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On Fsgb-Connectedness and Fsgb-Disconnectedness in Fuzzy Topological Spaces

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Abstract: The theme of this article is to introduce and investigate a new type of fuzzy strongly generalized b-connectedness namely fsgb-connectedness and fsgb-disconnectedness. Some of their properties and characteristics have been determined.
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I. Introduction:

Several real-world issues in economics, medicine, engineering and social science contain imprecise data, and their solutions rely on uncertainty. L.A.Zadeh[17] established the concepts of fuzzy sets and fuzzy operations to deal with such uncertainty. C.L.Chang[6] , who introduced fuzzy topological spaces , presented the analytical aspect of fuzzy set theory practically. The theory of fts was developed by several authors. The concept of b-open sets in general topology was first developed by Andrejevic [1].

Jenifer and Megha introduced the fsgb-closed sets concepts in [9], the concept of fsgb-continuous , fsgb-irresolute, fsgb-open and fsgb-closed mappings in [10] and some new forms of fsgb-continuous maps namely fuzzy strongly generalized b-continuous functions namely strongly fsgb-continuous, perfectly fsgb-continuous and completely fsgb-continuous mappings in fts [12].Also a new weaker form of continuous functions known as upper fsgb-continuous multifunctions and lower fsgb-continuous multifunctions in [13].In this article, the concepts of fsgb-connectedness and fsgb-disconnectedness are introduced and their properties are investigated.

II. Preliminaries:

Throughout this study (L,τ),(M,σ) and (N,γ)(or simply L, M and N) are fuzzy topological spaces (in-short as fts). The interior, closure and compliment of a fuzzy subset P of (L,τ) are denoted by Int(P), Cl(P) and P^c respectively. Unless specifically specifies, no separation axiom are expected.

2.1 Definition[9] A fuzzy set(in short f-set) P in a fts L is called fb-open iff $P \leq (\mathrm{IntCl}(P) \vee \mathrm{ClInt(P)})$.

2.2 Definition[9] Fb-interior and Fb-closure of a fuzzy set P is as follows

(i)bInt(P) =$\vee\{Q: Q$ is a fb-open set of L and $P \geq Q\}$.

(ii) bCl(P) =$\wedge\{R: R$ is a fb-closed set of L and $R \geq P\}$.

2.3Defintion [9] A f-set P in an fts L is known as fuzzy generalized closed set (in short(fg-CS) if $\mathrm{Cl}(P) \leq Q$, whenever $P \leq Q$ and Q is f-OS in L.

2.4Definition [9]A fuzzy open set (in short f-OS) P in a fts L is called a fsgb-CS that is fsgb-closed set if $\mathrm{bCl}(P) \leq Q$, whenever $P \leq Q$ and Q is fg -open set in L.

2.5Definition [9]A f-OS P in a fts L is called a fsgb-open set(in short fsgb-OS) if $\mathrm{bInt}(P) \geq Q$, whenever $P \geq Q$and Q is fg -open set in L.

2.6Defintion[10]A mapping $f: L \to M$ is said to be fsgb-continuous if $f^{-1}(P)$ is fsgb -closed set in L, for every fuzzy closed set P in M.

2.7Definition[10]A map $g: L \to M$ is known as fsgb-irr that is fsgb-irresolute map, if $g^{-1}(P)$ is fsgb-CS in L for every fsgb-CS P in M.

2.8Definition[14] A fuzzy point $l_p \in Q$ is known as quasi-coincident with f-set Q denoted by l_pqQ iff $p + Q(l) > 1$. A f-set Q is quasi-coincident with a f-set R denoted by $Q_q R$ iff there exists $l \in L$ such that $(l) + R(l) > 1$. If Q and R are not quasi-coincident the we denote it as $Q_{\bar{q}}$R.Note that $Q \leq R \leftrightarrow Q Q_{\bar{q}}(1 - R)$.

III. Fuzzy Strongly Generalized b-Connectedness in fts.

Definition 3.1. A fuzzy topological spaces(L,τ) is known as fsgb-CdS that is fuzzy strongly generalized b-connected space iff 0 and 1 are the only f-sets which are fsgb-closed and fsgb-open (in short fsgb-clopen) sets.

Definition 3.2. A fts (L,τ) is known as fsgb-Cds between f-sets P and Q if there does not exist fsgb-clopen set R in L such that $P \leq R$ and $R_{\bar{q}}Q$.

Theorem 3.3. A fts(L,τ) is fsgb-connected iff (L,τ) is fsgb-CdS between each pair of its non-zero f-sets.

Proof. Consider P and Q are pair of non-zero f-sets of L. Let (L,τ) is not fsgb-Cds between P and Q. Then there exist a fsgb-clopen set R of L such that $P \leq R$ and $R_{\bar{q}}Q$. As P and Q are non-zero f-sets and R is proper fsgb-clopen set of L. Hence(L,τ) is not fsgb-CdS, which the contradicts the hypothesis.

Conversely, consider(L,τ) is not fsgb-CdS. Then there is a proper f-set R of L that is fsgb-clopen set. Thus(L,τ) is not fsgb-CdS between R and $1-R$, which contradicts the hypothesis.

Theorem 3.4. A fts(L,τ) is fsgb-connected iff (L,τ) is fsgb-CdS between P and Q iff there is no fsgb-clopen set R such that $P \leq R \leq 1-Q$.

Proof: It is evident.

Theorem 3.5. If a fts (L,τ) is fsgb-CdS between f-sets P and Q such that $P \leq P_1$ and $Q \leq Q_1$,then (L,τ) is fsgb-Cds between P_1 and Q_1.

Proof. Consider (L,τ) is not fsgb-CdS between P_1 and Q_1 .Then there exists a fsgb-clopen set R of L such that $P_1 \leq R$ and $R_{\bar{q}}Q_1$. Thus $P \leq R$. Now $R_{\bar{q}}Q$ .If $R_{\bar{q}}Q$, then there exists a point $a \in L$ such that $R(a)+Q(a)>1$. Hence $R(a)+Q_1(a)>R(a)+Q(a)>1$ and $R_{\bar{q}}Q_1$, which contradicts the hypothesis.

Theorem 3.6. If a fts (L,τ) is fsgb-CdS between f-sets P and Q, then P and Q are non-zero.

Proof. Assume that $=0$, then P is fsgb-clopen set of L such that $P \leq P$ and $P_{\bar{q}}Q$ .Thus (L,τ) cannot be a fsgb-CdS, which contradicts the hypothesis.

Theorem 3.7. Every fsgb-CdS is f-CdS.

Proof. Consider (L,τ) be fsgb-CdS. Let (L,τ) is not f-CdS and so $\exists$ a proper f-set $P(P \neq 0, P \neq 1)$ such that P is f-clopen set. As every f-CS is fsgb-CS. Thus (L,τ) is not fsgb-CdS, which contradicts the hypothesis. Therefore (L,τ) is f-CdS.

Theorem 3.8. A fts (L,τ) is fsgb-CdS iff (L,τ) has no non-zero fsgb-OS P and Q such that $P+Q=1$.

Proof. Consider (L,τ) is fsgb-CdS. If(L,τ) has 2 non-zero fsgb-OS P and Q such that $+Q=1$, so P is a proper f-set that is fsgb-clopen set of L. Thus(L,τ) is not fsgb-CdS, which contradicts the hypothesis.

Conversely, consider (L,τ) is not fsgb-CdS , then ithas a proper f-set P of L that is fsgb-clopen set. Thus $=1-P$, is a fsgb-OS of L so that $P+Q=1$, which contradicts hypothesis.

Remark 3.9. A fts (L,τ) is fsgb-CdS iff it has no non-zero f-set P and Q such that $P+Q=1$, fsgb-Cl$(P)+Q=P+$fsgb-Cl$(Q)=1$.

Theorem 3.10. Consider$g:(L,\tau)\rightarrow(M,\sigma)$ is fsgb-irr, surjection and L is fsgb-Cds, then M is fsgb-CdS.

Proof. Consider L be a fsgb-CdS.Let M is not fsgb-CdS and then there is a proper f-set P of $M(P \neq 0, P \neq 1)$ such that P is fsgb-clopen set. As g is fsgb-irr,$g^{-1}(P)$ is fsgb-clopen set of Lsuch that $g^{-1}(P) \neq 0$ and $g^{-1}(P) \neq 1$. Therefore (L,τ) is not fsgb-CdS, which contradicts the hypothesis. Thus (M,σ) is fsgb-CdS.

Theorem 3.11. Consider $g:(L,\tau)\rightarrow(M,\sigma)$ is fsgb-$\mathbb{CN}$ map , surjection and L is fsgb-CdS, then M is fsgb-CdS.

Proof. Consider L be a fsgb-CdS. Let M is not fsgb-CdS and then there is a proper f-set P of M $(P \neq 0, P \neq 1)$ such that P is fsgb-clopen set. As g is fsgb-$\mathbb{CN}$map,$g^{-1}(P)$ is fsgb-clopen set of Lsuch that $g^{-1}(P) \neq 0$ and $g^{-1}(P) \neq 1$.Therefore (L,τ) is not fsgb-CdS, which contradicts the hypothesis. Thus (M,σ) is fsgb-CdS.

Theorem 3.12. Consider(L,τ) be fsgb$T_{1/2}$space and f-CdS then (L,τ) is fsgb-CdS.

Proof. Consider(L,τ) is fsgb$T_{1/2}$ space and f-CdS. Let(L,τ) is not fsgb-CdS and then $\exists$ a proper f-set P of L $(P \neq 0, P \neq 1)$ such that P is fsgb-clopen set. As(L,τ) is fsgb$T_{1/2}$space,P is f-clopen set. Thus(L,τ) is not f-CdS, which contradicts the hypothesis. Therefore(L,τ) is fsgb-CdS.

Theorem 3.13. Every fsgb-CdS is fb-CdS (fgb-Cd and fbg-Cd)

Proof. Consider (L,τ) be a fsgb-CdS. Suppose that (L,τ) is not fb-cd (fgb-Cd and fbg-Cd) and then there exists a fuzzy set P $(P \neq 0, P \neq 1)$ so that P is fb-open (fgb-open and fbg-open) and also fb-close (fgb-close and fbg-close). Since every fb-close (fgb-close and fbg-close) is fsgb-close, (L,τ) is not fsgb-cd, which contraducts the assumption. Thus (L,τ) is fb-connected (fgb-close and fbg-close).

The inverse implication is untrue, as it can be seen from the below illustrations.

Example 3.14. Consider L= {x, y, z} .Let the fuzzy sets be $P=\{(x,0.4),(y,0.3),(z,0.5)\}$ $Q=\{(x,0.2),(y,0.6),(z,0.1)\}$

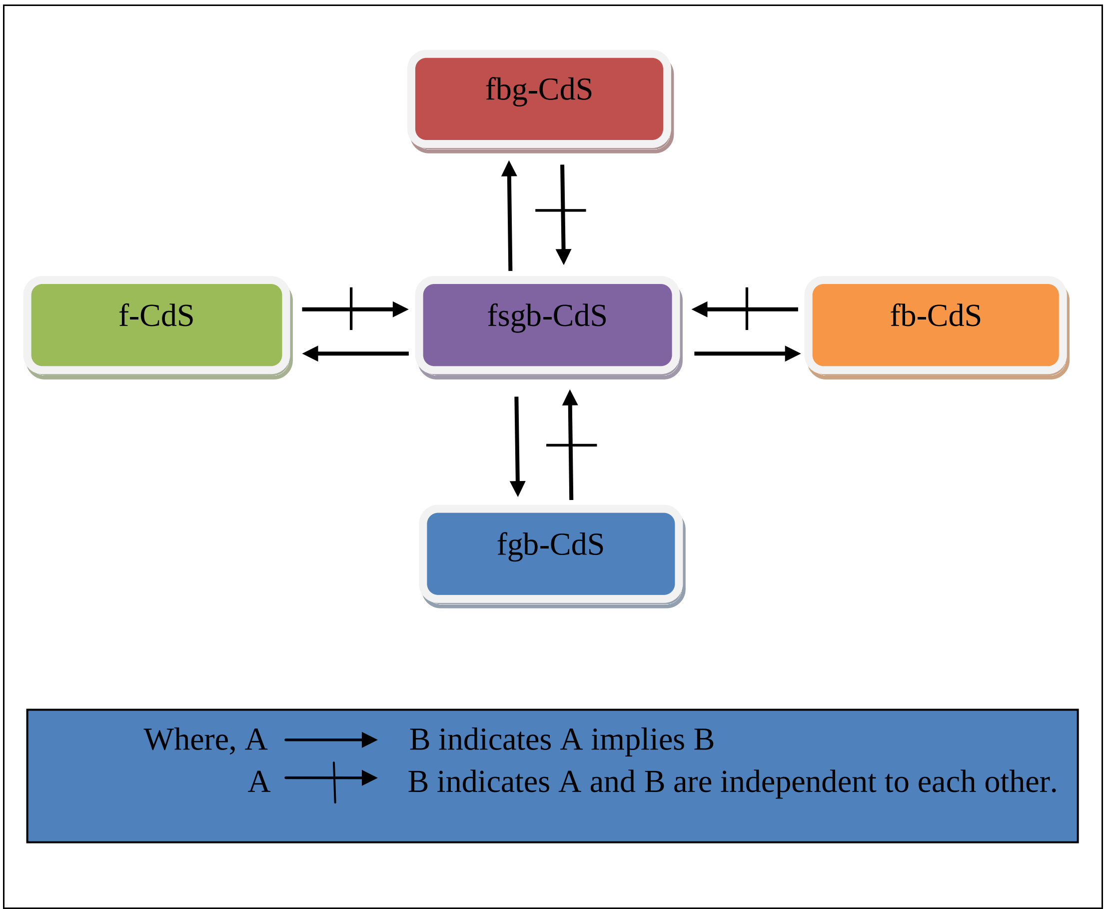
Consider $\tau=\{0,P,1\}$, then the fuzzy sets Q is not a f-OS and f-CS of L.

Thus (L,τ) is f-CdS but not fsgb-CdS.

Example 3.15. Consider L={x, y, z} . Let the fuzzy set be $P=\{(x,0.3),(y,0.6),(z,0.2)\}$ $Q=\{(x,0.1),(y,0.4),(z,0.5)\}$ $R=\{(x,0.2),(y,0.5),(z,0.3)\}$

Consider $\tau = \{0. P, Q, 1\}$, then the FS R is not a fb-OS and a fb-CS of L.
Thus (L, τ) is fb-cd but not fsgb-CdS.
Example 3.16. Consider L= {x, y, z}.
Also consider the fuzzy sets $P = \{(x, 0), (y, 1), (z, 0)\}$ $Q = \{(x, 1), (y, 1), (z, 0)\}$ $R = \{(x, 0), (y, 1), (z, 1)\}$.
Let $\tau = \{0, P, Q, 1\}$, then the fuzzy set R is fsgb-CS but not fsgb-OS of L.
Thus (L,τ) is fsgb connected.

Fig. 3.1. Interrelations of fsgb-connected spaces in fts.



IV. Extremally fuzzy strongly generalized b-disconnectedness.

Definition 4.1. A fts (L, τ) is called as extremally fsgb-disconnected (briefly e-fsgb-d) if fsgb-$cl(P)$ is fsgb-OS, whenever P is fsgb-OS.

Theorem 4.2. For a fts (L, τ) the following statements are equivalent.

(i) (L, τ) is e-fsgb-d.

(ii) For every fsgb-CS P, fsgb-$int(P)$ is fsgb-CS.

(iii) For every fsgb-OS P, we have fsgb-$cl(P)$ + fsgb-$Cl[$1-fsgb-$Cl(P)] = 1$.

(iv) For each pair of fsgb-OS P and Q in (L, τ) with fsgb-$Cl(P) + Q = 1$, we have fsgb-$Cl(P)$ + fsgb-$Cl(Q) = 1$.

Proof.

(i)→(ii)

Consider P be any fsgb-CS. Let us prove that fsgb-$int(P)$ is fsgb-CS. Now $1 -$ fsgb-$int(P)$=fsgb-$Cl(1 - P)$. As P is fsgb-CS, $1 - P$ is fsgb-OS and so by assumption (i) fsgb-$Cl(1 - P)$ is fsgb-OS, which implies that $1 -$ fsgb-$int(P)$ is fsgb-OS. Thus fsgb-$int(P)$ is fsgb-CS.

(ii)→(iii)

Let P be any fsgb-OS. Now $1 -$ fsgb-$Cl(P)$ = fsgb-$int(1 - P)$. Thus, fsgb-$Cl(P)$ + fsgb-$Cl[1 -$ fsgb-$Cl(P)]$ = fsgb-$Cl(P)$ + fsgb-$Cl[$fsgb-$int(1 - P)]$
= fsgb-$Cl(P)$ + fsgb-$int(1 - P)$ by (ii)

$=$ fsgb-$Cl(P) + 1 -$ fsgb-$Cl(P) = 1$.
(iii) →(iv)
Let P and Q be any two fsgb-OS such that
fsgb-$Cl(P) = 1$ ------------(1).
Then by (iii) fsgb-$Cl(P) +$ fsgb-$Cl[1 -$ fsgb-$Cl(P)]$ --------------- (2).
But from (1) $Q = 1 -$ fsgb-$Cl(P)$
and from (1) and (2),
$1 -$ fsgb-$Cl(P) =$ fsgb-$Cl[1 -$ fsgb-$Cl(P)]$
i.e., $1 -$ fsgb-$Cl(P) =$ fsgb-$Cl(Q)$.
Thus fsgb-$Cl(P) +$ fsgb-$Cl(Q) = 1$.
(iv) →(i)
Let P be any fsgb-OS in (L, τ)
Put $Q = 1 -$ fsgb-$Cl(P)$ ----------------- (3)
Now by assumption (iv) fsgb-$Cl(P) +$ fsgb-$Cl(Q) = 1$
i.e., fsgb-$Cl(Q) = 1 -$ fsgb-$Cl(P)$ -------------(4)
From (3) and (4), $Q =$ fsgb-$Cl(Q)$.
Hence Q is fsgb-CS and so fsgb-$cl(Q)$ is fsgb-CS. Then $1 -$ fsgb-$Cl(Q)$ is fsgb-OS and from (4) fsgb-$Cl(P)$ is fsgb-OS in (L, τ). Therefore, (L, τ) is e-fsgb-d.

Theorem 4.3. A fts (L, τ) is an e-fsgb-d space iff fsgb-$Cl(P) =$ fsgb-int[fsgb-$Cl(P)$] for each $P \in$ fsgb-$O(L, \tau)$.

Proof. Consider P be a fsgb-OS in e-fsgb-d space (L, τ). Then fsgb-$cl(P)$ is a fsgb-OS in (L, τ). Therefore fsgb-$Cl(P) =$ fsgb-int[fsgb-$Cl(P)$].

Conversely, if P be a fsgb-OS then fsgb-$Cl(P) =$ fsgb-int[fsgb-$cl(P)$]. Thus fsgb-$Cl(P)$ is a fsgb-OS. Hence (L, τ) is a e-fsgb-d space.

Theorem 4.4. A fts (L, τ) is a e-fsgb-d space iff fsgb-$int(Q) =$ fsgb-Cl[fsgb-$int(Q)$] for every $Q \in$ fsgb-$C(L, \tau)$.

Proof. Consider Q be a fsgb-CS in e-fsgb-d space (L, τ). Then$(1 - Q)$ is a fsgb-OS and fsgb-$Cl(1 - Q)$ is fsgb-OS in (L, τ). Thus , fsgb-$Cl(1 - Q) =$ fsgb-int[fsgb-$Cl(1 - Q)$]. This implies that $1 -$ fsgb-$Cl(1 - Q) = 1 -$ fsgb-int[fsgb-$Cl(1 - Q)$]. Therefore, fsgb-$int(Q) =$ fsgb-Cl[fsgb-$int(Q)$].

Conversely, if Q is a fsgb-OS then $1 - Q$ is fsgb-CS in L and by hypothesis we get fsgb-$int(1 - Q) =$ fsgb-Cl[fsgb-$int(1 - Q)$]
and $1 -$ fsgb-$int(1 - Q) = 1 -$ fsgb-Cl[fsgb-$int(1 - Q)$].
Thus, fsgb-$Cl(Q) =$ fsgb-int[fsgb-$Cl(Q)$]. Hence, (L, τ) is a e-fsgb-d space.

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