IOSR Journal of Engineering (IOSRJEN) **www.iosrjen.org**
ISSN (e): 2250-3021, ISSN (p): 2278-8719
Vol. 12, Issue 1, January. 2022, || Series -I || PP 59-74

Performances of Reconstruction Algorithm on Compressive Sensing Monostatic and Bistatic Mimo Radar

Marie Emile RANDRIANANDRASANA[1], Radiarisainana Sitraka RAKOTONDRAMANANA[2], Paul Auguste RANDRIAMITANTSOA[3], Andry Auguste RANDRIAMITANTSOA[4]

Department of Telecommunication, Antsirabe Vankinankaratra High Education Institute,
University of Antananarivo, Madagascar
Department of Telecommunication, Antsirabe Vankinankaratra High Education Institute,
University of Antananarivo, Madagascar
Department of Telecommunication, High School Polytechnic of Antananarivo,
University of Antananarivo, Madagascar
Department of Telecommunication, High School Polytechnic of Antananarivo,
University of Antananarivo, Madagascar
Received 17 January 2022; Accepted 31 January 2022

Abstract: The RADAR uses multiple technique to achieve more precision and more best processing. The first technic use on RADAR is Multiple Input Multiple Output (MIMO) RADAR for switching fast direction of arrival of target's searching. New technic appears to having more flexibility of signal sent to the target using the Compressive Sensing (CS). The signal to transmitted will be acquired and compress simultaneously using sparse signal multiply by matrix sensing and signal should respect the Restricted Isometry Property (RIP). At the receiver, algorithm of reconstruction of signal compressed should be done. This article shows how to process MIMO radar with Compressing Sensing and how to reconstruct them. For thus, three algorithms are used: Compressive Sampling Matching Pursuit (CoSAMP), Look Ahead Orthogonal Matching Pursuit (LAOMP) and Orthogonal Matching Pursuit (OMP) on monostatic and bistatic RADAR. For having good performance, the number of antennas should be more than 16x16. The algorithm of CoSAMP is not usable for bistastic RADAR because it offers correlation between signal sent and signal recovered less than 50% but it's not a complex implementation indeed on monostatic RADAR. The two variants LAOMP and OMP offers correlation more than 80% until 99% on bistatic and monostatic RADAR but have more complex implementation.
Keywords: Radar MIMO, CoSAMP , LAOMP, OMP, Compressive Sensing, Isometry

II. INTRODUCTION

RADAR MIMO is made up of several transmitting antennas each emitting a distinct waveform (beam) and numerous receiving antennas. The beams emitted by the transmitting systems are noisy by various attenuations, fading and interference. The CS technique is used to reconstruct the information in this antenna system. The signal must first be acquired or measured using electronic equipment and devices. Generally speaking, compressive sensing never predicts that reconstruction from widely sampled non-adaptive measurements is possible, even using recovery and compression algorithms. This article evaluates the possibility of obtaining a compressed version of the signal more directly and of evaluating the performance of the reconstruction algorithms.

III. TYPES DE RADAR

MIMO radars could be divided into two types [1]:

- Monostatic MIMO radars (collocated MIMO radars): the target is point like in a conventional primary radar;
- Bistatic MIMO radars (MIMO radars with widely spaced antennas): the target is located, illuminated and monitored by different transmitters and receivers.

In these two types of MIMO radar, we can find the following systems:

- Primary radars: emit microwave signals which are reflected by targets. So, this radar receives the reflected part of its own signal.

- Secondary radars: are placed on board an aircraft, the purpose of which is to respond to radar interrogations by generating a coded signal. This response may contain a lot of information such as altitudes, identification code, radio communication failure.
- Imaging or non-imaging radars: Imaging radars are used to present an image of the area or of an object in a space. They are used to map the Earth, other planets, and other celestial objects. Non-imaging radars are used to measure the properties of reflections from the region or object being observed.
- Pulse radars, continuous wave radars.
- Continuous modulated wave radars: the transmitted signal is constant in amplitude but modulated in frequency.
- Unmodulated continuous wave radars: the transmitted signal is constant in amplitude and frequency.

IIII. COMPRESSIVE SENSING

3.1 Acquisition process

Take m linear measurements of a signal $x \in \mathbb{C}^N$, then, multiply this signal to a matrix $\mathcal{A} \in \mathbb{C}^{m \times N}$; this matrix is called the measurement matrix or sensing matrix [2] [3] [4].

$$y = Ax. \tag{1}$$

The vector $y \in \mathbb{C}^m$ is called the measure vector. The main interest is in the largely under-sampled case $m \ll N$. Without no more information, it's very difficult to recover x by y car because linear equation (1) is highly indeterminate and has infinity of solution. So, the following hypothesis must be verified:

- The signal has a sparse representation in frequency domain or base named $\mathcal{B} \epsilon \mathbb{C}^{N \times N}$.
- The sparse signal x is directly captured with compressed format and less than Nyquist criteria, during phase named phase of measurement or phase of acquisition. The signal is multiplied by sensing matrix $\boldsymbol{\mathcal{A}} \in \mathbb{C}^{m \times N}$

The equation (1) becomes:

$$y = Cz \tag{2}$$

With, $C = \boldsymbol{\mathcal{A}}\boldsymbol{\mathcal{B}} \in \mathbb{C}^{m \times N}$

$z \in \mathbb{C}^N$ is the transformation of x in the domain $\boldsymbol{\mathcal{B}}$, with $\|z\|_0 = k$ et $k \ll N$.

By considering the additive noise during the phase of acquisition, equation (2) becomes:

$$y = \boldsymbol{\mathcal{A}}x + b \tag{3}$$

Where $b \in \mathbb{C}^m$ is a vector representing additive noise.

3.2 Sparse

A signal is sparse when it contents only some significant elements not nulls. In the case of signal represented by vector with finite dimension and discrete value $x \in \mathbb{C}^m$, sparse is norm l_0 equal to k. k, $\|x\|_0 = k$ et $k \ll N$. [3][5][6]

A signal x has sparse representation with domain $B \epsilon \mathbb{C}^{N \times N}$ if the transformation $z = Bx$ is sparse. So, $\|x\|_0 = k$ and $k \ll N$. The degree of sparse r of signal x is defined by:

$$r = \frac{k}{N} \tag{4}$$

Where k and N represent respectively the number of non nulls elements and the dimensions of x for measuring the compressibility degree: Less value of r, gives high value of compressibility. Depending of signal's nature; many domains transformation could be used like Discrete Fourier Transform (DFT), Discrete Cosine Transform (DCT), Discrete Wavelet Transform (DWT).

The sparse is not only a process exploited by compressed acquisition but also on many compression algorithms.

3.3 Compressibility

All notations used are: [N] set formed by {1, 2…., N} and card(S) for the cardinality of set S. And, we will note S the complement [N] \ S of set S in [N]. [2] [7] [8]

Definition 1: support

The vector support r $x \in \mathbb{C}^N$ is the set of index with his input not null. So,

$$\text{supp}(x) := \{j \in [N]: x_j \neq 0\} \cdot$$

The vector $x \in \mathbb{C}^N$ *is* s-sparse if the input s is most not null anywhere; So,

$$\|x\|_0 := card(supp(x)) \leq s$$

The equation could be expressed also by:

$$\|x\|_{\not{f}}^{\not{f}} = \sum_{j=1}^{N} |x_j|^{\not{f}} = card(\{j \in [N]: x_j \neq 0\}). \tag{5}$$

Definition 2

For $\not{f} > 0$; the $\eta_{\not{f}}$ – error of s-term of approximation with a vector $x \in \mathbb{C}^N$ is defined by:

$$\eth_s(x)_{\not{f}} = inf\{\|x - z\|_{\not{f}}; z \in \mathbb{C}^N\}. \tag{6}$$

Proposition 2

For all $q > \not{p} > 0$ and for all $x \in \mathbb{C}^N$,

$$\eth_s(x)_q \leq \frac{1}{s^{\frac{1}{\not{p}}-\frac{1}{q}}} \|x\|_{\not{p}}$$

So, $\eth_S(x)_q^q$ becomes:

$$\eth_S(x)_q^q \leq \left(\frac{1}{s}\|x\|_{\not{p}}^{\not{p}}\right)^{\frac{q-\not{p}}{\not{p}}} \|x\|_{\not{p}}^{\not{p}} = \frac{1}{s^{\frac{q}{\not{p}}-1}} \|x\|_{\not{p}}^q \tag{7}$$

$$\eth_s(x)_q \leq \frac{C_{\not{p},q}}{s^{\frac{1}{\not{p}}-\frac{1}{q}}} \|x\|_{\not{p}}$$

With,

$$C_{\not{p},q} = \left[\left(\frac{\not{p}}{q}\right)^{\frac{\not{p}}{q}}\left(1-\frac{\not{p}}{q}\right)^{1-\frac{\not{p}}{q}}\right]^{\frac{1}{\not{p}}} \leq 1 \tag{8}$$

Definition 3

For $\not{p} > 0$; the equation (9) is expressed by:

$$\|x\|_{\not{p},\infty} = \inf\{T \geq 0 : card(\{j \in [N] : |x_j| \geq t\}) \leq \frac{T^{\not{p}}}{t^{\not{p}}}\ pour\ tout\ t > 0\}. \tag{9}$$

For all $q > \not{p} > o$ and for all $x \in \mathbb{C}^N$,

$$\boldsymbol{\eth_s(x)_q} \leq \frac{\boldsymbol{d_{\not{p},q}}}{s^{\frac{1}{\not{p}}-\frac{1}{q}}} \|\boldsymbol{x}\|_{\not{p},\infty}$$

With,

$$\boldsymbol{d_{\theta,q}} = \left(\frac{\not{p}}{\boldsymbol{q}-\not{p}}\right)^{\frac{1}{q}} \tag{10}$$

3.4 Measurement's minimal number

In this section, the goal is to find the measurement's minimal number for being reconstructed to s-*sparse* vector. The solution could have 2 significations, in one case if the arrangement of measure considers all reconstruction of s-*sparse* vector of $x \in \mathbb{C}^N$ simultaneously or in other case, if s-*sparse* vector of $x \in \mathbb{C}^N$ of the measurement's arrangement consider specific vector. In the first impression, the second scenario seems artificial but it's needed to guarantee the reconstructing phase when the matrix A is chosen randomly and the vector *sparse* of x is fixed [2] [3] [9]

The minimal number m of measurement depends of considered arrangement which is equal to 2s in the first case and s+1 in the second case. But, the arrangement of reconstruction must be also stable. Then, the number of minimal required measurement implies the factor of ln $\left(\frac{N}{s}\right)$ and the arrangement is never stable with only 2s measurement.

Before separating the two discussed arrangement, the equivalence of the following two properties should be noticed for the s sparse and matrix sensing $\mathcal{A} \in \mathbb{C}^{mXN}$ and *s-sparse* of $x \in \mathbb{C}^N$ gave.

(a) The vector x is the unique solution s-sparse of equation $\boldsymbol{\mathcal{A}}$z = y with y = $\boldsymbol{\mathcal{A}}$x i.e.,

$$\{ z \in \mathbb{C}^N : \boldsymbol{\mathcal{A}}z = \boldsymbol{\mathcal{A}}x,\ \|z\|_0 \leq s \} = \{x\} \tag{11}$$

(b) The vector x could be reconstructed with solution unique by:

$$\underset{z\in\mathbb{C}^N}{minimum}\|z\|_0 \quad \text{with } \boldsymbol{\mathcal{A}}z = y. \tag{12}$$

In fact, if *s-sparse of* $x \in \mathbb{C}^N$ is the unique *s-sparse* solution of the equation $\boldsymbol{\mathcal{A}}$z = y with y = $\boldsymbol{\mathcal{A}}$x, so, solution $x'$of the equation (12) is s-*sparse* and satisfy the equation $\mathcal{A}x' + y$ verifying $x' = x$. This shows that (a) implies (b), so (b) to (a) is evident.

3.5 Coherence

Definition 4

Let $\mathcal{A} \in \mathbb{C}^{mXN}$ matrix with columns $\boldsymbol{a_1}$, . . ., $\boldsymbol{a_N}$, are $l_2 - normalized$,i.e.,$\|a_i\|_2 = 1$ for i ∈ $[N]$. The coherence $\mu = \mu(\mathcal{A})$ of matrix $\boldsymbol{\mathcal{A}}$ is defined by [2] [3] [10]:

$$\mu = \underset{1 \leq i \neq j \leq N}{max}|\langle a_i, a_j\rangle| \tag{13}$$

Definition 5

Let $\mathcal{A} \in \mathbb{C}^{mXN}$: their columns a_1 , . . ., a_N,are $l_2 - normalized$,i.e.,$\|a_i\|_2 = 1$ for i ∈ $[N]$. The function said $l_2 - cohérence\ \mu_1$ of matrix $\boldsymbol{\mathcal{A}}$ is defined by:

For all $s \in [N-1]$,

$$\mu_1(s) = \underset{i\in[N]}{max}\left[max\{\textstyle\sum_{j\in S}|\langle a_i, a_j\rangle|, S \subset [N], card(S) = s, i \notin S\}\right] \tag{14}$$

Theorem 1
Let $\mathcal{A} \in \mathbb{C}^{mXN}$: their columns are $l_2 - normalized$ and let $s \in [N]$. For all vector s-*sparse* $x \in \mathbb{C}^N$,

$$(1 - \mu_1(s-1)\|x\|_2^2 \leq \|\mathcal{A}x\|_2^2 \leq (1 + \mu_1(s-1))\|x\|_2^2 \qquad (15)$$

Proposition 3: Let the matrix $\mathcal{A} \in \mathbb{C}^{mXN}$ and their columns are $l_2 - normalized$ and one integer s≥ 1 exists, if we admit this equation:

$$\mu_1(s) + \mu_1(s-1) < 1$$

Then, for sub-set $S \subset [N]$ with $card(S) \leq 2s$, the matrix $A_S^* A_S$ is inversible, and the matrix A is injective. We could conclude so:

$$\mu_1 < \frac{1}{2s-1}$$

3.6 Restricted Isometry Property
Definition 6
The s-th constant of restricted isometry $\delta_s = \delta_s(\mathcal{A})$ of the matrix $\mathcal{A} \in \mathbb{C}^{mXN}$ is the smallest element $\delta \geq 0$ verify: [2] [3] [11]

$$(1-\delta)\|x\|_2^2 \leq \|\mathcal{A}x\|_2^2 \leq (1+\delta)\|x\|_2^2$$

For all vector s-*sparse* $x \in \mathbb{C}^N$, We have:

$$\delta_s = \max_{S \subset [N], card(S) \leq s} \|\mathcal{A}_S^* \mathcal{A}_S - I\|_{2\to 2} \qquad (16)$$

Definition 7
The constant of restricted orthogonality $\varkappa_{s,t} = \varkappa_{s,t}(\mathcal{A})$ of the matrix $\mathcal{A} \in \mathbb{C}^{mXN}$ is the smallest $\varkappa \geq 0$ verifying:

$$|\langle \mathcal{A}_u, \mathcal{A}_V \rangle| \leq \varkappa \|u\|_2 \|v\|_2$$

For the disjoint element supporting s-*sparse* and t-*sparse* of vector $u, v \in \mathbb{C}^N$, we could have:

$$\varkappa_{s,t} = max\{\|\mathcal{A}_S^* \mathcal{A}_S\|_{2\to 2}, S \cap T = \emptyset, card(S) \leq s, card(T) \leq t\} \qquad (17)$$

Proposition 4
The constant of restricted isometry and constant of restricted orthogonality have a relation expressed by:

$$\varkappa_{s,t} \leq \delta_{s+t} \leq \frac{1}{s+t}(s\delta_s + t\delta_t + 2\sqrt{st\varkappa_{s,t}})$$

Proposition 5
Let the integers $m, n, t \geq 1$ with $t \geq n$, So:

$$\varkappa_{t,m} \leq \sqrt{\frac{t}{n}} \varkappa_{n,r} \text{ ; and } \delta_t \leq \frac{t-d}{n}\delta_{2n} + \frac{d}{n}\delta_n \quad \text{avec } d = pgcd(n,t)$$

IV. TRANSFORMATIONS

4.1 Cosine transform
The Discrete Cosine transform is very used on images compression and even on videos compression. We could find some versions of DCT, like example, DCTv1, DCTv2, DCTv3ou DCTv4. The version 2 and 4 are the most used on data compression. The DCT could also expressed in many dimensions: one dimensions (1D), two dimensions (2D) and three dimensions (3D). The matrix DCTv2(1D) is constructed [5] like this:

$$\Psi(N \times N) = \left[\mathcal{P}_{ij} = \hbar * \cos\left(i(1+2j)\frac{\pi}{2N}\right)\right] \qquad (18)$$

Where i and j represents respectively the numbers of line and columns of matrix. It varies between 0 and N-1. *h* is a constant defined by:

$$\hbar = \begin{cases} \sqrt{\frac{1}{N}} \; si \; i = 0 \\ \sqrt{\frac{2}{N}} \; si \; i \neq 0 \end{cases} \qquad (19)$$

Knowing the matrix DCTv2 is orthogonal, the Inverse Discrete Matrox is obtained by the transpose only of the matrix DCTv2. The DCT transform the signal to be sparse by grouping the information of low frequency. The high frequency component has a low value and becomes less important. It could so suppress the visual loose.

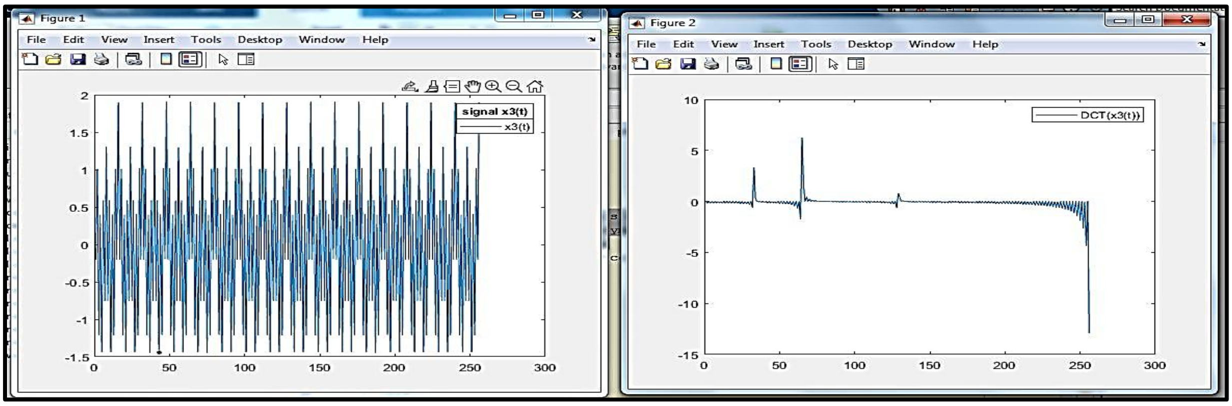

Figure 01: Discrete cosine transform of signal

4.2 Wavelet transforms

The wavelet transform uses high level of multiple transformations. Like on figure 02, the coefficient is generated with hierarchy using multiple low pass filter h and high pass filter g followed by sampling methods. The resultant coefficient is respectively called approximations and details. The impulse response of filter change following the function of wavelet type like Haar Wavelet Transform (HWT) or Daubechies Wavelet Transform (DWT) [5].

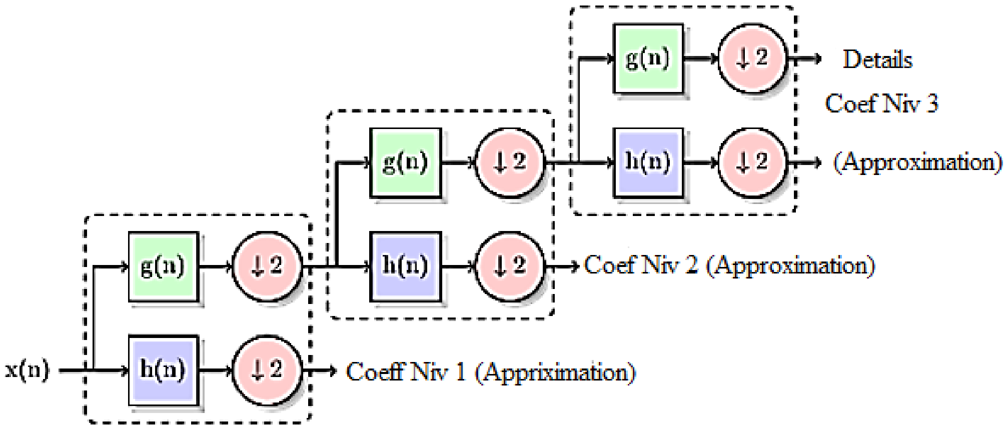

Figure 02: Wavelet Transform

4.3 Fourier Transform

The Fourier Transform (FT) of signal *x(t)* is defined by [5]:

$$F\{x(t)\} = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \tag{20}$$

$$F\{x(t)\} = X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \tag{21}$$

And the inverse of Fourier transform is defined by:

$$F^{-1}\{X(\omega)\} = x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega \tag{22}$$

$$F^{-1}\{X(f)\} = x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df \tag{23}$$

4.4 Z transform

The transform Z is an adaptation of Laplace transform for the study of transient response concerning numeric signal. The Z transform could be compared like numeric series. It permits the signal processing and system sampling, similar with Laplace transform for the analogue signal. In practice, the table of Z transform is used for calculating all Z transform of signal. [3] [11].

Let F*(p) the Laplace transform of sampling signal $\mathfrak{z}^*(t)$ represented by:

$$\mathfrak{z}^*(t) = \mathfrak{z}_0\delta(t) + \mathfrak{z}_1\delta(t - T_e) + \cdots \tag{24}$$

The Laplace transform is expressed by:

$$F^*(p) = \sum_{k=0}^{+\infty} \mathfrak{z}_k e^{-kpT_e} \tag{25}$$

To express the Z transform, a simple change of variable is done:

z = e^{pTe} , where p is the variable of Laplace, this variable is general and a complex variable (i.e. could be wrote by p = a + jb).

The Z transform is consequently defined by:

$$Z(\mathfrak{z}(t)) = F(z) = \sum_{k=0}^{+\infty} \mathfrak{z}_k Z^{-k} \tag{26}$$

V. TECHNIQUE CS

On Acquisition system, all sampling is acquired firstly. The number of samples acquired can vary from a few thousand to millions of samples. Then, this process is followed by the compression which takes advantage of the redundancy present in the signal to represent it in a domain where the majority of its coefficients can be eliminated, with negligible or no loss in quality. Finally, a massive amount of information should be collected, while only a small fraction of this collected information will actually be used to represent the signal.. [8] [9] [10]

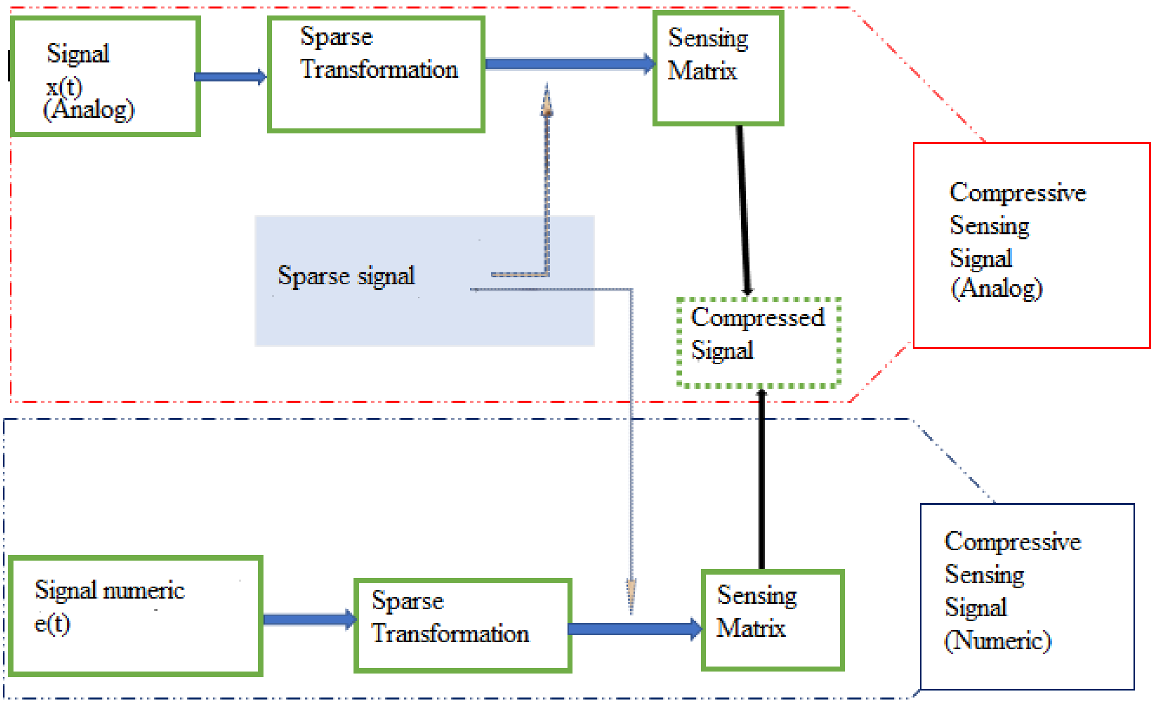

Figure 03: Simplified diagram of a compression technique process

Based on the equations (1), (2), (16) et (17), the process of CS technique can be summarized as follows:

- ✓ Convert the signals to be sparse using transforms such as DCT, DFT, or DWT.
- ✓ Choose the compression ratio or Compressive Ratio (CR) for the M × N size image to compress, we note:

$$m = CR \times M \tag{27}$$

- ✓ Generate the measurement matrix Φ and A of dimension m × N, this matrix will be used as a sensing matrix.
- ✓ Perform the compressive sensing of the signals by realizing the product of the coefficients of the signals by that of the sensing matrix:

$$y_{m\times N} = \Phi_{m\times N} \times x_{N\times N} \tag{28}$$

- ✓ This results in a compressed and encoded signal is then stored or shared over a network.

VI. RECONSTRUCTION ALGORITHM

6.1 Orthogonal Matching Pursuit OMP

This algorithm begins with an initial index S^0 and with [4] [9] [10]:

$$x^0 = argmin\{\|y - \mathcal{A}z\|, supp(z) \subset S^0\}, \tag{29}$$

And iteratively we could have:

$$S^{n+1} = S^n \cup \mathcal{L}_1(\mathcal{A}^*(y - \mathcal{A}x^n)) \tag{30}$$

$$x^{n+1} = argmin\{\|y - \mathcal{A}z\|_2, supp(z) \subset S^{n+1}\} \tag{31}$$

Where $\mathcal{L}$ is the maximum length of the vector. The algorithm of reconstruction OMP is represented like this:

Input:

Transformation matrix $\mathbf{\Psi}$, measurement matrix $\mathbf{\Phi}$

CS matrix **A**: $\mathbf{A} = \mathbf{\Phi\Psi}$

Measurement vector $\mathbf{y}$

Output:

Estimated signal $\tilde{x} \leftarrow \mathbf{x}^K$

Measurement approximation of y by $\mathbf{a}^K$

Residual $\mathbf{r}^K = \mathbf{y} - \mathbf{a}^K$.

Pose ΩK I the position of non-null elements $\tilde{x}$.

(1) $\mathbf{r}_0 \leftarrow \mathbf{y}$, $\Omega_0 \leftarrow \varnothing$, $\mathbf{\Theta}_0 \leftarrow []$

(2) **For** $i = 1, \ldots, K$

(3) $\omega_i \leftarrow \arg\max_{j=1\ldots,N} |\langle \mathbf{r}i{-}1, Aj\rangle|$ (*Column's maximal correlation*)

(4) $\Omega i \leftarrow \Omega_{i-1} \cup \omega_i$ (*indices' update*)

(5) $\mathbf{\Theta} i \leftarrow [\mathbf{\Theta}_{i-1}\ A\omega_i]$

(6) $\mathbf{x}i = \arg\min_{\mathbf{x}} \|\boldsymbol{r}_{i-1} - \mathbf{\Theta}_i\mathbf{x}\|_2^2$

(7) $\mathbf{a}_i \leftarrow \mathbf{\Theta}_i\mathbf{x}_i$ (*new approximation*)

(8) $\mathbf{r}_i \leftarrow \mathbf{y} - \mathbf{a}_i$ (*residue's update*)

(9) **End For**

(10) **return** $\mathbf{x}^K$, $\mathbf{a}^K$, $\mathbf{r}^K$, Ω^K

6.2 Compressive Sampling Matching Pursuit (CoSaMP)

The algorithm *CoSaMP* starts on an initial s-sparse vector $x^0 \in \mathbb{C}^N$, typically $x^0 = 0$ and produces sequence (x^n) defined respectively by [4] [9] [10] :

$$U^{n+1} = supp(x^n) \cup \mathcal{L}_{2s}(\mathcal{A}^*(y - \mathcal{A}x^n)) \quad (32)$$

$$u^{n+1} = argmin\{\|y - \mathcal{A}z\|_2, supp(z) \subset U^{n+1}\} \quad (33)$$

$$x^{n+1} = H_s(u^{n+1}) \quad (34)$$

Where, $\mathcal{L}$ is the maximum length of the vector.

Result of the algorithms: The RIP condition $\delta_{4s} = \delta_{4s}(\mathcal{A})$ of the matrix $\mathcal{A} \in \mathbb{C}^{mXN}$ satisfied:

$$\delta_{4s} < \sqrt{\frac{\sqrt{11/3} - 1}{2}} \approx 0{,}4782$$

Then, for $x \in \mathbb{C}^N, b \in \mathbb{C}^m$ and $S \subset [N]$ with $card(S) = s$, the sequence (x^n) defined by (32), (33) and (34) with $y = \mathcal{A}x + b$, satisfied:

$$\|x^n - x_s\|_2 \leq \xi^n \|x^0 - x_s\|_2 + \tau \|\mathcal{A}x_{\bar{s}} + e\|_2 , \quad (35)$$

With $0 < \xi < 1$ and $\tau > 0$

The algorithm CoSaMP is summarized by for steps:

Input

Transformation matrix Ψ, measurement matrix Φ

CS matrix A: $A = \Phi\Psi$

Measurement vector y

K sparse signal level

Number of wanted iteration

Imposed criteria

Output

K-signal of estimated sparse $\hat{x}$ du signal original

(1) $x_0 \leftarrow 0$, $r \leftarrow y$, $i \leftarrow 0$

(2) While imposed criteria

(3) $i \leftarrow i + 1$

(4) $z \leftarrow A*r$ *(signal indexer)*

(5)$\Omega \leftarrow supp(z^{2K})$ *(Choose support with the best 2K-sparse approximation)*

(6)$T \leftarrow \Omega \cup supp(x_{i-1})$ (support's fusion)

(7) $x = \arg\min_{\tilde{x} : supp(\tilde{x}) = T} \|A\tilde{x} - y\|_2^2$ *(least squares resolution)*

(8) $x_i \leftarrow \bar{x}^K$ *(Size : K-sparse approximation)*

(9) $r \leftarrow y - Ax_i$ *(Update of known sampling)*

(10) End While

(11) $\hat{x} \leftarrow x_i$

(12) return $\hat{x}$

Identification: It finds the largest components 2s of sparse signal.
Support's fusion: It merges support (form) of original signal with support of the solution of the previous iteration.
Estimation: It estimates a solution using the least squares method where the solution lies below a support T. [4] [9] [10]
Size (sizing): It takes the solution estimate and compresses it to a required medium.

6.3 Look Ahead Orthogonal Matching Pursuit (LAOMP)

To apply reconstruction LAOMP algorithm, the sensing matrix must respect RIP property.
Let M diagonal matrix and A sensing matrix, verifying [4] [9] [10]:

$$M_i = \frac{1}{\|\mathcal{A}_i\|_2^2} \tag{36}$$

Then, some transformation expressed by:

$$Ny = b\,; N = \mathcal{A}M$$
$$y = M^{-1}x$$

And,

$$\begin{cases} min\|y\|_0 \\ \|Ny - b\|_2 < \varepsilon \end{cases}$$

So, the equation (37) is verified as:

$$x = My \tag{37}$$

It's difficult to find exact solution of this equation. That's why, LAOMP use greedy algorithm for having approximative solution of this equation. Firstly, the residual vector is defined by:

$$r_k = b - N_{I_k} y_{I_k} \tag{38}$$

With: I_k is the data support which contains the selected columns of index. The residual vector and the support are initialized as:

$$\begin{cases} r_0 = b \\ I_k = 0 \end{cases}$$

At each iteration, the LAOMP algorithm finds the column of N which is maximum correlated in the regularized residual vector, formulated by:

$$\begin{cases} i_k = \arg max_{(j\in(U-I_{k-1}))}\, g(N_j, r_{k-1}) \\ I_k = I_{k-1} \cup i_k \end{cases} \tag{39}$$

Where, U represents the indexes for all columns, g (a, b) represents the internal product of a and b. The iteration stops when the number of the column in the holder has ended at a sparse level. The LAOMP algorithm is similar to OMP.

In any iteration, the amplitude components L of the internal product of N with the residual vector r are put in time, hence:

$$T_k = \max\left(g\left(N_{U-I_{k-1}}, r_{k-1}\right), \mathcal{L}\right) \tag{40}$$

Where, T_k being the time at k-th iteration and $\mathcal{L}$ is the maximum length of the vector N.

Algorithm of LAOMP reconstruction:

Input:

The sparse signal b and CS matrix: A
(Normalized the CS matrix with B matrix)

Output:

Estimated signal x^*
(1) Ny ← b,
(2) N ← AB, y ← B^{-1}x
(3) y_0 ← 0, r_0 ← b, I_0 ←∅, U← distance (1,col(N)), k← 0
(4) Repeat k=k+1
(5) T_k=arg max (($N_{U-I_{k-1}}$, r_{k-1}),L)
(6) I_{k-1}=arg $min_{j\in T_k}$ $(look\_ahead\_residue(N_j))$
(7) $I_k = union\ (I_{k-1}, i_k)$
(8) until k> K
(9) $y = N^{+}_{I_k} b, x^* = BN^{+}_{I_k} b$
(10) return x^*

VII. MIMO RADAR WITH COMPRESSIVE SENSING

Consider a MIMO radar system consisting of N_r antennas of RX and N_t antennas of TX. In the far domain, the antennas must be estimated K targets. For simplicity's sake, we'll assume that the targets are not moving, so the only parameters that need to be estimated are the target azimuth angles. θ_k , k = 1 ..., K.

7.1 CS applied in a collocated MIMO radar system

Suppose the antennas of TX and RX are closely spaced and randomly distributed over a small area, with the antenna of the i^{th} TX/RX placed upright $(\rho_i^t, \theta_i^t)/(\rho_i^r, \theta_i^r)$ (in a polar coordinate system). M denotes the number of spaced samples of the transmitted waves with a period T_e. In m^{th} receiver, the received signal is linearly compressed by the measurement matrix Φ. Where Φ has dimension L x M. [12]

The waveform (far field and narrowband) is assumed that all targets are located in the same cell of same frequency. The baseband signal received at the m^{th} antennas could be expressed by:

$$r_m \cong \sum_{k=1}^{L} \beta_k e^{\frac{j2\pi f}{c}\Gamma_m^r(\alpha_k)}\, \Phi X V_t(\alpha_k) + \Phi n_m = \Phi \psi_m s + z \tag{41}$$

Where $X \in \mathbb{C}^{L\times M_t}$ is a matrix whose columns contain the transmitted waves; β_k is the reflection coefficient of k^{th} target ; $V_t(\alpha_k) = \left[e^{j\frac{2\pi f}{c}\Gamma_1^t(\alpha_k)}, \dots, e^{j\frac{2\pi f}{c}\Gamma_{M_t}^r(\alpha_k)}\right]^T$ is a vector associated by the angle α_k ; $\Gamma_1^{t/r}(\alpha_k) = \rho_i^{t/r}\cos(\alpha_k - \theta_i^{t/r})$; and n_m is the interference in the m^{th} receiver (interference and thermal noise).

By stacking the data received from each antenna into a long vector, we form y, for which it stands:

$$y = \left[r_1^T, \dots, r_{N_r}^T\right]^T = \Psi s + z \tag{42}$$

With $s = [s_1, \dots, s_N]^T$ is a sparse vector.

If the number of targets is small compared to N, then s is a sparse vector, with the locations of its non zero elements providing information on target angles. A variety of CS methods can be applied to recovery from s. By the CS, formulation Ψ (a matrix) is the sensor and ψ_m is a matrix base of m^{th} antennas.

7.2 CS applied in a largely separate MIMO radar system

We consider an antenna configuration with the multiple antennas of TX and RX which are arbitrary located in a large area. We assume that there are K targets in the search space, each one consists of Q independent and isotropic diffusers. Let $(x_{ti}; y_{ti})$ et $(x_{ri}; y_{ri})$ the coordinate of i^{th} antennas TX and of i^{th} antennas RX, respectively, and $(x_{qk}; y_{qk})$ denotes the location of q^{th} diffusor of k^{th} target at the initial period T of the drawdown. All places in this section are given in Cartesian coordinates. [13] [14] [15] The distance between the antenna of i^{th} TX/RX and the q^{th}diffusor of k^{th} target at instant t equals:

$$d_{qki}^{t/r}(t) = \sqrt{\left(x_{qk} - x_i^{t/r}\right)^2 + \left(y_{qk} - y_i^{t/r}\right)^2} \tag{43}$$

In particular, for a stationary target the parameters to be estimated are x_{qk} and y_{qk} avec $q = 1, \dots, Q \; et \; k = 1, \dots, K$.

The baseband signal arrived at the j^{th} antenna receiver emitted by i^{th} antenna issuer is expressed by:

$$z_{ij}(t) = \sum_{k=1}^{K} \sum_{q=1}^{Q} \sigma_{qk}^{ij} x_i \left(t - \frac{d_{qki}^{t}(t) + d_{qki}^{r}(t)}{c}\right) + n_{ij}(t), j = 1, \dots, M_r \tag{44}$$

Where the $\sigma_{qk}^{ij}$is the attenuation coeffiecient linked to q^{th} diffusor of k^{th} target and the pair of antennas TX/RX, and $n_{ij}(t)$ denotes interference and noise. It is assumed that the antennas transmit on different channels, at different times, and each antenna set TX/RX widely separated receives signals from targets.

We must then discretize the target space in points of N grid, i.e.., $[(x_n; y_n)]$ with n = 1,…,N. Let ε_n^{ij} the associate coefficient at n-th grid point for an antenna pair of TX/RX. The equation (44) could be rewrite as combination of reflected signal target on all point of grid i.e. :

$$\begin{aligned} sr_{ij}(t) &= \sum_{n=1}^{N} \varepsilon_n^{ij} x_i\left(t - \tau_{in} - \tau_{nj}\right) + n_{ij}(t) \\ &= A_{ij}^T(t) S^{ij} + n_{ij}(t), \end{aligned} \tag{45}$$

With $A_{ij}^T(t) = \left[x_i\left(t - \tau_{i1} - \tau_{1j}\right), \dots, x_i\left(t - \tau_{iN} - \tau_{Nj}\right)\right]^T$ and $S^{ij} = \left[\varepsilon_1^{ij}, \dots, \varepsilon_N^{ij}\right]^T$.

If the target is located at the coordinate $(x_n; y_n)$, the coefficient ε_n^{ij} is equal to the channel gain associated with the corresponding target and with the antenna peers (i,j) ; somewhere else , ε_n^{ij} worth zero.

Let y_{ij} the compressible measurement vector received from $j^{ème}$antenna receiver and it could be expressed by :

$$y_{ij} = \Phi\left[sr_{ij}(T_s), \dots, sr_{ij}((L-1)T_s)\right]^T \tag{46}$$

Let pose $\Psi_{ij} = \left[A_{ij}(T_s), \dots, A_{ij}((L-1)T_s)\right]^T$, $n_{ij} = \left[n_{ij}(T_s), \dots, n_{ij}((L-1)T_s)\right]^T$and Φ the measurement matrix which is used to compress the received data.

Samples from all receiving antennas are merged to obtain a vector y of length $M_t, N_r \; and \; L$, i.e.,

$$\begin{aligned} y &= \left[y_{11}^T, \dots, y_{1N_r}^T, y_{M_t1}^T, \dots, y_{M_tN_r}^T\right]^T \\ &= diag\left\{\left[\Theta_{11}, \Theta_{11}, \dots, \Theta_{M_tN_r}\right]\right\} S + n \end{aligned} \tag{47}$$

Knowing that : $y_{ij} = \Phi \Psi_{ij} S^{ij} + \Phi n_{ij}$

With $\Theta_{ij} = \Phi \Psi_{ij}$, $S = [(S^{11})^T, \dots, (S^{M_tN_r})^T]^T$ and $n = \left[\Phi(n_{11})^T, \Phi(n_{12})^T, \dots, \Phi\left(n_{M_tN_r}\right)^T\right]^T$

7.3 use of the Gaussian random matrix

Let $x_i(t)$ signal emitted by the TX (rang *i*) and the radar MIMO system, $x_i(t)$ could be expressed by :

$$x_i(t) = A(t).\cos(\omega_i t) * \sum_{k=0}^{\infty} rect_\tau(t - kT) \tag{48}$$

This expression could be expressed by:

$$x_i(t) = A(t).\cos(\theta_i t)$$

x_i is a carrier signal f_i (pulsation ω_i), with his width τ and his period *T*. *A(t)* represents the amplitude modulation. [15]

VIII. RESULT FOR THE COMPARISON OF RECONSTRUCTION ALGORITHMS

8.1 MIMO RADAR reconstruction

The block diagram of the reconstruction of an analog signal in the general case is shown in Figure 04 shows the result.

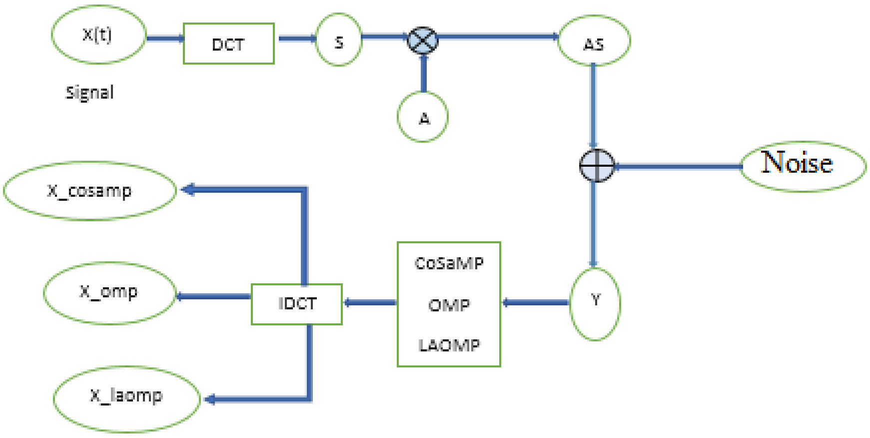

Figure 04 : Compression and reconstruction of signal

X(t) : original signal
S : sparse signal
A : sensing matrix
AS : compressed matrix
(product of the sensing matrix with the sparse signal)

Y : noisy signal
x_cap : reconstructed signal (use of the CoSaMP method)
x_omp : reconstructed signal (use of the OMP method)
x_laomp : reconstructed signal (use of the LAOMP method)

This diagram shows the reconstruction of signal at the input, using the three reconstruction algorithms, we obtain the three signals x_cosamp, x_omp et x_laomp at the output.

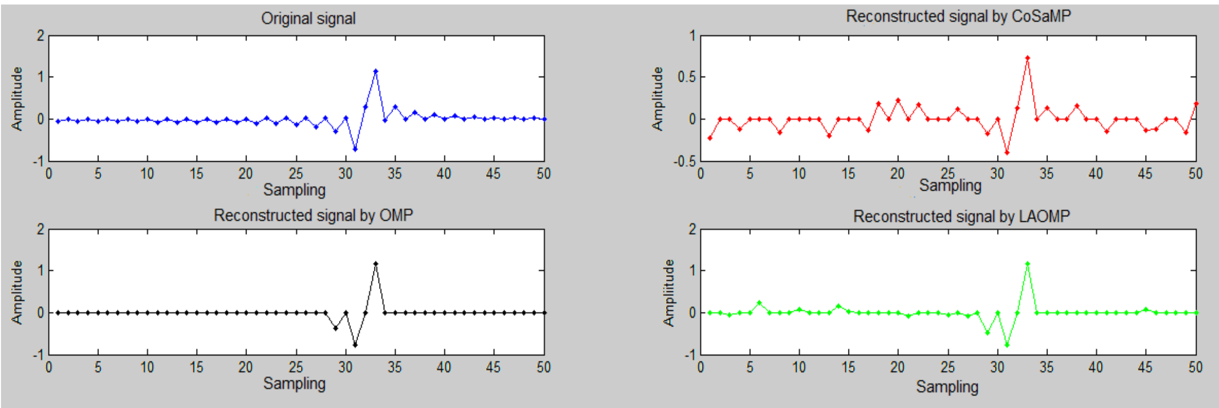

Figure 05 : Reconstruction of signals in a MIMO radar system 4X4

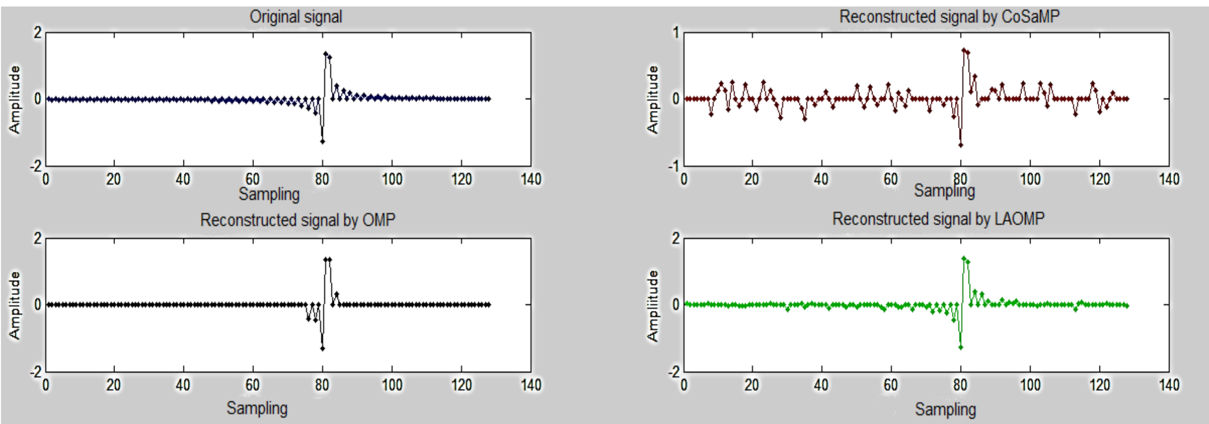

Figure 06 : Reconstruction of signals in a MIMO radar system 8X8

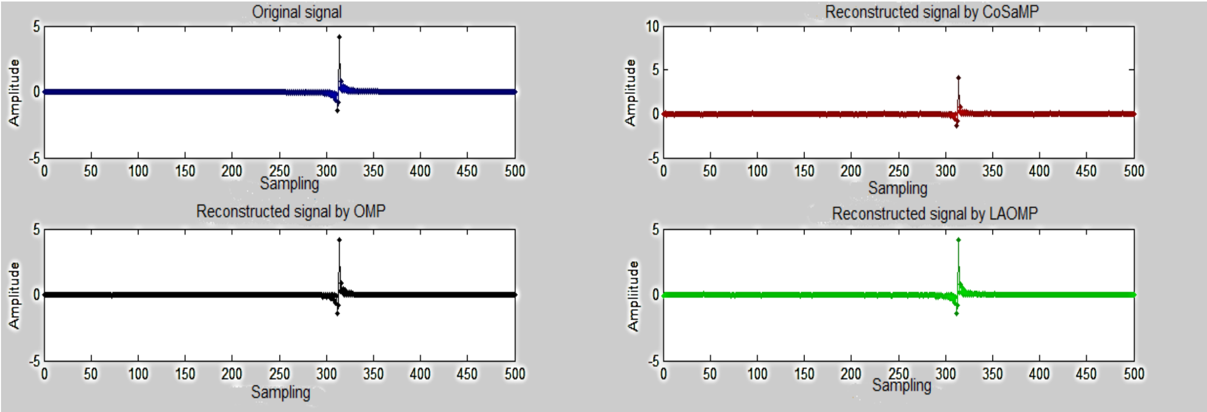

Figure 07: Reconstruction of signals in a MIMO radar system 16X16

Interpretation:

In 4X4 MIMO radar (figure 05) the original signal is reconstructed in the receiver with correlation rates of 51.34% if we use the CoSaMP reconstruction algorithm, of 85.71% that of the OMP and of 90, 76% that of LAOMP. The signal compression rate is 75%.

The correlation rates between the transmitted signal and that received by 8X8 MIMO radar can be evaluated as follows: 70% to 80% for CoSaMP, 87% to 95% for OMP, 88% to 98% for LAOMP. The compression ratio is 75% (figure 06).

In 16X16 MIMO radar (figure 07), the correlation between the transmitted signal and the reconstructed one received is 98% to 99%, with a compression ratio of 75%.

Note that the quality of the reconstructed signal is good in 16X16 MIMO radar.

8.2 Monostatic RADAR reconstruction

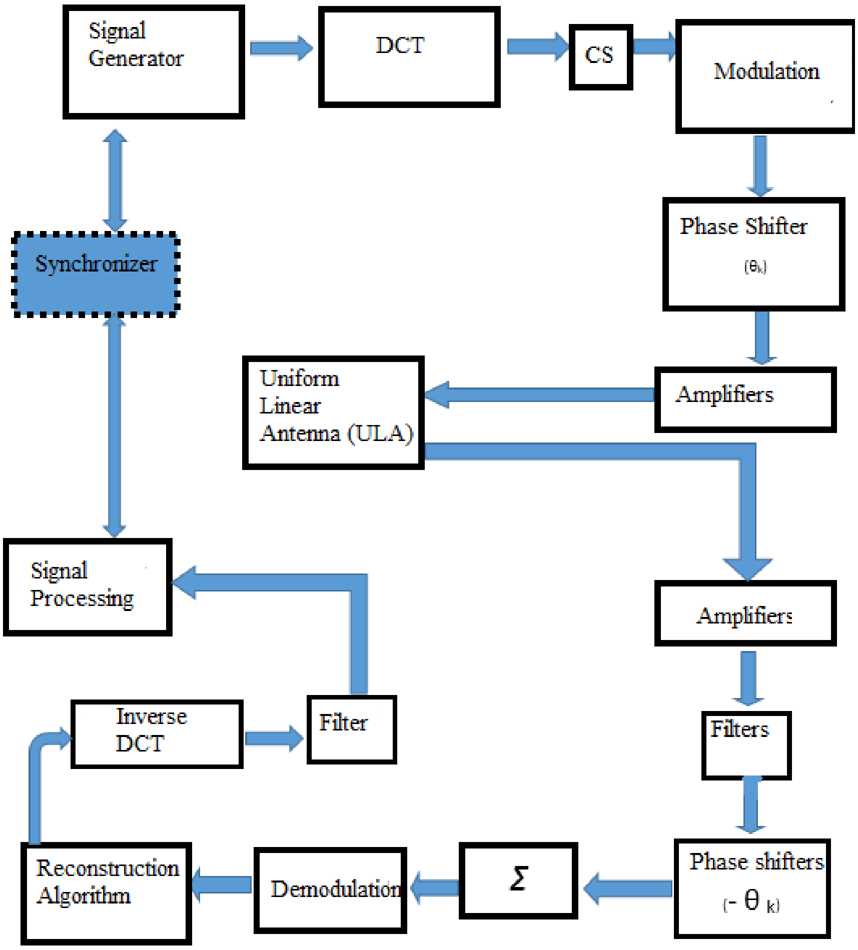

Figure 08 : CS applied in monostatic MIMO radar

Let H be the transfer matrix of this system whose elements are generated randomly of the compression matrix, N_e being the number of antenna TX et N_r being the number of antenna RX. The signal will be generated and trasnlated to frequency domain and multiply by compression matrix for have signal compressed and acquired. The signal will be shifted on the phase and added for beaming on specific angular using ULA technique. On The receiver, all process will be reversed.
Figure 08 shows the application of two techniques: beamforming technique and compressive sensing technique in a monostatic MIMO radar system.

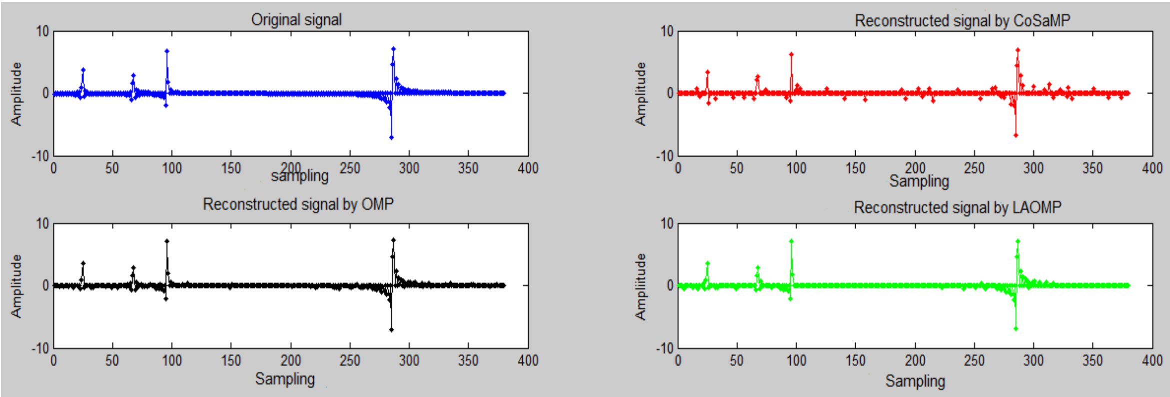

Figure 09 : Signal reconstruction in monostatic MIMO radar

From Figure 09, signal reconstruction with CoSaMP is quite good compared to other methodologies.
The monostatic MIMO radar system having the OMP and LAOMP reconstruction algorithm is very stable depending on the uniformity of the signal reconstruction. In our simulation, the signal compression rate ranges from 55% up to 90%.

8.3 Bistatic RADAR Reconstruction

Let B be the transfer matrix of this MIMO radar system whose elements are generated randomly and A the compression matrix, N_e being the number of antenna TX and N_r being the number of antenna RX.
The distance between TX and RX is equal to 300km, and it is assumed that the target to be detected is stationary and supports a secondary radar on board. Waves travel at the speed of light.

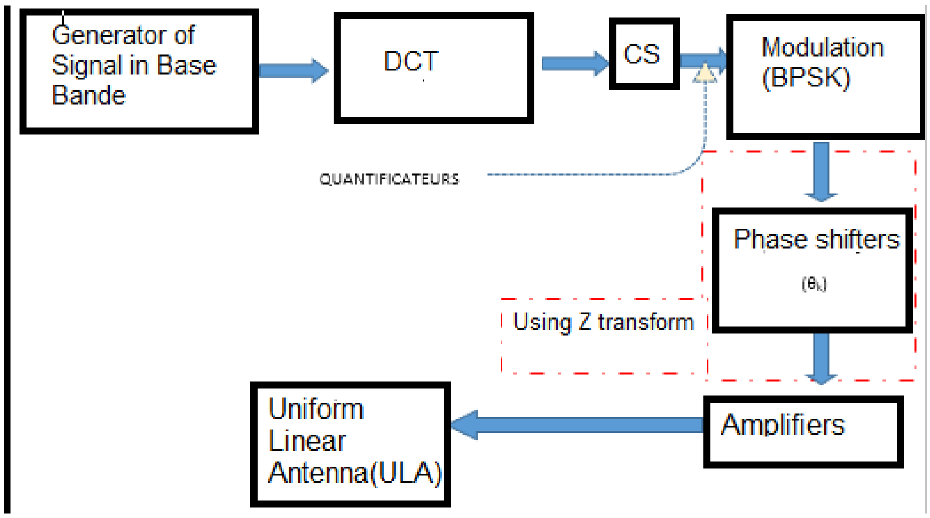

Figure 10 : Block diagram of the bistatic MIMO radar emission

The signal emitted by the TXs (earth stations) depends on the parameters f_i (frequency of i-th TX), distance d_{qr} (distance between q target and r receiving antenna) and d_{qt} (distance between q target and r transmitting antenna).
The modulated signal at the output of the modulator (BPSK) being transformed into Z then phase shifted by the phase shifter. The antenna array configuration is uniform linear.
Figure 12 shows a reconstruction of the signals in bistatic multicarrier 8X8 MIMO radar using OMP as the reconstruction algorithm;

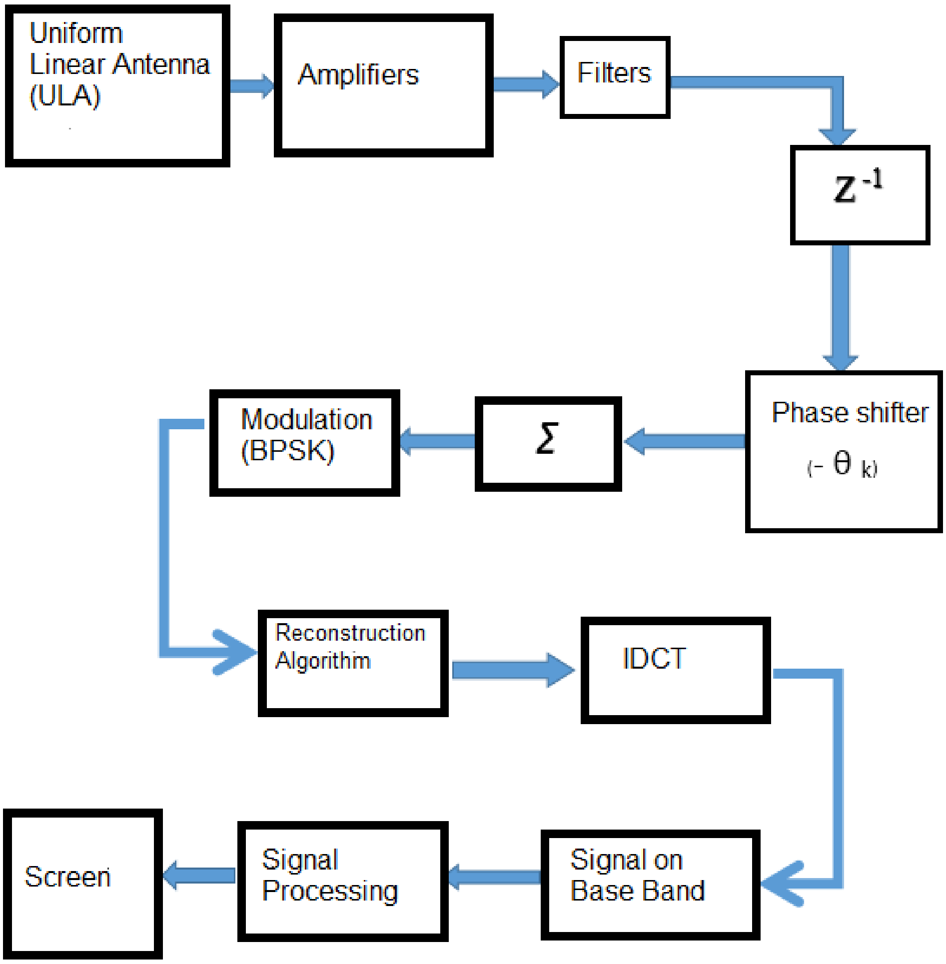

Figure 11 : Simplified diagram in bistatic MIMO radar receivers

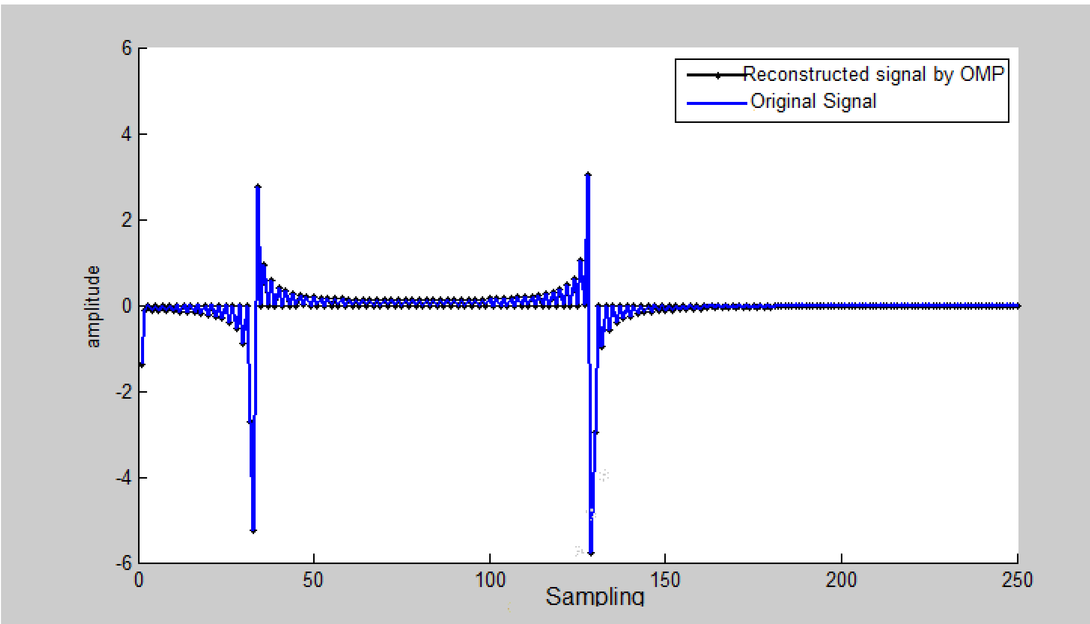

Figure 12 : Signal reconstruction in bistatic 8X8 MIMO radar

The signal generated by the signal generator is transformed so that it has become parsimonious, then this signal is compressed and changed into a digital signal, then, the signal thus obtained must pass the various blocks before being transmitted in the channel of propagation. The compression ratio is in the range of 55% to 80%.

From Figure 12, the reconstructed received signal at the receiver is similar to the original transmitted signal. In this figure the reconstruction algorithm is the OMP seen from its stability and the quality of the reconstructed signal when used in various simulations made. The correlation between the original signal and that reconstructed on reception is 96% to 99%.

8.4 Comparative result between monostatic and bistatic RADAR

Table 01 : Comparaison between monostatic and bistatic RADAR with CS

	Reconstruction Algorithm	Compression rate	Correlation between sent and reconstructed signal
Monostatic	CoSAMP	55-80%	60% - 80%
	LAOMP	55-80%	88% - 98%
	OMP	55-80%	87% - 95%
Bistatic	CoSAMP	80%	<50%
	LAOMP	80%	88-98%
	OMP	80%	96-98%

The table shows the comparative result between the reconstruction methodologies using the compression sensing algorithm in MIMO Radar. Using a 16x16 antenna, the CoSAMP methodology can be used for monostatic RADAR but not usable for bistastic RADAR. The algorithm offers a less complex reconstruction compared to the OMP variants but offers a poor quality of less than 50% for the case of bistatic RADAR. The reconstruction rate of the two OMP variants is therefore suitable for reconstruction in bistastic and mononstatic RADA with a correlation of greater than 88%. However, the realization of the two OMP variants are much more complex than the CoSAMP algorithm.

IX. CONCLUSION

Therefore, To have a good reconstruction quality, the antenna used in Compressing Sensing MIMO Radar must be greater than 16x16. The CoSAMP algorithm is easier to design but is not applicable in bistatic MIMO RADAR because the algorithm may have a correlation rate of less than 50% between sent signal and reconstructed signal. The OMP and LAOMP algorithm is more difficult to design but applicable in bistatic and mono-Static MIMO RADAR with a correlation greater than 80%. Our study focuses mainly on the comparison of existing algorithms on the reconstruction of compressing sensing signals without having tested the different compressing sensing methodologies such as using the DFR method (Fourrier and Random Diagonalization).

REFERENCES

[1]. M. Stefano Fortuaniti, Luca Sanguineti,Fulvio Gini,Maria S.Greco,Braham Himed, *« Massive MIMO radar for target detection»,* IEEE,arXiv:1906.06191,15 Janvier 2020.

[2]. M. Simon Foucart, Holger Rauhut *« A Mathematical introduction to compressive sensing »*, Drexel University Philadelpia et RWTH Aachen University,2010.

[3]. M. Irena Orovic, Vladan Papic, Cornel Ioana, Xiumei Li, et Srdjan Stankovic *« Compressive Sensing in signals processing: algorithms and transform domain formulations »*, Hindawi Publishing Corporation-Mathematical Problems in Engineering,Article ID 7616393, 2 Aout 2016.

[4]. M. Ravelomanantsoa Andrianiaina *« Approche déterministe de l'acquisition comprimée et la reconstruction des signaux issus de capteurs intelligentes distribués »*, Université de Lorraine, 9 Novembre 2015.

[5]. M. Zhen Gao, Linglong Dai, Wei Dai,Byonghyo Shim, et Zhaocheng Wang *« Structured compressive sensing-based spatio-temporal joint channel estimation for FDD massive MIMO »,* IEEE transactions on communications,Vol.64,NO.2,Fevrier 2016.

[6]. M. Neda Rojhani, *« Assessment of Compressive Sensing 2X2 MIMO Antenna Design for Millimeter-Wave Radar Image Enhancement »*, Université de Florence, 9 Avril 2020.

[7]. M. Rauhut Holger *« Analysis of Compressive Sensing in Radar »*, RWTH Aachen University, 8 Decembre 2015.

[8]. M. Jungang Yang,Tian Jin, Chao Xiao, et Xiaotao *Huang « Compressed sensing radar imaging : fundamentals, challenges, and advances »,* College of Electronic Science and Technology, National University of Defense Technology, 13 Juillet 2019.

[9]. M. Christopher Alan Rogers *« MIMO radar waveform design and sparse reconstruction for extended target detection in clutter »,* Old Dominion University, Mai 2014.

[10]. M. Magron Paul *« Reconstruction de phase par modèles de signaux : application à la séparation de sources audio »,* Institut des Sciences et Technologie de Paris,2 Décembre 2016.

[11]. M. Peng Chen, Zhenxin Cao, Zhimin Chen, Xianbin Wang *« Off-grid DOA estimation using sparse Bayesian learning in MIMO radar with unknown mutual coupling »*, IEEE transactions on signal processing, arXiv:1804.04460v3,7 Decembre 2018.

[12]. M. Abdollah Ajorlo,Arash Amini, Mohamad H.Bastani *« A compressive sensing based colocated MIMO radar power allocation and waveform design »*, Journal of LATEX class files,Vol.14,NO.8, Août 2015.

[13]. M. Shunqiao Sun *« MIMO radars with sparse sensing » ,* The State University of New Jersey, Janvier 2016.

[14]. M. Weike Feng, Jean-Michel Friedt, Giovanni Nico, et Motoyuki Sato *« Three-dimensional ground based imaging radar based on C-band cross-MIMO array and tensor compressive sensing »*, IEEE Geoscience and Remote Sensing Letters, Novembre 2019, pp.1585-1589, hal-02382673.

[15]. M. Andrew Jason Duly *« Transmit signal design for MIMO radar and massive MIMO channel estimation »*, Purdue University,12 Juillet 2013.

[16]. M. Weike Feng, Jean-Michel Friedt, Giovanni Nico, et Motoyuki Sato *« Three-dimensional ground based imaging radar based on C-band cross-MIMO array and tensor compressive sensing »*, IEEE Geoscience and Remote Sensing Letters, Novembre 2019, pp.1585-1589, hal-02382673.