{dR} - W_1R^2\theta(R) = 0 \qquad (13)$$

For convenience, was defined

$$K_2 = 2;\ \beta^2 = W_1\ \ and\ \ R' = \beta R \qquad (14)$$

In this case,

$$R'^2\frac{d^2\theta(R')}{dR'^2} + R'\frac{d\theta(R')}{dR'} - R'^2\theta(R') = 0 \qquad (15)$$

The Equation (15) has a singular regular point in R'= 0 and By Georg Frobenius (1849-1917),Euler-Mascheroni[22, pag.243], Kreyzig[17, pag.190], Arpaci[18, pag.231], Hildebrand[19, pag.143], Schneider[20, pag.46-59], Carslaw and Jaeger[21, pag.374-376]:

$$\theta(R') = \sum_{n=0}^{\infty} a_n R'^{n+s} \qquad (16)$$

$$\theta'(R') = \frac{d\theta(R')}{dR'} = \sum_{n=1}^{\infty} a_{n-1}(n+s-1)R'^{n+s} \qquad (17)$$

$$\theta''(R') = \frac{d^2\theta(R')}{dR'^2} = \sum_{n=2}^{\infty} a_{n-2}(n+s-2)(n+s-3)R'^{n+s} \qquad (18)$$

Then

$$\mathbb{P}(R)\sum_{n=2}^{\infty} a_{n-2}(n+s-2)(n+s-3)R^{n+s} + \mathbb{Q}(R)\sum_{n=1}^{\infty} a_{n-1}(n+s-1)R^{n+s}$$
$$- \mathbb{W}(R)\sum_{n=0}^{\infty} a_n R^{n+s} = 0 \qquad (19)$$

By algebraic manipulation, the following indicial equation was obtained:

$$a_0[(s^2-s)+s]R'^{s} = 0 \qquad with \qquad a_0 \neq 0 \ and \ s = 0 \qquad (20)$$

The roots of the indicial equation are equal to zero, and the recurrence rule is given by

$$a_n = \frac{a_{n-2}}{n^2} \qquad (21)$$

or

$$a_2 = \frac{a_0}{2^2};\ a_4 = \frac{a_0}{2^2 4^2}\ ;\ a_6 = \frac{a_0}{2^2 4^2 6^2}\cdots \qquad (22)$$

For the situation in analysis, two equal roots, there are two linearly independent solutions, which constitute a fundamental system of solution Kreyzig [17]. The first is:

$$\theta_1(R) = 1 + \sum_{m=1,2,3..}^{\infty} a_{2m}(\beta R)^{2m};\quad a_{2m} = \frac{1}{2^{2m}(m!)^2} \qquad (23)$$

The second linearly independent solution contains a logarithmic term and has a form:

$$\theta_2(R) = [\ln(\beta R)]\theta_1(R) + \sum_{m=1,2,3\ldots}^{\infty} A_m(\beta R)^m \qquad (24)$$

By Carslaw and Jaeger[21], and Boyce and Diprima[22] the more convenient expression is

$$\theta_2(R) = -\left[\ln\left(\frac{\beta R}{2}\right)+\gamma\right]\theta_1(R) + \sum_{m=1,2,3\ldots}^{\infty} a_{2m}H_m(\beta R)^{2m} \qquad (25)$$

At where

$$H_m = \frac{1}{m} + \frac{1}{m-1} + \cdots + \frac{1}{2} + 1 \quad and\ \gamma \cong 0.5772 \qquad (26)$$

γ is known as the Euler-Mascheroni[22, pag.247] constant.
Then

$$\theta(R) = a_0\theta_1(R) + a_1\theta_2(R) \qquad (27)$$

$$\theta(R) = a_0[1 + \sum_{m=1,2,3..}^{\infty} a_{2m}(\beta R)^{2m}] + a_1\left[-\left[\ln\left(\frac{\beta R}{2}\right)+\gamma\right]\theta_1(R) + \sum_{m=1,2,3\ldots}^{\infty} a_{2m}H_m(\beta R)^{2m}\right] \qquad (28)$$

The first boundary condition is defined by Cotta and Mikhailov[16]:

$$\theta(0) = 1 \quad \rightarrow \quad a_0 = 1 + a_1\left[\ln\left(\frac{\beta R_b}{2}\right)+\gamma\right] \qquad (29)$$

Finally,

$$\theta(R) = \theta_1(R) + a_1\left\{\left[\ln\left(\frac{\beta R_b}{2}\right)+\gamma\right]\theta_1(R) + \theta_2(R)\right\} \qquad (30)$$

$$\theta'(R) = \theta_1{}'(R) + a_1\left\{\left[\ln\left(\frac{\beta R_b}{2}\right)+\gamma\right]\theta_1{}'(R) + \theta_2{}'(R)\right\} \qquad (31)$$

Where (Figure 2)

$$R_b = R \rightarrow 0 \qquad (32)$$

For the second boundary conditions:

$$\theta'(1) = -B_{i2}K\theta(1) \qquad (33)$$

In this case,

$$a_1 = \frac{-[\theta_1(1) + B_{i2}K\theta_1'(1)]}{\left[\ln\left(\frac{\beta R_b}{2}\right)+\gamma\right][\theta_1(1) + B_{i2}K\theta_1'(1)] + [\theta_2(1) + B_{i2}K\theta_2'(1)]} \qquad (34)$$

The total exchange heat transfer is given by

$$\dot{q} = \frac{-kA_b(T_b - T_\infty)\theta'(0)}{L_0} \qquad (35)$$

The dimensionless exchange heat transfer is written in the form, by definition

$$Q_b = \frac{\dot{q}}{h_2A_b(T_b - T_\infty)} \quad \rightarrow \quad Q_b = \frac{-1}{B_{i2}K}\left(\frac{d\theta}{dR}\right)_{R=0} \qquad (36)$$

A_b *and* T_b *are the base area and the base temperature respectively*
Efficiency is given by

$$\eta = \frac{-1}{B_{i2}K(1+K)}\left(\frac{d\theta}{dR}\right)_{R=0} \quad (37)$$

And, the efficacy

$$\varepsilon = \frac{2Q_b}{B_{i2}K} \quad (38)$$

IV. RESULTS AND DISCUSSION

Results for temperature profile, heat transfer rate, efficiency, and effectiveness were obtained for different values of aspect ratio ($K = \frac{L_o}{w/2}$) and two different materials used in electric motors (Cast Iron and Aluminum).

The parameter R_b(equations 29 – 31) has a great influence on the generation of the results obtained by the model and, in general, it is strongly dependent on the heat transfer coefficient (h_2) and the aspect ratio. Figure 3, below, presents for the high amplitude of the heat transfer coefficient and a wide range of values for the aspect ratio, the relationship of R_b to the heat transfer coefficient external to the motor surface.

Figure 4 shows temperature distribution as a function of the radial position, for a wide range of values of the external heat transfer coefficient and high value for the aspect ratio. For a high value of the heat transfer coefficient by external convection ($h_2 = 1000$), the temperature, along the entire length of the fin, tends to the temperature of the medium. In contrast, the fin temperature tends to the surface temperature of the electric motor when the heat transfer coefficient decreases ($h_2 = 50$).

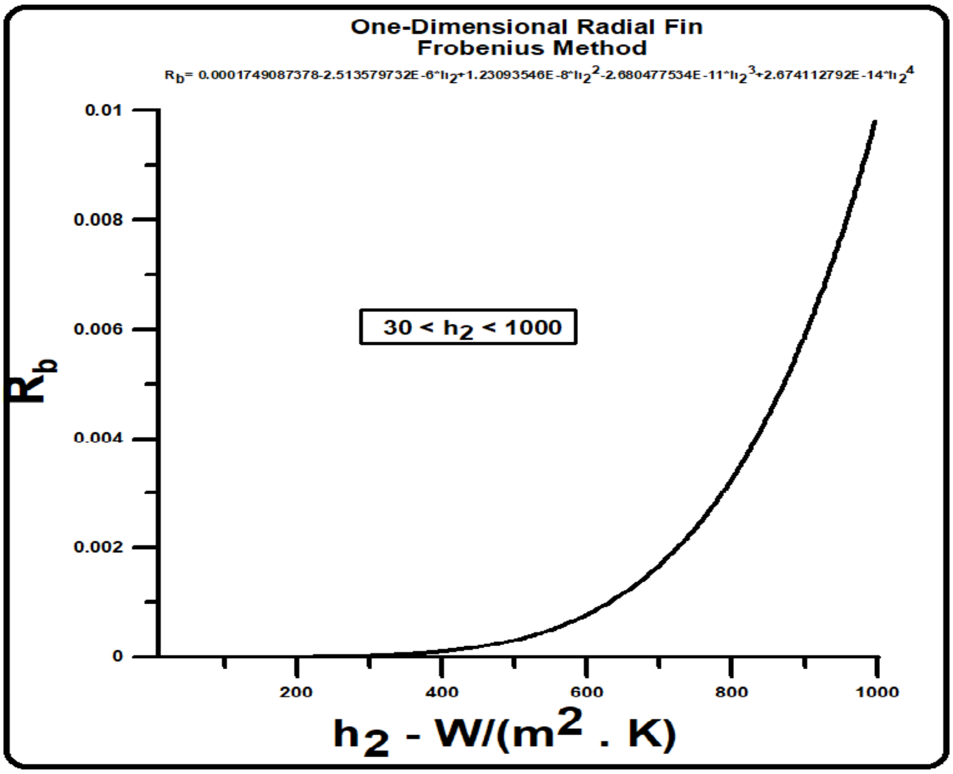

FIGURE 3– R_b versus convection heat transfer coefficient h_2

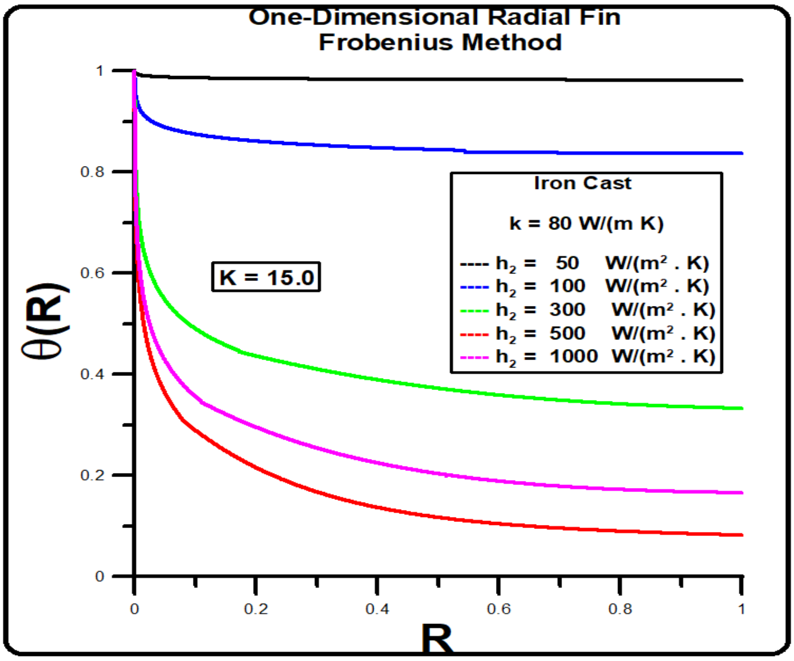

FIGURE 4–Dimensional temperature versus radial fin position

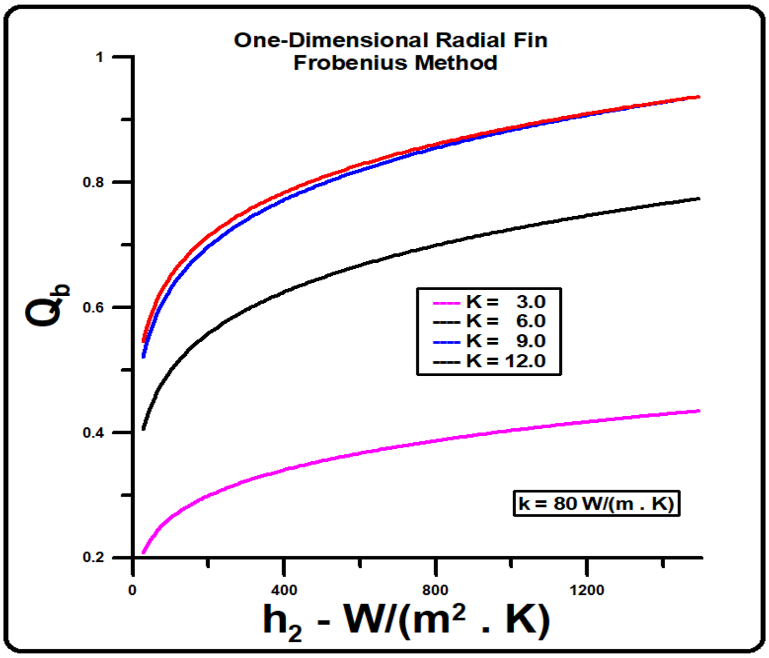

FIGURE 5–Dimensional heat transfer rate versus heat transfer coefficient h_2

The dimensionless heat transfer rate, as a function of the heat transfer coefficient by external convection, is represented by Figure 5, for different values of the aspect ratio and Cast Iron as the material of the fin. The results demonstrate that the thermal performance does not increase indefinitely with the increase of the aspect ratio. For relatively high values for aspect ratio, the heat transfer rate remains stagnant, with a tendency to decrease. This fact can be justified by referring to Figure 4 when the temperature tends to the value of the surface temperature of the electric motor for high values of the aspect ratio and high values for the external heat transfer coefficient.

Figure 6 presents results for efficiency as a function of the external heat transfer coefficient for two types of materials for the fin of the electric motor, Cast Iron, and Aluminum. For Cast Iron, efficiency is justified, above 60%, for values of the external heat transfer coefficient below 150 W / (m^2. K). On the other hand, for Aluminum, values below 400 W / (m^2. K) enable efficiency above 60% for the fin of the electric motor. The fact to be emphasized, concerning the efficiency, is that the aspect ratio is relevant for the high external heat transfer coefficient. For low values of the external heat transfer coefficient, increasing the aspect ratio does not have a significant influence on the efficiency. The difference, observable, concerning Cast Iron and Aluminum, can be justified according to the values of the conduction heat transfer coefficient of the materials: k = 80 W / (m. K) for Cast Iron and k = 237 W / (m. K) for Aluminum.

Values for efficacy, considering Cast Iron and Aluminum, are shown in Figure 7. The aspect ratio does not have much influence on the results, and the difference between the heat conduction coefficients is very relevant. To efficacy, as values below 2 do not justify the placement of fins, values above 200 W / (m^2 K) are not recommended for cast iron, and values above 400 W / (m^2 K) is not recommended for Aluminum.

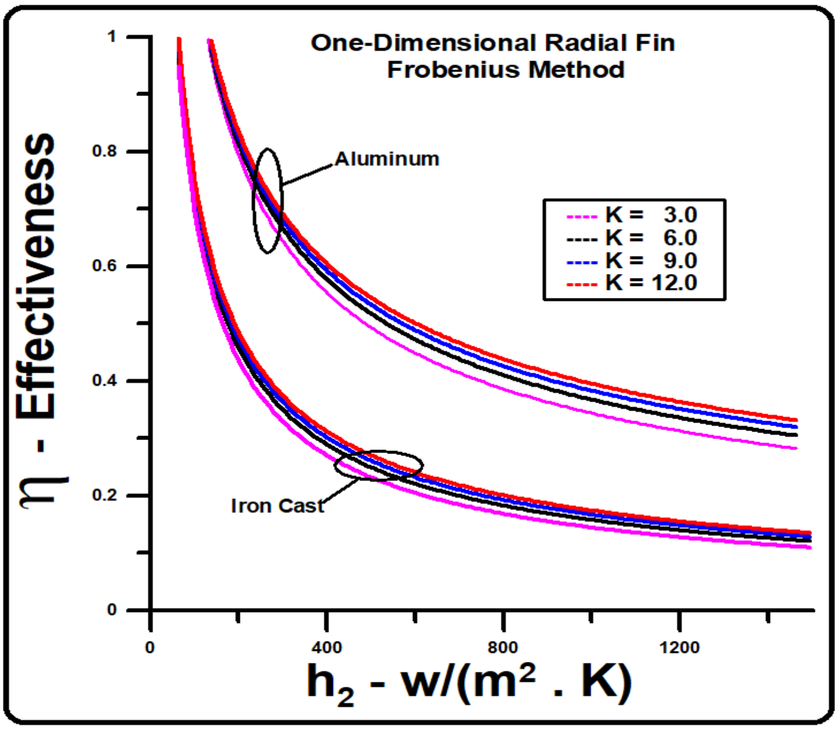

FIGURE 6 –Effectiveness versus heat transfer coefficient h_2

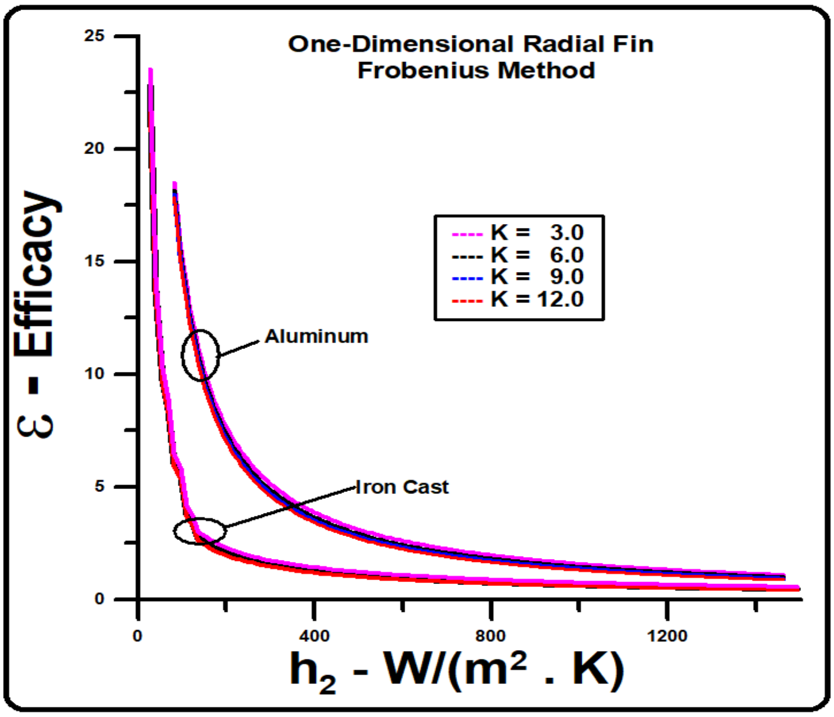

FIGURE 7 -Efficacy versus heat transfer coefficient h_2

V. CONCLUSIONS

The main conclusions reached in this work are described below:

- The thermal performance does not increase indefinitely with the increase of the aspect ratio.
- For relatively high values for aspect ratio, the heat transfer rate remains stagnant, with a tendency to decrease.
- Regarding efficiency, the aspect ratio is only relevant for high values of the external heat transfer coefficient.
- For Cast Iron, efficiency is justified for values of the external heat transfer coefficient below 200 W / (m^2. K).
- For Aluminum, values below 400 W / (m^2. K) enable efficiency above 60% for the fin of the electric motor.
- The aspect ratio does not have much influence on the results of the efficacy, but the difference between the heat conduction coefficients, Cast Iron and Aluminum, is very relevant.

The synthesis of the results obtained lead us to conclude that the aspect ratio is relevant for values less than and equal to 9, and for values of the external heat transfer coefficient less than 200 W / (m^2 K) for Cast Iron, and less than 400 W / (m^2 K) for Aluminum.

REFERENCES

[1]. Suárez, Felipe; Keegan, Sergio D.; Mariani, Ensor J.; Barreto, Guilhermo F. (2019). **"A Novel One-Dimensional Model to Predict Fin Efficiency of Continuous Fin-Tube Heat Exchangers."**Applied Thermal Engineering, Volume 149, 25 February, Pages 1192-1202. https://doi.org/10.1016/j.applthermaleng.2018.12.125

[2]. Soares, Marcus V. F. &Nogueira, Élcio (2019). "**Finned Electric Motor with Prescribed Heat Flux and Influence of the Internal and External Heat Convection Coefficients on the Temperature of the Core."** Global Journal of Researches in Engineering: A Mechanical and Mechanics Engineering, Volume 19, Issue 3, Version 1.0.

[3]. Santos, Tatiana de A. M. (2017).**"Thermal Performance Analysis of Finned Electric Motors: Bidimensional Rectangular Fin Solution with Prescribed Base Temperature."** Completion of course work. (Degree in Mechanical Engineering) - Oswaldo Aranha University Center - Volta Redonda. Advisor: Élcio Nogueira.

[4]. Freire, Denise; Novais, Ariane; Nogueira, Elcio (2012) **"Rectangular profile fin analytical solution: comparison of thermal performance between aluminum and cast iron in electric motors."** Cadernos UniFOA (Printed), v. 20, p. 43.

[5]. Yuri; Freire, Denise.; Nogueira Elcio (2014).**"Aluminum and cast iron in the production of finned electric motor carcasses: efficiency, costs, operational and environmental aspects."** Cadernos UniFOA (Printed), v. IX, p. 11-19.

[6]. Novais, Ariane; Chagas, R. D. F.; Nogueira, Élcio (2014) **"Theoretical analysis of the thermal performance of finned electric induction motors."** Cadernos UniFOA (Online), v. IX, p. 19-34.

[7]. Campo, Antonio and Kundu, Balaram (2017) **"Exact Analytic Heat Transfer from an Annular Fin with Stepped Rectangular Profile."** American Journal of Heat and Mass Transfer, Vol. 4 No. 4, pp. 146-155 DOI:10.7726/ajhmt.2017.1013.

[8]. Cotta, R.M. and Ramos, R. (1993) **"Error analysis and improved formulations for extended surfaces."** Proceedings of the NATO - Advanced Study Institute on Cooling of Electronic Systems, NATO ASI Series E: Applied Sciences, vol.258, pp. 753±787.

[9]. Sommers, A. D. and Jacobi, A. M. (2006) **"An Exact Solution to Steady Heat Conduction in a Two-Dimensional Annulus on a One-Dimensional Fin: Application to Frosted Heat Exchangers with Round Tubes."** Journal of Heat Transfer APRIL 2006, Vol. 128.

[10]. KRAUS, A.D. (1993) **"Analysis of Extended Surface Arrays for Air-Cooled Electronic Equipment."** Advanced Study Institute on Cooling of Electronic Systems, NATO ASI Series E: Applied Sciences, vol.258.

[11]. Aparecido, J. B. and Cotta, R. M. (1988). **"Improved One-Dimensional Fin Solutions."** Heat Transf. Eng., V. 11, no. 1, 49-59.

[12]. Aparecido, J.B. and Cotta, R.M. (1988**).**"Modified one-dimensional analysis of radial fins."**** Proceedings of the Second National Meeting of Thermal Sciences, ENCIT, pp. 225±228.

[13]. OKTAY, S. (1993) **"Beyond Thermal Limits in Computer Systems Cooling."** Advanced Study Institute on Cooling of Electronic Systems, NATO ASI Series E: Applied Sciences, vol.258.

[14]. Nogueira, Élcio (2019). **"One-Dimensional Radial Fin by Frobenius Method versus Two-Dimensional Straight Radial Fin."**International Journal of Engineering Innovation & Research Volume 8, Issue 4, ISSN: 2277 – 5668.

[15]. Nogueira, Élcio (2019). **"Exact Two-Dimensional Rectangular Fin Solution Versus Improved Classical One-Dimensional Radial Fin with Application in Electrical Engine."** International Journal of Advances in Engineering & Technology, August.
[16]. Cotta, R. M. and Mikhailov, M. D. (1997**). "Heat Conduction: Lumped Analysis, Integral Transforms, Symbolic Computation."** J0hn Wiley & Sons, New York.
[17]. Kreyszig, E.(1969**)."Advanced Engineering Mathematics."** Technical and Scientific Books Publisher, Rio de Janeiro.
[18]. Arpaci, V.S. (1966). **"Conduction Heat Transfer."** Addison-Wesley Publishing Company, Inc., Printed in the United States of America, Library of Congress Catalog, No. 66-25602.
[19]. Hildebrand, Francis B. (1962**)."Advanced Calculus for Applications."** Prentice-Hall, INC. New Jersey.
[20]. Schneider, P. J.(1957**)."Conduction Heat Transfer."** Addison-Wesley Publishing Company, Inc., Printed in the United States of America, Library of Congress Catalog, No. 55-5025.
[21]. Carslaw, H. S., Jaeger, J. C. (1948**)."Conduction of Heat in Solids."** Oxford of Clarendon Press, Glasgow.
[22]. Boyce, William E. and Diprima, Richard C. (1986**)."Elementary Differential Equations and Boundary Value Problems."** John Willey & Sons, New York.

Élcio Nogueira. "Influence Of The Aspect Ratio On The Thermal Performance Of Axial Extended Surface In Electric Motor By The Generalized One-Dimensional Radial Fin Using The Frobenius Method." *IOSR Journal of Engineering (IOSRJEN)*, 10(3), 2020, pp. 32-40.