The State Space Modeling of Single, Two and Three ALFC of Power System Using Integral Control and Optimal LQR Control Method

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ABSTRACT:
The modeling of electric power system on the state space with single, two and three area with integral and optimal control system known as Linear Quadratic Regulator(LQR) for designing the load frequency control system are realized in this paper. The fluctuations on voltage and frequency must be reduced to minimum level against load changes for obtaining high quality electric energy on electric power system. In this paper this is achieved by designing the integral controller and LQR. By using state-space analysis a state equation for three area load frequency is obtained. The proposed optimal LQR load frequency has been compared with integral control Simulink using MATLAB.

Keywords: Area Control Error (ACE), Linear Quadratic Regulator (LQR), State Space, Integral Control

I. INTRODUCTION
The main purpose of operating the load frequency control is to keep uniform the frequency changes during the load changes. During the power system operation rotor angle, frequency and active power are the main parameters to change [1]. In multi area system a change of power in one area is met by the increase in generation in all areas associated with a change in the tie-line power and a reduction in frequency. In the normal operating state the power system demands of areas are satisfied at the nominal frequency. A simple control strategy for the normal mode is to operates in such a way that

1. Keep frequency approximately at nominal value.
2. Maintain the tie-line flow at about schedule.
3. Each area should absorb its own load changes.

Controller must be sensitive against changes in frequency and load. To analyze the control system, the mathematical model must be established. There are two models which are widely used [3],

1. Transfer function model, 2. state variable approach.

In above methods the non-linearity of the system is to be converted in to Linearized system with suitable hypothesis and complete block diagram of the power system is established.

II. MODELLING OF POWER SYSTEM
II.1 SINGLE AREA SYSTEM ANALYSIS [2, 3]:
Fig.1 gives the schematic diagram of load frequency control (p-f) of a turbo-generator. In this control method a frequency sensor senses the change in frequency and gives the signal \( \Delta f \). The p-f controller senses the change in frequency signal and the increments in tie-line real powers \( \Delta P_{tie} \), which will indirectly provide information about incremental state error. These sensor signals are amplified, mixed and transformed into a real-power control signal \( \Delta P_c \). The valve control mechanism takes \( \Delta P_c \) as the input signal and provides the output signal, which will change the position of the inlet valve of the prime mover. [2]
From the above Fig.1, the state space equation of single area is developed as under.

\[
\frac{d}{dt}(Δf) = \frac{1}{\tau_{ps}} \left[-Δf + K_{ps}ΔP_G - K_{ps}ΔP_R\right] \tag{1}
\]

\[
\frac{d}{dt}(ΔX_E) = \frac{1}{\tau_{sg}} \left[-ΔX_E + ΔP_C - Δf / R \right] \tag{2}
\]

\[
\frac{d}{dt}(ΔP_R) = \frac{1}{\tau_f}[−ΔP_R + ΔX_E] \tag{3}
\]

From above three equations the state space matrix is obtained as under.

\[
\begin{bmatrix}
ΔX_E \\
ΔP_R \\
Δf
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{\tau_t} & 0 & -\frac{1}{\tau_{sg}} \\
0 & \frac{1}{\tau_t} & 0 \\
0 & 0 & \frac{1}{\tau_{ps}}
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2 \\
X_3
\end{bmatrix}
+ \begin{bmatrix}
1/\tau_{sg} \\
0 \\
0
\end{bmatrix} u
+ \begin{bmatrix}
0 \\
0 \\
-\frac{K_{ps}}{\tau_{ps}}
\end{bmatrix} p
\]

In the above matrix u is system input and p is the disturbance.

II. 2 TWO AREA SYSTEM ANALYSIS:

Similarly to the single area modeling, the two area state space equation is developed. [2]

\[
\frac{d}{dt}(Δf_1) = \frac{1}{\tau_{ps1}} \left[-Δf_1 + K_{ps1}ΔP_{G1} - K_{ps1}ΔP_{D1} - K_{ps1}ΔP_{TL}\right] \tag{4}
\]

\[
\frac{d}{dt}(Δf_2) = \frac{1}{\tau_{ps2}} \left[-Δf_2 + K_{ps2}ΔP_{G2} - K_{ps2}ΔP_{D2} + K_{ps2}ΔP_{TL}\right] \tag{5}
\]

\[
\frac{d}{dt}(ΔX_{E1}) = \frac{1}{\tau_{sg1}} \left[-ΔX_{E1} + ΔP_{C1} - Δf_1 / R_1\right] \tag{6}
\]

\[
\frac{d}{dt}(ΔX_{E2}) = \frac{1}{\tau_{sg2}} \left[-ΔX_{E2} + ΔP_{C2} - Δf_2 / R_2\right] \tag{7}
\]

\[
\frac{d}{dt}(ΔP_{G1}) = \frac{1}{\tau_{t1}}[−ΔP_{G1} + ΔX_{E1}] \tag{8}
\]

\[
\frac{d}{dt}(ΔP_{G2}) = \frac{1}{\tau_{t2}}[−ΔP_{G2} + ΔX_{E2}] \tag{9}
\]

\[
\frac{d}{dt}(ΔP_{TL}) = 2\pi T_{12}^0[Δf_1 - Δf_2] \tag{10}
\]
II.3 THREE AREA SYSTEM ANALYSIS:

Similarly to the two area modeling, the three area state space equation is developed as under [2].

\[
\frac{d}{dt}(\Delta f_1) = \frac{1}{\tau_{ps1}} \left[ -\Delta f_1 + K_{ps1} \Delta P_G1 - K_{ps1} \Delta P_D1 - K_{ps1} \Delta P_{TL1} \right]
\]

(11)

\[
\frac{d}{dt}(\Delta f_2) = \frac{1}{\tau_{ps2}} \left[ -\Delta f_2 + K_{ps2} \Delta P_G2 - K_{ps2} \Delta P_D2 - K_{ps2} \Delta P_{TL2} \right]
\]

(12)

\[
\frac{d}{dt}(\Delta f_3) = \frac{1}{\tau_{ps3}} \left[ -\Delta f_3 + K_{ps3} \Delta P_G3 - K_{ps3} \Delta P_D3 - K_{ps3} \Delta P_{TL3} \right]
\]

(13)

\[
\frac{d}{dt}(\Delta X_{E1}) = \frac{1}{\tau_{sg1}} \left[ -\Delta X_{E1} + \Delta P_C1 - \Delta f_1 / R_1 \right]
\]

(14)

\[
\frac{d}{dt}(\Delta X_{E2}) = \frac{1}{\tau_{sg2}} \left[ -\Delta X_{E2} + \Delta P_C2 - \Delta f_2 / R_2 \right]
\]

(15)

\[
\frac{d}{dt}(\Delta X_{E3}) = \frac{1}{\tau_{sg3}} \left[ -\Delta X_{E3} + \Delta P_C3 - \Delta f_3 / R_3 \right]
\]

(16)

\[
\frac{d}{dt}(\Delta P_G1) = \frac{1}{\tau_{c1}} \left[ -\Delta P_G1 + \Delta X_{E1} \right]
\]

(17)

\[
\frac{d}{dt}(\Delta P_G2) = \frac{1}{\tau_{c2}} \left[ -\Delta P_G2 + \Delta X_{E2} \right]
\]

(18)

\[
\frac{d}{dt}(\Delta P_G3) = \frac{1}{\tau_{c3}} \left[ -\Delta P_G3 + \Delta X_{E3} \right]
\]

(19)

\[
\frac{d}{dt}(\Delta P_{TL1}) = 2\pi T_{12}^0 [\Delta f_1 - \Delta f_2] + 2\pi T_{13}^0 [\Delta f_1 - \Delta f_3]
\]

(20)

\[
\frac{d}{dt}(\Delta P_{TL2}) = 2\pi T_{12}^0 [\Delta f_1 - \Delta f_2] - 2\pi T_{23}^0 [\Delta f_2 - \Delta f_3]
\]

(21)

\[
\frac{d}{dt}(\Delta P_{TL3}) = 2\pi T_{13}^0 [\Delta f_1 - \Delta f_3] + 2\pi T_{23}^0 [\Delta f_2 - \Delta f_3]
\]

(22)
The integral control is composed of a frequency sensor and an integrator. The frequency sensor measures the frequency error $\Delta f$ and this error signal is fed into the integrator. The input to the integrator is called the Area Control Error (ACE). The ACE is the change in area frequency, which when used in an integral-control loop, forces the steady-state frequency error to zero.

Fig. 2 shows the proportional plus integral control of LFC of a single-area system.

III. INTEGRAL CONTROL [2]

The integral control is composed of a frequency sensor and an integrator. The frequency sensor measures the frequency error $\Delta f$ and this error signal is fed into the integrator. The input to the integrator is called the Area Control Error (ACE). The ACE is the change in area frequency, which when used in an integral-control loop, forces the steady-state frequency error to zero. Fig. 2 shows the proportional plus integral control of LFC of a single-area system.

![Fig. 2 PI control of LFC of a single-area system.](image-url)
The integrator produces a real-power command signal $\Delta P_c$ and is given by

$$
\Delta P_c = -K_i \int \Delta f \, dt
$$

$$
= -K_i \int (ACE) \, dt
$$

$\Delta P_c$ = input of speed–changer  
$K_i$ = integral gain constant.

The value of $K_i$ is given by below equation. [4]

$$
K_i = \frac{1}{4} \tau_p \frac{K_p}{\tau_s} (1 + \frac{K_p}{R})^2 = K_{crit}
$$

The value of $K_i$ is so selected that the response will be damped and non-oscillatory. In this case

$$
K_i < K_{crit}.
$$

In our analysis the $K_i$ is selected by above equation. It increases the system type by 1 which forces the final frequency deviation to zero.

### IV. OPTIMAL CONTROL DESIGN FOR THE LOAD FREQUENCY CONTROL AGAINST TO LOAD CHANGES. [1, 3, 5, 6]

Optimal control is a branch of modern control theory that deals with designing controls for dynamic systems by minimizing a performance index that depends on the system variables.

Consider the plant described by

$$
\dot{X} = AX(t) + Bu(t)
$$

The problem is to find the vector $K(t)$ of the control law

$$
u(t) = -K(t)x(t)
$$

This minimizes the value of a quadratic performance index $J$ of the form

$$J = \int_{t_0}^{t_f} (x^TQx + u^TRu) \, dt$$

The constraint problem is solved by using an $n$-vector of Lagrange multipliers, $\lambda$. The problem reduces to the minimization of the following unconstrained function.

$$J(x, \lambda, u, t) = [x^TQx + u^TRu] + \lambda^T[Ax + Bu - \dot{x}]
$$

The optimal values (denoted by subscript $^*$) are found by equating the partial derivatives to zero.

$$
\frac{\partial J}{\partial \lambda} = AX^* + Bu^* - \dot{x} = 0 \quad \Rightarrow \quad \dot{x}^* = AX^* + Bu^*
$$

$$
\frac{\partial J}{\partial u} = 2Ru^* + \lambda B = 0 \quad \Rightarrow \quad u^* = -\frac{1}{2}R^{-1}\lambda B
$$

$$
\frac{\partial J}{\partial x} = 2x^*Q + \dot{\lambda} + \lambda A = 0 \quad \Rightarrow \quad \dot{\lambda} = -2Qx^* - A\lambda
$$

Assume that there exists a symmetric, time varying positive definite matrix $p(t)$ satisfying

$$
\lambda = 2p(t)x^*
$$

Obtaining derivative of above equation

$$
\dot{\lambda} = 2(\dot{p}x^* + p\dot{x}^*)
$$

Substituting values gives the optimal closed-loop control law

$$
u^*(t) = -R^{-1}Bp(t)x^*
$$
Finally equating, we obtain

$$\dot{p}(t) = -p(t)A - A'p(t) - Q + p(t)BR^{-1}B'p(t)$$

(25)

For linear time-invariant systems, since $\dot{p} = 0$, when the process is of infinite duration, that is $t_f = \infty$, equation reduces to the algebraic Riccati equation

$$pA + A'p + Q - pBR^{-1}B'p = 0$$

In solution of the above equation, Riccati equation solution is used. From Riccati equation matrix optimal feed-back gaining and state response of the system at the beginning are solved. For the solution of Riccati equation, $[k, p] = \text{lqr}(A, B, Q, R)$ function in Matlab Control Toolbox is used.

In (The modeling of electric power system on the state space and controlling of optimal LQR load frequency), journal of electrical and electronics engineering 2009, volume 9, number 2 preferred Q and R parameters to design the optimal LQR.[7] Using the Riccati equations, K feed-back gaining matrix is selected. If the system response is not stable, the new Q and R weight matrices are determined.

V. RESULTS

By using MATLAB Simulink software following results are obtained for single area, two area and three area systems. All the three types of system are tested for different operating conditions which includes 1) without any change in the load, 2) with the change in load, 3) with ACE controller and 4) with LQR controller. For more than one system the performance is observed for the same systems or with different systems.

V.1 SINGLE AREA SYSTEM:

![Fig.5.1.1. WITHOUT LOAD CHANGE](image1)

![Fig.5.1.2. LOAD CHANGE](image2)

![Fig.5.1.3. WITH ACE CONTROLLER](image3)

![Fig.5.1.4. WITH LQR CONTROLLER](image4)
V.2. TWO AREA SYSTEM:

Fig. 5.2.1 TWO SAME SYSTEMS WITHOUT LOAD CHANGE

Fig. 5.2.2 TWO DIFFERENT SYSTEMS WITHOUT LOAD CHANGE

Fig. 5.2.3 TWO SAME SYSTEMS WITH LOAD CHANGE

Fig. 5.2.4 TWO DIFFERENT SYSTEMS WITH LOAD CHANGE

Fig. 5.2.5 TWO SAME SYSTEMS WITH ACE

Fig. 5.2.6 TWO DIFFERENT SYSTEMS WITH ACE

Fig. 5.2.7 TWO SAME SYSTEMS WITH LQR

Fig. 5.2.8 TWO DIFFERENT SYSTEMS WITH LQR
V.3. THREE AREA SYSTEM:

Fig. 5.3.1 THREE SAME SYSTEMS WITHOUT LOAD CHANGE

Fig. 5.3.2 THREE SAME SYSTEMS WITHOUT LOAD CHANGE

Fig. 5.3.3 THREE SAME SYSTEMS WITH LOAD CHANGE

Fig. 5.3.4 THREE DIFFERENT SYSTEMS WITH LOAD CHANGE

Fig. 5.3.5 THREE SAME SYSTEMS WITH ACE

Fig. 5.3.6 THREE DIFFERENT SYSTEMS WITH ACE

Fig. 5.3.7 THREE SAME SYSTEMS WITH LQR

Fig. 5.3.8 THREE DIFFERENT SYSTEMS WITH LQR
VI. DISCUSSION

From the above results the frequency deviation of all the areas is given in the below TABLE I.

<table>
<thead>
<tr>
<th>EFFECTS</th>
<th>SINGLE AREA</th>
<th>TWO AREA SAME SYSTEM</th>
<th>TWO AREA DIFFERENT SYSTEM</th>
<th>THREE AREA SAME SYSTEM</th>
<th>THREE AREA DIFFERENT SYSTEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>WITHOUT LOAD CHANGE</td>
<td>-2.45 (see Fig.5.1.1)</td>
<td>-2.45 (see Fig.5.2.1)</td>
<td>-2.45 (see Fig.5.2.2)</td>
<td>-2.48 (see Fig.5.3.1)</td>
<td>-2.58 (see Fig.5.3.2)</td>
</tr>
<tr>
<td>LOAD CHANGE</td>
<td>-5.4 (see Fig.5.1.2)</td>
<td>-5.6 (see Fig.5.2.3)</td>
<td>-5.99 (see Fig.5.2.4)</td>
<td>-5.0 (see Fig.5.3.3)</td>
<td>-5.79 (see Fig.5.3.4)</td>
</tr>
<tr>
<td>ACE CONTROLLER</td>
<td>-3.91 (see Fig.5.1.3)</td>
<td>-3.94 (see Fig.5.2.5)</td>
<td>-4.49 (see Fig.5.2.6)</td>
<td>-3.93 (see Fig.5.3.5)</td>
<td>-4.23 (see Fig.5.3.6)</td>
</tr>
<tr>
<td>LQR CONTROLLER</td>
<td>-0.0265 (see Fig.5.1.4)</td>
<td>-0.0826 (see Fig.5.2.7)</td>
<td>-0.0896 (see Fig.5.2.8)</td>
<td>-0.0199 (see Fig.5.3.7)</td>
<td>-0.0303 (see Fig.5.3.8)</td>
</tr>
</tbody>
</table>

It is seen from the above table that as the area is increased with the same system the frequency deviation is increased little bit. And for different system its deviation is again increased. The deviation is controlled by ACE controller up to minor value its affect on areas and system is taken. In case of LQR controller, as the load is changed the frequency according to the controller with load placement becomes stable in a short time. The load frequency is not affected much with the changes in the system parameter. Results are more satisfied in LQR controller.

VII. CONCLUSION

In this study the ACE controller and optimal LQR controller is implemented as supplementary controller in each area of a three are interconnected power system for the cases with and without load changes. The positive effects of LQR controller is shown in all cases of different areas. Results of all the cases are compared. It shows the improved dynamic response of LQR controller compared to the conventional ACE controller. LQR design procedure is simpler and clearer than the traditional control design. With this high quality and performance controller, no modification is made in the controller structure against changing parameters and loads. This demonstrates us that optimal LQR controller is more robust against changes that occur in the system than the other traditional controllers.

APPENDIX

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>SYSTEM 1</th>
<th>SYSTEM 2</th>
<th>SYSTEM 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power system gain constant, ( K_{ps} )</td>
<td>105</td>
<td>120</td>
<td>100</td>
</tr>
<tr>
<td>Power system time constant, ( \tau_{ps} )</td>
<td>22</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>Speed Regulation R</td>
<td>2.5</td>
<td>2.5</td>
<td>3</td>
</tr>
<tr>
<td>Normal frequency, ( f )</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Governor time constant, ( \tau_{sg} )</td>
<td>0.3</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>Turbine time constant, ( \tau_{t} )</td>
<td>0.5</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>Integration time, constant, ( K_i )</td>
<td>( \frac{1}{4\tau_p} K_{ps} \left( 1 + \frac{K_{ps}}{R} \right)^2 = \frac{K_{crit}}{K_{ps}} )</td>
<td>0.15</td>
<td>0.1</td>
</tr>
</tbody>
</table>

\[ T12 = 0.08, T23 = 0.08 \text{ and } T13 = 0.07. \]
\[ \Delta X_E = \text{Change in valve position.} \]
\[ \Delta P_G = \text{Change in generation.} \]
\[ \Delta f = \text{Change in frequency.} \]
\[ \Delta P_{TL} = \text{Change in tie line power.} \]
REFERENCES


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