IOSR Journal of Engineering (IOSRJEN) **www.iosrjen.org**
ISSN (e): 2250-3021, ISSN (p): 2278-8719
Vol. 14, Issue 9, September 2024, ||Series -1|| PP 14-23

Liquid blowdown from a gas-pressurized cylindrical vessel: a simple model

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Received 03 September 2024; Accepted 16 September 2024

Abstract: Vessel blowdown is common in industrial processes and can take place under a variety of operating conditions. Basically, it consists of the discharge of a pressurized tank or circuit, containing gas (or vapor), liquid or both. It is of interest the characterization of the transient leading to the complete (or, at least, partial) discharge of the content. This paper considers a tank partially filled with a liquid, which is discharged by suitable pressurization of the gas on the top. A simple model is developed to describe a slow discharge (quasi-steady approximation). The equations are formulated in dimensionless form to identify and discuss the influence of the major parameters affecting physics and dictating the engineering design of the process. The discharge of a gas-pressurized vessel is simulated, and the effect of the relevant parameters is represented graphically. Besides, the effect of the liquid on the gas temperature is taken into consideration. Eventually, the generalization of the proposed model to non-ideal operating conditions, involving e.g. friction, lack of equilibrium, etc., is discussed.
Keywords: Blowdown; Pressurized Vessel; Liquid Drainage; Tank Discharge.

I. INTRODUCTION

The discharge of fluids (liquids, gases, and vapor-liquid mixtures) from pressurized vessels is often encountered in several industrial contexts either as a step of the process or as an accidental procedure. If the pressure is not regulated during the discharge, the latter is preferably referred to as blowdown [1]. As significant examples of the variety of applications reported in the literature, the following are worth mentioning.

Boiler blowdown [2], is required for the continuous operation of a boiler, since enables both removing precipitated solid materials and controlling the concentration of dissolved minerals in the water. Since in this case, boiler blowdown represents an energy loss, it is of utmost importance proper management of the frequency and duration of the procedure.

Charge and discharge of liquid propellants to the rocket engines [3] is another important field of application, in which pressure transients play a primary role depending on the specific application (e.g.: military, aerospace, etc.) so that careful evaluation of the choice of a regulated or a blowdown feed system is needed.

Various accidental events leading to the failure or rupture of pressurized tanks and pipelines may cause the blowdown. A recent review [4] presents some of the available models and tools for the optimum design of the blowdown process to identify their potential and limitations. This study highlights that a general approach or a universally applicable model cannot be found owing to the specificity of the problem and the relative operating conditions. Moreover, validation data are still lacking, which makes it difficult to properly evaluate the deviations from the ideal behaviors often assumed in the models.

A common feature of literature works is that seldom a model is developed starting from scratch but most often some earlier formulation is assumed as a starting point. Though it is reasonable to avoid repetitions of assessed developments, it is nevertheless important to make clear the fundamental principles and the major assumptions lying at the basis of a formulation since the blowdown process may occur in such a variety of systems, processes and operating conditions that not all the available approaches are useful and can be adopted as alternatives. This problem arises because most of the published literature addresses a single specific problem (or category of problems) rather than a general approach with some exceptions listed below.

Liquid discharge from vessels is addressed in [5] taking into account different geometries and layouts. Fluid discharge characteristics are determined for orifices located at any location on the vessel. However, the major limitation of this paper is that it only considers tank vented to the atmosphere, i.e. non-pressurized.

Single-phase or multiphase blowdown is considered in [6] through a mechanistic homogeneous equilibrium model. One-dimensional mass and momentum balances are written and numerically integrated for three subsystems, namely, the tank, the pipe and the nozzle. The tank is considered isobaric and isothermal throughout its volume, steady-state flow takes place in the pipe, where all the frictional effects are considered and the nozzle, which is treated as frictionless. Choked flow or discharge at atmospheric pressure is assumed based on the instantaneous tank pressure. Multi-compositional equations of state are used to describe the thermodynamics of either isothermal or isentropic blowdown process. Two-phase flow is described though the homogeneous model, that is using mass weighted averages of the vapor and liquid properties. The model shows appreciable agreement with the data relative to the discharge of gases and vapors from bottles and coiled tubes.

The complex problem of the blowdown of pressure liquified gases from creep and knife induced cracks has been studied in [7], where a model is presented for the unsteady compressible chocked flow through openings of varying areas by implementing a crack opening model based on the plastic displacement theory.

However, none of the above works, though stating general problems, can be applied as it is to the problem of the blowdown of a liquid partially filling a suitably pressurized tank, from which origins the goal of the present paper. A simple model based on the mass and energy conservation equations is developed into detail, highlighting all the simplifying assumptions, to describe the transient corresponding to the complete discharge of the liquid. The model equations are then written into dimensionless form to get a compact mathematical formulation and, most important, to identify similarity parameters useful to generalize the results as much as possible. A simple scheme based on finite differences is proposed for numerical integration. Eventually, a discussion about the major causes of deviation from ideality is provided together with indications to include corrective parameters in the model.

II. MATERIALS AND METHODS

1. System description and main assumptions

The system under analysis, consisting of a vertical cylindrical vessel partially filled with a liquid, is shown in Fig.1. The gas cap is pressurized to speed up the drainage of the liquid from an orifice locate on the bottom. Thus, the flow of the liquid is due to the action of the gravity on the liquid and the expansion of the gas in the cap. Accordingly, the initial pressure of the gas ($p_{g,i}$) is the one needed to achieve a complete drainage of the liquid, such that the final pressure of the gas in the tank is the external pressure (p_0).

The discharge process is studied under the following assumptions:

1. Perfect liquid (i.e. incompressible and non-viscous).
2. Perfect gas (i.e. ideal gas with constant specific heat capacities)
3. Quasi-steady flow of the liquid.
4. Quasi-static expansion of the gas.
5. Thermal insulation towards the environment.

Assumptions 3 and 4 correspond to a slow process (quasi-steady approximation), which enables an analytical treatment. Comments about deviations from this behavior, i.e. a fast discharge, will be made in Section III.

2. Model description

Taking as a control volume the vessel, the discharge process is described at first with a transient mass balance [8]:

$$\frac{dm}{dt} = -\dot{m}_l \tag{1}$$

Where m is the total mass and $\dot{m}_l$ is the liquid mass flow rate.

Since only the mass of the liquid in the vessel changes, due to the discharge, $dm = dm_l = \rho_l dV_l$. On the other hand, the volume of the vessel is constant ($dV_l + dV_g = 0$), leading to:

$$\rho_l \frac{dV_g}{dt} = \dot{m}_l \tag{2}$$

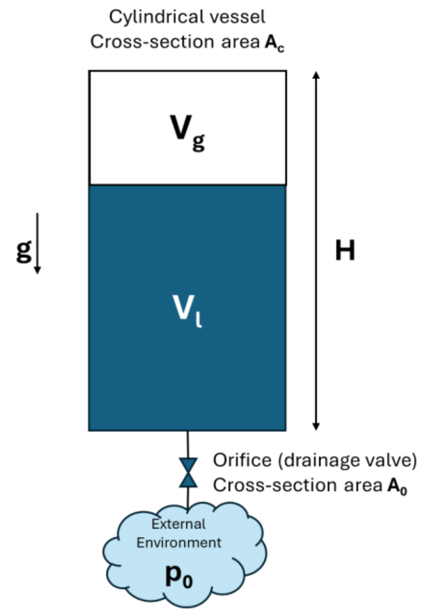

Figure 1. Schematic of the cylindrical vessel.

Where ρ_l is the liquid density. The meaning of (2) is as follows: the drainage of the liquid is due to the expansion of the gas.

The liquid mass flow rate is expressed as:

$$\dot{m}_l = \rho_l U_0 A_0 \tag{3}$$

Where U_0 is the average velocity of the liquid through the drainage valve, the cross-section of which has an area A_0.

The velocity is derived from the Bernoulli equation (mechanical energy balance) [8] across the drainage valve (which in the simplest representation is just a hole). It is assumed that the velocity of the liquid in the tank (at the inlet of the valve) is negligible compared to U_l.

$$\frac{p_l}{\rho_l} = \frac{p_0}{\rho_l} + \frac{U_0^2}{2} \tag{4}$$

Where p_l is the pressure of the liquid at the bottom of the tank, which differs from the pressure of the liquid at the liquid-gas interface owing to the different height. Accordingly, the Stevin law states:

$$p_l = p_g + \rho_l g H_l \tag{5}$$

Where p_g is the pressure of the gas in the cap and g is the acceleration due to gravity. Owing to the small gas density, the effect of the height is neglected for the gas, thus the gas pressure is uniform in the cap.

Replacing (5) in (4) it obtained the average drainage velocity of the liquid as:

$$U_0 = \sqrt{2\frac{p_g + \rho_l g H_l - p_0}{\rho_l}} \tag{6}$$

Notice that if the vessel is open on the top [5], and hence the gas is not pressurized ($p_g = p_0$), (6) reduces to the Torricelli equation [5], [8].

The liquid mass flow rate (3) is then expressed through (6) as:

$$\dot{m}_l = A_0\sqrt{2\rho_l\left(p_g + \rho_l g H_l - p_0\right)} \tag{7}$$

Both the gas pressure and the liquid height are a function of the gas volume, as follows.
The quasi-static expansion of the gas is described by a polytropic process [8]:

$$p_g V_g^n = p_0 V^n \tag{8}$$

Recall that the gas reaches the external pressure when it fills the whole volume of the vessel, i.e. all the liquid has been discharged. The polytropic exponent n is expected to take values between 1 (isothermal expansion) and $\gamma = c_p/c_v$ (adiabatic expansion). More on this subject, which depends on the occurrence of heat transfer between the liquid and the gas will be said in a dedicated section.

The liquid heigh is easily related to the vessel height (see Fig. 1) and finally to the gas volume, knowing the vessel cross-section A_c:

$$H_l = H - H_g = \frac{V - V_g}{A_c} \tag{9}$$

Replacing Equations no. 8 and 9 into Equation no. 7, the mass balance expressed by (2) becomes a first order differential equation which describes the transient of the gas volume:

$$\rho_l \frac{dV_g}{dt} = A_0\sqrt{2\rho_l\left[p_0\left(\frac{V}{V_g}\right)^n + \rho_l g \frac{V - V_g}{A_c} - p_0\right]} \tag{10}$$

The differential problem is completed by the initial condition: $t = 0, V_g = V_{g,i}$, which depends on the initial filling of the vessel.

3. Dimensionless formulation of the governing equation

(10) is conveniently set into dimensionless form to achieve a more compact formulation and to generalize its meaning by highlighting the relative magnitude of the various terms.

First, the dimensionless gas volume is defined as $\tilde{V}_g = V_g/V$, so that the first and the second term inside the square brackets become, respectively: $p_0\tilde{V}_g^{-n}$ and $\rho_l g V\left(1 - \tilde{V}_g\right)/A_c = \rho_l g H\left(1 - \tilde{V}_g\right)$.

Second, the terms inside the square brackets are set into dimensionless form dividing by p_0, which leads to the following expression of the right-hand term of Equation no. 10:

$$A_0\sqrt{2\rho_l p_0} \cdot \sqrt{\tilde{V}_g^{-n} + \frac{\rho_l g H}{p_0}\left(1 - \tilde{V}_g\right) - 1} = A_0\sqrt{2\rho_l p_0} \cdot f\left(\tilde{V}_g\right) \tag{11}$$

Notice that the group:

$$N_{To} = \frac{\rho_l g H}{p_0} \tag{12}$$

Represents the ratio between the relative pressure due to the liquid column and the external pressure. Being the order of magnitude of the term that represents the Torricelli equation (as shown above) it will be called Torricelli number.

Then, it is evident that (10) can be set into dimensionless form dividing by $A_0\sqrt{2\rho_l p_0}$, which leads to the following expression of the left-hand term:

$$\frac{\rho_l V}{A_0\sqrt{2\rho_l p_0}}\frac{d\tilde{V}_g}{dt} \tag{13}$$

Accordingly, the factor multiplying the time derivative of the dimensionless gas volume has the dimensions of a time, and can be considered a characteristic time (τ) for the process under consideration:

$$\tau = \frac{\rho_l V}{A_0\sqrt{2\rho_l p_0}} \tag{14}$$

Equation no. 14 can be made more meaningful by the following manipulations:

$$\tau = \frac{\rho_l V}{A_0\sqrt{2\rho_l p_0}} \cdot \frac{A_c}{A_c} = \frac{H\frac{A_c}{A_0}}{\sqrt{2\frac{p_0}{\rho_l}}} = \frac{\lambda}{\vartheta} \tag{15}$$

Where λ is a characteristic length resulting from the vessel height and a contraction factor (ratio between the vessel area and the discharge valve area), and ϑ is a characteristic velocity, i.e. the velocity if the drainage would take place at the reference (external) pressure. The characteristic time clearly increases if λ increases and ϑ decreases, which makes clear the dynamics of the drainage process.

In particular, the characteristic length increases if:

1. The vessel volume increases.
2. The drainage valve area decreases.

The characteristic velocity increases in case of:

1. Low-density liquids.
2. High-pressure discharge.

It is useful to define a dimensionless mass flow rate of the discharged liquid as $\tilde{m}_l = \dot{m}_l/\rho_l\vartheta A_0 = U_0/\vartheta = f(\tilde{V}_g)$.

Finally, the dimensionless time $\tilde{t} = t/\tau$ is replaced to get the dimensionless formulation of the differential problem:

$$\begin{cases} \dfrac{d\tilde{V}_g}{d\tilde{t}} = \sqrt{\tilde{V}_g^{-n} + N_{To}(1-\tilde{V}_g) - 1} \\ \tilde{t} = 0, \tilde{V}_g = \tilde{V}_{g,i} \end{cases} \tag{16}$$

Where the initial value of the dimensionless gas volume is the initial gas fraction.
Once the value of the polytropic exponent n is determined based on the considerations exposed in the next section, the differential problem of Equation no. 16 can be solved by numerical integration.

4. Thermal considerations

According to the assumption of thermal insulation of the vessel, one might be tempted to assume adiabatic the quasi-static expansion undergone by the gas, which would result in an isentropic process. However, the gas-liquid interface is not adiabatic, hence heat transfer takes place across it. Moreover, also mass transfer (due to evaporation) is likely to take place, which is assumed to give a negligible contribution.

Turning then the attention to sensible heat exchange between the gas and the liquid across the interface, the energy balance states that the same amount of heat gained by the gas must be lost by the liquid (or vice versa):

$$m_g c_x \Delta T_g + m_l c_l \Delta T_l = 0 \tag{17}$$

Where c_x is the specific heat capacity of the gas undergoing the polytropic process. Its relationship with the polytropic exponent is well-known [8]:

$$n = \frac{c_x - c_p}{c_x - c_v} \tag{18}$$

Assuming a complete lack of thermal equilibrium between the gas and the liquid, $\Delta T_l = 0$, which causes the gas expansion to be adiabatic, i.e. $c_x = 0$ or $n = \gamma = c_p/c_v$.

On the other hand, in case of thermal equilibrium between the phases ($\Delta T_g = -\Delta T_l$):

$$c_x = \frac{m_l}{m_g} c_l = \frac{V_l}{V_g}\frac{\rho_l}{\rho_g} c_l = \frac{1-\tilde{V}_g}{\tilde{V}_g}\frac{\rho_l}{\rho_g} c_l \tag{19}$$

Which happens to vary along the process, and obviously causes the variation of the polytropic exponent in turn. Notice that only if $\tilde{V}_g$ is small, the expansion tends to be isothermal ($\tilde{V}_g \to 0, c_x \to \infty, n \to 1$) whereas increasing $\tilde{V}_g$, the expansion tends to become adiabatic ($\tilde{V}_g \to 1, c_x \to 0, n \to \gamma$).

Finally, noticing that the mass of the gas in the vessel is constant, the specific heat capacity can be more easily calculated replacing $V_g \rho_g = V_{g,i} \rho_{g,i}$. Clearly, the initial density $\rho_{g,i}$ depends also on the initial temperature of the gas in the cap. Considerations about the behavior of the temperature will be made in the discussion of the results.

5. Numerical integration

The differential problem (16) is solved by a finite difference scheme [9]. Denoting the right-hand term of the governing equation as $f(\tilde{V}_g)$, the range of variation of $\tilde{V}_g$ (between $\tilde{V}_{g,i}$ and 1) is divided into N equal intervals, denoted by the subscript k such that the equation is written as:

$$\frac{\tilde{V}_{g,k+1} - \tilde{V}_{g,k}}{\tilde{t}_{k+1} - \tilde{t}_k} = f(\tilde{V}_{g,k}) \tag{20}$$

From which the evolution of the transient is computed as:

$$\begin{cases} \tilde{t}_{k+1} = \tilde{t}_k + \dfrac{\tilde{V}_{g,k+1} - \tilde{V}_{g,k}}{f(\tilde{V}_{g,k})} \\ k = 0, \tilde{t}_k = 0, \tilde{V}_{g,k} = \tilde{V}_{g,i} \end{cases} \tag{21}$$

III. RESULTS AND DISCUSSION

The major results can be summarized in plots representing the transient of the most significant quantities, i.e. the dimensionless gas volume, the dimensionless mass flow rate of the discharged liquid (dimensionless drainage rate), the dimensionless pressure and temperature for the two limiting cases of adiabatic and isothermal expansion. The initial filling ratio and the Torricelli number are taken as parameters.

1. Adiabatic expansion

The initial pressurization of the gas required to get a complete discharge of the liquid is determined solely by the initial liquid filling fraction of the vessel ($V_{l,i}/V = 1 - \tilde{V}_{g,i}$). Fig. 2 represents the initial pressure

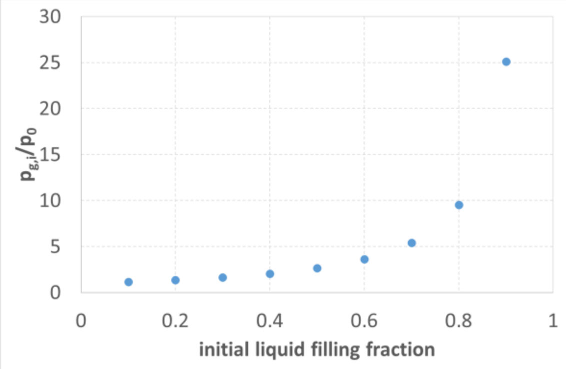

Figure 2. Initial gas pressurization versus initial liquid filling fraction.

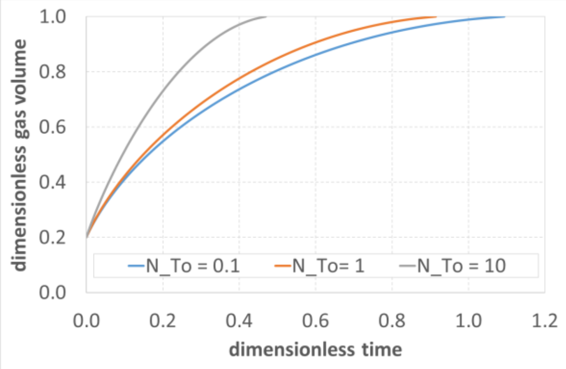

Figure 3. Transient of the dimensionless gas volume for different Torricelli numbers.

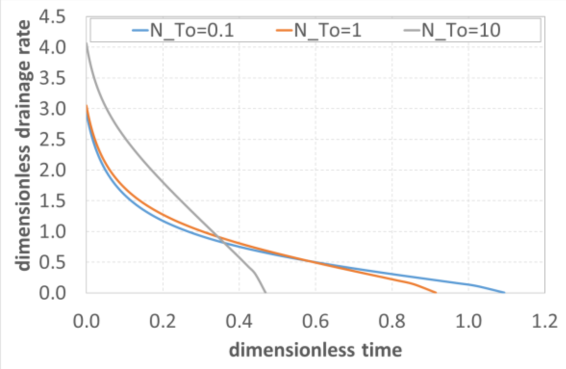

Figure 4. Transient of the dimensionless drainage rate for different Torricelli numbers.

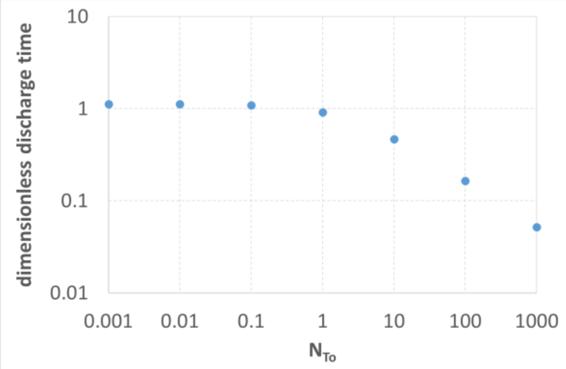

Figure 5. Influence of the Torricelli number of the dimensionless discharge time.

ratio ($p_{g,i}/p_0$) as a function of $1 - \tilde{V}_{g,i}$. The higher the initial liquid filling (i.e. the percentage of the vessel volume initially occupied by the liquid) the higher the gas pressure required for the blowdown.

The transient can be represented either reporting either the dimensionless gas volume or the dimensionless drainage rate against the dimensionless time. Here, two parameters are relevant, namely, the initial liquid filling fraction and the Torricelli number. As an example, Fig. 3 and Fig. 4 report the behavior of $\tilde{V}_g$ and $\tilde{\dot{m}}_l$ for an initial liquid filling fraction of 0.8 (corresponding to $\tilde{V}_{g,i}$ = 0.2). It is assumed that the gas is diatomic (corresponding to dry air or nitrogen, the latter being a very common inert gas). The higher the Torricelli number, the faster the blowdown. This result is explained keeping in mind the meaning of the parameter, which quantifies the effect of the gravity (though the height of the liquid column). At the same initial pressurization (same $\tilde{V}_{g,i}$, same $p_{g,i}/p_0$), the higher the liquid column, the faster the liquid discharge.

Moreover, it is interesting to note that the effect of the gravity is significant only for Torricelli numbers higher than 1, as shown in Fig. 5, where it is seen that for $N_{To} < 1$ the dimensionless discharge time is not significantly affected by this parameter, thus the transient is dominated by the dynamics of the gas expansion.

The maximum benefit of the pressurization in terms of the reduction in the discharge time is then obtained for small values of N_{To}, i.e. when the action of the gravity is negligible. Fig. 6 compares the transient of a pressurized vessel and a non-pressurized one (for which the Torricelli equation rules) for $N_{To} = 0.1$ and $N_{To} = 10$, respectively. It is seen that the pressurization significantly shortens the transient for the smaller N_{To} whereas the benefit for the larger N_{To} is marginal.

To highlight the benefit of the pressurization at small values of the Torricelli number, it is useful to define the ratio between the discharge time of a non-pressurized vessel and the one of a pressurized vessel, which is reported in Fig. 7 as a function of the initial liquid filling. The transient of a pressurized vessel is between about 3.75 and 5 times faster than the one of a non-pressurized vessel. Moreover, the lower the liquid filling, the higher the benefit.

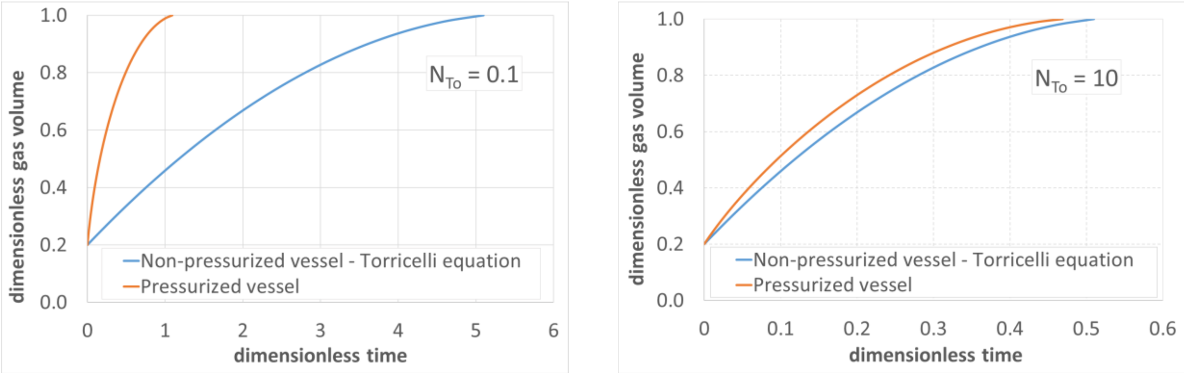

Figure 6. Comparison of the transient of the dimensionless gas volume for N_{To} = 0.1 (left) and 10 (right).

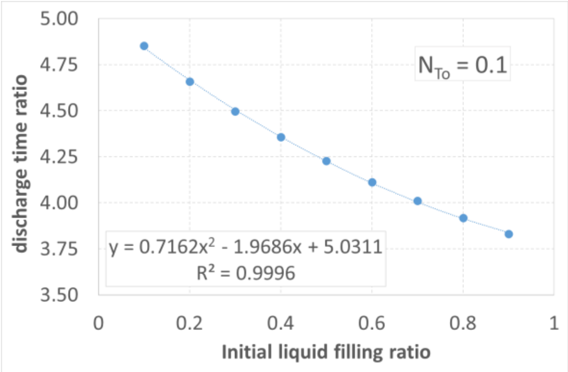

Figure 7. Discharge time ratio versus initial liquid filling ratio for small Torricelli number.

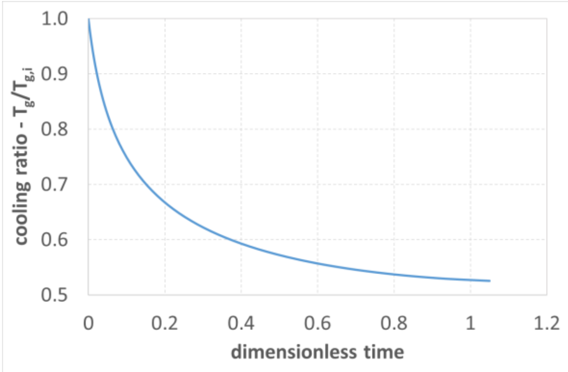

Figure 8. Dimensionless gas temperature transient for an adiabatic expansion.

A final remark is about the gas temperature. The adiabatic expansion determines a temperature drop as depicted in Fig. 8 showing the time behavior of the ratio $T_g/T_{g,i}$, which can be interpreted as a cooling ratio, for the same operating conditions of Fig. 3. Notice that the final (absolute) temperature of the gas is almost halved (for $n = 1.4$). Thus, depending on the initial temperature, the adiabatic expansion may lead to intense cooling, which may affect either the vessel wall (e.g. embrittlement) or the gas-liquid interface (e.g. condensation/freezing).

2. Non-adiabatic expansion

The change in the polytropic exponent, previously discussed, impacts on the dynamics of the discharge, which slows down. Actually, in the adiabatic expansion all the internal energy variation of the pressurized gas is converted into useful work to push the liquid out of the vessel, whereas in a non-adiabatic expansion part of the internal energy variation of the pressurized gas is exchanged as heat between the gas and the liquid, which also causes the initial gas pressurization to be lower.

As an example, Fig. 9 shows the dimensionless discharge time and the initial pressurization as a function of the polytropic exponent in the allowable range for the same filling conditions of Fig. 3. A Torricelli number of 0.1 has been chosen to neglect the effect of gravitational drainage. It is seen that $p_{g,i}/p_0$ monotonically increases with n, which is accompanied by a decrease of the dimensionless discharge time, noting that the transient for the adiabatic blowdown ($n = 1.4$) is about 20% faster than for the isothermal one ($n = 1$).

In case of non-adiabatic expansion, the isothermal process is the most significant as suggested by inspection of (19). Considering applications related to water, oils or liquid fuels storage, the density ratio ρ_l/ρ_g has an order of magnitude of 10^3. The specific heat capacity c_l takes values in the range between about 1 to 4 kJ kg^{-1} K^{-1} (e.g., from [10]: Acetone, 2.15; Alcohol propyl, 2.37; Benzene, 1.8-1.92; Fuel Oil, 1.67-2.09; Gasoline, 2.22; Kerosene 2.01; Water, 4.19). Accordingly, the factor $\rho_l c_l / \rho_g$ is of an order of magnitude of 1 MJ kg^{-1} K^{-1}.

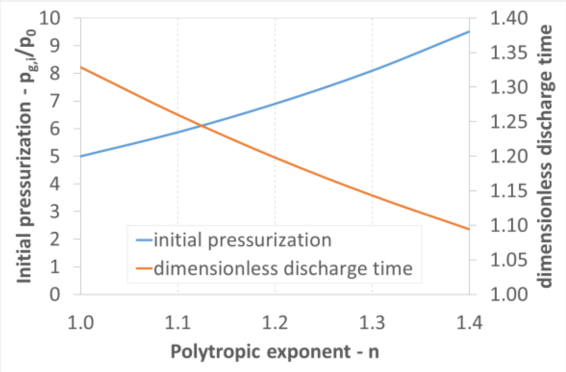

Figure 9. Influence of the polytropic exponent on the initial gas pressurization and on the discharge time.

As a result, except in the very final stage of the expansion, where $1 - \tilde{V}_g$ approaches 0, the resulting values of c_x are so higher than both c_p and c_v that the corresponding n is practically equal to 1.

3. Deviations from ideality

The results discussed in the previous section are affected by deviations from some of the ideal assumptions stated at the beginning. Some of these deviations can be included in the model resulting in a more detailed formulation and/or a more articulated procedure of calculation. Some others, however, require a change in the modelling strategy and cannot be included in the present analysis. In the following, the three major deviations from ideality are identified and briefly discussed:

1.1. Real liquid behavior.

Since the liquid compressibility is negligible in a very wide range of the pressurization, real effects are accountable to viscosity, which determines frictional effects. The most important is the concentrated pressure loss at the discharge valve, where an (almost) sudden contraction of the cross-section area of the flow takes place. This aspect can be introduced in the model through a discharge coefficient $C_d < 1$ to reduce the kinetic energy of the flow in Equation no. 4, as follows:

$$\frac{p_l}{\rho_l} = \frac{p_0}{\rho_l} + C_d \frac{U_0^2}{2} \tag{22}$$

The effect will be an increase in the discharge time due to a lower discharge velocity. In [ref] the discharge coefficient is taken constant for the sake of simplicity but C_d is generally a function of the Reynolds number (and hence of the drainage velocity U_0), as shown in [11], which provides models for both Newtonian and non-Newtonian fluids. Clearly, the dependence on the drainage velocity makes the solution of the problem iterative.

1.2. Fast discharge.

Such deviation implies the failure of the quasi-steady approximation to describe the transient. On one hand, the lack of thermodynamic equilibrium between the gas and the liquid makes the gas expansion practically adiabatic. On the other hand, the polytropic (and, specifically, the isentropic) model for the (adiabatic) expansion is no longer realistic. Frictional effects in both the gas and the liquid determine three-dimensional pressure and temperature fields, which calls for a transient CFD simulation of the process. However, for practical use, an empirical approach could still be adopted, considering an average gas pressure and temperature, and fitting experimental data to find an apparent polytropic exponent in the range $1 < n < \gamma$. In any case, such an empirical extension of the model requires careful consideration, and its validity is limited to the tested conditions.

IV. CONCLUSION

A simple model to study the liquid blowdown of a gas-pressurized vessel has been derived based on mass and energy balances. A dimensionless formulation has been introduced to highlight the order of magnitude of the competing phenomena, i.e. the pressurization and gravity. The resulting first-order differential equation has been solved numerically through a finite difference scheme, leading to the following major results:

1. The higher the Torricelli number (gravitational effect), the faster the blowdown.
2. The effect of the gravity is, however, not significant for Torricelli numbers lower than 1: in this case, the transient is mainly dominated by the dynamics of the gas expansion.
3. For small Torricelli numbers, the transient of a pressurized vessel is between about 3.75 and 5 times faster than the one of a non-pressurized vessel. Moreover, the lower the liquid filling, the higher the benefit.
4. The adiabatic expansion may lead to intense cooling, depending on the expansion ratio.
5. In the case of non-adiabatic expansion, the isothermal process represents the limits for a complete thermal equilibrium between the gas and the liquid.
6. The transient of the isothermal expansion is slower than for the adiabatic expansion.

Finally, possible deviations from the idealizations introduced in the model have been considered suggesting simple ways to account for real effects (e.g. friction).

ACKNOWLEDGEMENTS

The author is grateful to his student Mr. Lorenzo Rizzetto for drawing his attention to the problem discussed in this paper.

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