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A Control Method for a Discrete System with Power Functions

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Abstract :- **In this paper we study a certain control method for a discrete system described with power functions. Especially we show existence of control inputs which stabilize systems of which state vectors vary explosively after vibration with low amplitude during several steps. These control inputs are made by technique for controlling chaos.**

Keywords :– *discrete system, discrete control, difference equation, power function, controlling chaos*

I. INTRODUCTION

In this paper we treat the following system (1) with power functions;

$$\begin{aligned} &x_1(k+1) = a - x_1(k)^6 + x_2(k)^3 + u(k), \quad && a > 0 \\ &x_2(k+1) = x_1(k)^3, && k = 1, 2, \ldots \qquad (1) \\ &x_1(0) = x_2(0) = 0. \end{aligned}$$

The right-hand sides of the system (1) are the right-hand sides of the Henon mapping (2) in which variates $x_1(k)$, $x_2(k)$ are changed to $x_1(k)^3$, $x_2(k)^3$, respectively, and b = 1.

$$\begin{aligned} &x_1(k+1) = a - x_1(k)^2 + bx_2(k) + u(k), \quad && a, b > 0 \\ &x_2(k+1) = x_1(k), && k = 1, 2, \ldots \qquad (2) \end{aligned}$$

If $x_1(k)$, $x_2(k)$ of the Henon mapping are changed to $x_1(k)^2$, $x_2(k)^2$, respectively, we only see immediately explosion of $x_1(k)$, $x_2(k)$. Then we decided to research for the system (1). Control inputs for the system (1) are made by technique for controlling chaos [1].

Controlling chaos is useful for electrical communication, and so on [2] [3]. Though applications of our system have been not founded, we will make use of them to control for phenomena whose states are suddenly changed explosively.

II. PROPERTIES OF THE SYSTEM WITH POWER FUNCTIONS

Let the input u(k) = 0 on the system (1). For 0<a<= 0.70385, $x_1(k)$ of the system (1) tends to a point as k tending to infinity, and $x_2(k)$ does so. For a >= 0.70386, $x_1(k)$ and $x_2(k)$ of the system (1) vibrate during several steps, and after vibration their absolute values tend to infinity as k tending to infinity.

III. STABILIZING INPUTS

In this section we set a = 0.8 on the system (1) and give control inputs stabilizing the system (3), using technique for controlling chaos [1] [3].

$$\begin{aligned} &x_1(k+1) = 0.8 - x_1(k)^6 + x_2(k)^3 + u(k), \\ &x_2(k+1) = x_1(k)^3, \qquad k = 1, 2, \ldots \qquad (3) \\ &x_1(0) = x_2(0) = 0. \end{aligned}$$

The fixed point of the system (3) is (α_1, α_2) = (0.715, 0.366). Though system equations are differentiated by $x_1(k)$, $x_2(k)$ for linearization on controlling chaos, we partially differentiate the system (3) by $x_1(k)^3$ and $x_2(k)^3$. The right hand sides partially differentiated by $x_1(k)^3$, $x_2(k)^3$ are the followings, respectively;

$-2x_1(k)^3$, 1 for the first equation of (3),
1, 0 for the second equation of (3).

We approximate the system (3) around the fixed point (α_1, α_2).

$$\begin{pmatrix} x_1(k) & 0 \\ 0 & x_2(k) \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = A\left\{\begin{pmatrix} x_1(k)^3 & 0 \\ 0 & x_2(k)^3 \end{pmatrix} - \begin{pmatrix} \alpha_1{}^3 & 0 \\ 0 & \alpha_2{}^3 \end{pmatrix}\right\}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \tag{4}$$

where a matrix A is the value of differential at (α_1,α_2), i.e.,

$$A = \begin{pmatrix} -2\alpha_1{}^3 & 1 \\ 1 & 0 \end{pmatrix}. \tag{5}$$

We set by λ_{max} and λ_{min} the maximal eigen value of A and the minimal eigen value of A, respectively. Then we have the next property of the system (3).

Fact 1. A control input

$$u(k) = K\left\{\begin{pmatrix} x_1(k)^3 & 0 \\ 0 & x_2(k)^3 \end{pmatrix} - \begin{pmatrix} \alpha_1{}^3 & 0 \\ 0 & \alpha_2{}^3 \end{pmatrix}\right\}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= 1.4305\,(x_1(k)^3 - 0.715\text{^}3)\text{-}(x_2(k)^3 - 0.366\text{^}3)\,, \tag{6}$$

where $K = -\lambda_{min}[1 \quad -\lambda_{max}] = [\alpha_1{}^3 + (\alpha_1{}^6 + 1)\text{^}(1/2) \quad -1] = [1.4305 \quad -1]$

for $A = \begin{pmatrix} -2\alpha_1{}^3 & 1 \\ 1 & 0 \end{pmatrix}$,

stabilizes the system (3).

For a matrix

$$A = \begin{pmatrix} -2\alpha_1 & 1 \\ 1 & 0 \end{pmatrix}, \tag{7}$$

of which the (1, 1)-element is different from one of the matrix (5), we obtain the next stabilizing input.

Fact 2. A control input

$$u(k) = K\left\{\begin{pmatrix} x_1(k)^3 & 0 \\ 0 & x_2(k)^3 \end{pmatrix} - \begin{pmatrix} \alpha_1{}^3 & 0 \\ 0 & \alpha_2{}^3 \end{pmatrix}\right\}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= 1.9446(x_1(k)^3 - 0.715\text{^}3) - (x_2(k)^3\text{-}0.366\text{^}3), \tag{8}$$

where $K = -\lambda_{min}(1 \quad -\lambda_{max}) = [\alpha_1 + (\alpha_1{}^2 + 1)\text{^}(1/2) \quad -1)] = [1.9446 \quad -1]$

for $A = \begin{pmatrix} -2\alpha_1 & 1 \\ 1 & 0 \end{pmatrix}$,

stabilizes the system (3).

IV. NUMERICAL EXPERIMENTS

Fact 1 and Fact 2 are seen in numerical experiments by Microsoft Excel 2007. On each figure the abscissa means values of k and the ordinate means values of $x_1(k)$ (solid lines), $x_2(k)$ (broken lines).

Fig.1 shows variation of $x_1(k)$ of the system (1) as a = 0.705 (then (α_1,α_2) = (0.650, 0.275)) and u (k) = 0 for all k. Fig. 2 shows variation of $x_1(k)$ of the system (3) (a = 0.8) as u (k) = 0 for all k. Fig. 3 shows $x_1(k)$ and $x_2(k)$ of the system (3) added the control input (6) on Fact 1 for k >= 3. Fig. 4 shows $x_1(k)$ and $x_2(k)$ of the system (3) added the control input (8) on Fact 2 for k >= 3. We can see stabilized states of the system with power functions.

V. CONCLUSION

We suggested a control method which stabilizes systems with power functions suddenly changed explosively. The following problems remain.

(i) Do there exist stabilizing inputs leading $(x_1(k), x_2(k))$ to a fixed point (α_1,α_2) as k tending to infinity for the systems (1) or (3) ?

(ii) Find conditions of a matrix A which constructs a control input stabilizing the systems (1) or (3).

(iii) For all 0.70385 < a < 0.70386, research convergence or explosion of the system (1).

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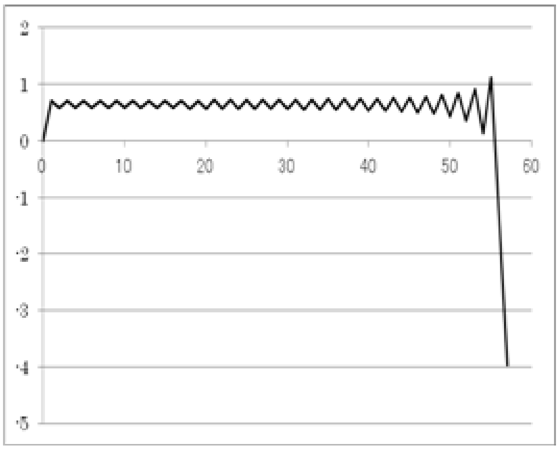

Figure 1: The system (1) (a = 0.705, k = 0, …, 57)

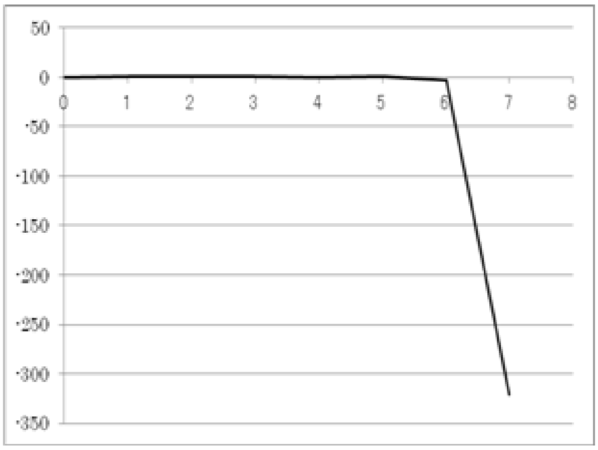

Figure 2: The system (1) (a = 0.8, k = 0, …, 7)

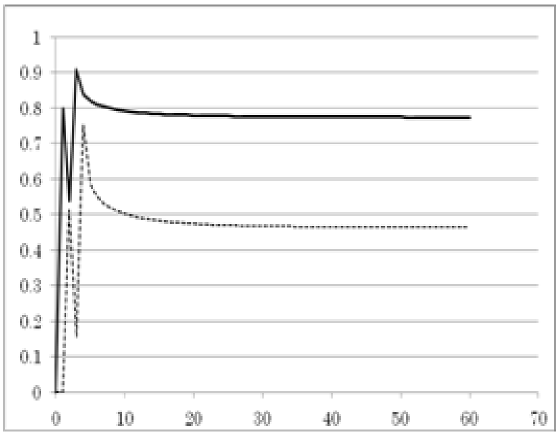

Figure 3: The system stabilized by Fact 1 (a = 0.8)

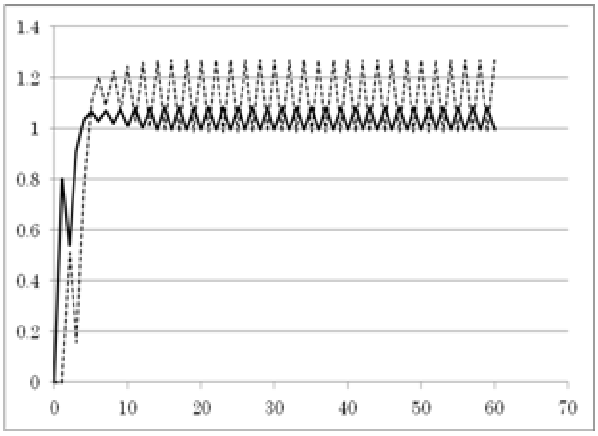

Figure 4: The system stabilized by Fact 2 (a = 0.8)

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