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# Analysis of diffusion and extraction in hollow cylinders for some boundary conditions

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***Abstract:*** *- Analysis of diffusion and extraction in hollow cylinders with different outer / inner radius ratio has been investigated. The first five roots,* $\alpha_n$ *of the two valuable equations,* $J_1(a\alpha)Y_0(b\alpha)-J_0(b\alpha)Y_1(a\alpha)=0$ *and* $J_0(a\alpha)Y_1(b\alpha)-J_1(b\alpha)Y_0(a\alpha)=0$ *, were derived and tabulated. Under the condition where diffusion coefficient is constant, the concentration profiles curves of diffusion and extraction for some cases have been demonstrated and discussed.*

***Keywords: -*** *Diffusion; Transport properties; Mechanical properties*

## I. INTRODUCTION

Because of a sensitive electrochemical method developed by Devanathan and Stachurski [1] and some mathematical solutions of the pertinent diffusion equation given by McBreen et al. [2], Kiuchi and McLellan [3], and Yen and Shih [4], measurements of the diffusion coefficient and the permeation rate of hydrogen through a metal membrane have been widely investigated. When critical hydrogen concentration induced cracking in a metal pipe (hollow cylinder) has become an important factor [5-6]. Therefore, the need to understand the concentration profile in a hollow cylinder might be urgent. Several decades ago, Ash et al. [7] provided a means of measuring the diffusion coefficient D for a material in the form of a hollow cylinder shell by the time lag method. Carslaw and Jaeger [8-9] also gave some solutions to the problem of heat conduction through a hollow cylinder shell with some initial and boundary conditions. Crank [10] applied the above mathematics to diffusion for hollow cylinder shells for some cases. However, a more general mathematical solution including steady, set up transient, and decay transient states of the concentration distribution and permeation rate in a hollow cylinder was investigated in our earlier study [11].

The objective of this study was to derive the mathematical solutions of diffusion and extraction in hollow cylinders with different outer / inner radius ratio K. In this paper, the concentration profiles for set up transient and decay transient states were given, respectively.

## II. MATHEMATICAL ANALYSIS

### *2.1. Diffusion equation*

Consider a long circular cylinder in which diffusion is everywhere radial. Concentration is then a function of radius r and time t only, and the diffusion equation becomes

$$\frac{\partial C}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(rD\frac{\partial C}{\partial r}\right) \tag{1}$$

### *2.2. Set up transient state*

Carslaw and Jaeger [8] have given the solution to the problem of diffusion into a hollow cylinder in which the concentration is initially zero and the boundary conditions on the two surfaces are

$$k_1\frac{\partial C}{\partial r}-k_2C=k_3, r=a$$
$$k_1^{'}\frac{\partial C}{\partial r}+k_2^{'}C=k_3^{'}, r=b \tag{2}$$

Carslaw and Jaeger [8] give the solution by the use of Laplace transformation. The final result is

$$C = \frac{-ak_3\left[k_1^{'} - bk_2^{'}\ln\left(\frac{r}{b}\right)\right] + bk_3^{'}\left[k_1 + ak_2\ln\left(\frac{r}{a}\right)\right]}{ak_2k_1^{'} + bk_1k_2^{'} + abk_2k_2^{'}\ln\left(\frac{b}{a}\right)} -$$

$$\pi\sum_{n=1}^{\infty}\frac{e^{-D\alpha_n^2 t}}{F(\alpha_n)} \times \left\{k_1^{'}\alpha_n J_1(b\alpha_n) - k_2^{'}J_0(b\alpha_n)\right\} \times$$

$$U_0(r\alpha_n) \times \begin{bmatrix} k_3\left\{k_1^{'}\alpha_n J_1(b\alpha_n) - k_2^{'}J_0(b\alpha_n)\right\} - \\ k_3^{'}\left\{k_1\alpha_n J_1(a\alpha_n) + k_2 J_0(a\alpha_n)\right\} \end{bmatrix} \quad (3)$$

where

$$F(\alpha_n) = \left(k_1^{'2}\alpha_n^2 + k_2^{'2}\right)\left[k_1\alpha_n J_1(a\alpha_n) + k_2 J_0(a\alpha_n)\right]^2 -$$

$$\left(k_1^2\alpha_n^2 + k_2^2\right)\left[k_1^{'}\alpha_n J_1(b\alpha_n) - k_2^{'}J_0(b\alpha_n)\right]^2 \quad (4)$$

$$U_0(r\alpha_n) = J_0(r\alpha_n)\left[k_1\alpha_n Y_1(a\alpha_n) + k_2 Y_0(a\alpha_n)\right] -$$

$$Y_0(r\alpha_n)\left[k_1\alpha_n J_1(a\alpha_n) + k_2 J_0(a\alpha_n)\right] \quad (5)$$

Here $\alpha_n$ are the roots of the following equation:

$$\left[k_1\alpha J_1(a\alpha) + k_2 J_0(a\alpha)\right]\left[k_1^{'}\alpha Y_1(b\alpha) - k_2^{'}Y_0(b\alpha)\right] -$$
$$\left[k_1\alpha Y_1(a\alpha) + k_2 Y_0(a\alpha)\right]\left[k_1^{'}\alpha J_1(b\alpha) - k_2^{'}J_0(b\alpha)\right] = 0$$
(6)

Consider the hollow cylinder with zero concentration initially and the boundary conditions on the two surfaces are

$$t \le 0, a < r < b, C = 0$$

$$t > 0, \quad r = a, C = C_0$$

$$t > 0, \quad r = b, C = C_0 \quad (7)$$

In order to satisfy the Eq. (7), the constant of the Eq. (2) can be derived as

$k_1 = 0$, $k_2 = 1$, $k_3 = -C_0$, $k_1^{'} = 0$, $k_2^{'} = 1$, $k_3^{'} = C_0$

Substituting these constants into Eqs. (4)-(6), one obtains

$$F(\alpha_n) = J_0^2(a\alpha_n) - J_0^2(b\alpha_n) \quad (8)$$

$$U_0(r\alpha_n) = J_0(r\alpha_n)Y_0(a\alpha_n) - Y_0(r\alpha_n)J_0(a\alpha_n) \quad (9)$$

$$J_0(a\alpha)Y_0(b\alpha) - Y_0(a\alpha)J_0(b\alpha) = 0 \quad (10)$$

where $\alpha_n$ are the positive roots of Eq. (10), as given in Table 1[8, 10]. Substituting

$$k_1 = 0, k_2 = 1, k_3 = -C_0, \quad k_1' = 0, \quad k_2' = 1, \quad k_3' = C_0$$

Eqs. (8)- (10) into Eq. (3), one obtains

$$C = C_0 - \pi C_0 \sum_{n=1}^{\infty} \frac{J_0(b\alpha_n)U_0(r\alpha_n)}{J_0(a\alpha_n) + J_0(b\alpha_n)} \exp\left(-D\alpha_n{}^2 t\right) \quad (11)$$

The normalized concentration of this case can be expressed as

$$\frac{C}{C_0} = 1 - \pi \sum_{n=1}^{\infty} \frac{J_0(ka\alpha_n)U_0(a\alpha_n r^*)}{J_0(a\alpha_n) + J_0(ka\alpha_n)} \exp[-(a\alpha_n)^2 \tau] \quad (12)$$

where

$$U_0(a\alpha_n r^*) = J_0(a\alpha_n r^*)Y_0(a\alpha_n) - J_0(a\alpha_n)Y_0(a\alpha_n r^*)$$

$$r^* = \frac{r}{a}, \quad K = \frac{b}{a}, \quad \tau = \frac{Dt}{a^2}$$

**2.3. Decay transient stat**

***2.3.1. Fast decay : Case A***

Carslaw and Jaeger [9] had given the solution to the problem of diffusion into a hollow cylinder with initial concentration and the two surfaces and kept at zero concentration. The general solution is

$$C = \frac{\pi^2}{2} \sum_{n=1}^{\infty} \frac{\alpha_n{}^2 J_0^2(a\alpha_n)}{J_0^2(a\alpha_n) - J_0^2(b\alpha_n)} \times \exp\left(-D\alpha_n{}^2 t\right) U_0(r\alpha_n) \int_a^b rf(r)U_0(r\alpha_n)dr \quad (13)$$

where

$$U_0(r\alpha_n) = J_0(r\alpha_n)Y_0(b\alpha_n) - Y_0(r\alpha_n)J_0(b\alpha_n) \quad (14)$$

Here $\alpha_n$ are the positive roots of Eq. (10), as given in Table 1[8, 10]. Consider, Eq. (12), the concentration of the steady state, as the initial condition i.e. $C=C_0$ . The boundary conditions are

$$t \le 0, a < r < b, C = C_0$$

$$t > 0, \quad r = a, C = 0$$

$$t > 0, \quad r = b, C = 0 \quad (15)$$

Substituting Eqs.(14) and (15) into the integral

$$\int_a^b rf(r)U_0(r\alpha_n)dr$$

in Eq. (13), one obtains

$$\int_a^b rf(r)U_0(r\alpha_n)dr = \int_a^b rC_0 U_0(r\alpha_n)dr = C_0 \int_a^b rU_0(r\alpha_n)dr \quad (16)$$

Since

$$\int_a^b rU_0(r\alpha_n)dr = -\frac{1}{\alpha_n^2}\left[r\frac{dU_0(r\alpha_n)}{dr}\right]_a^b = \frac{2\{J_0(a\alpha_n)-J_0(b\alpha_n)\}}{\pi\alpha_n^2 J_0(a\alpha_n)} \qquad (17)$$

Eq. (16) becomes

$$\int_a^b rf(r)U_0(r\alpha_n)dr = \frac{2C_0[J_0(a\alpha_n)-J_0(b\alpha_n)]}{\pi\alpha_n^2 J_0(a\alpha_n)} \qquad (18)$$

Substituting Eq. (18) into Eq. (13), one obtains

$$C = \pi C_0 \sum_{n=1}^{\infty} \frac{J_0(a\alpha_n)U_0(r\alpha_n)}{J_0(a\alpha_n)+J_0(b\alpha_n)} \exp\left(-D\alpha_n^2 t\right) \qquad (19)$$

Or the normalized concentration of this case is

$$\frac{C}{C_0} = \pi \sum_{n=1}^{\infty} \frac{J_0(a\alpha_n)U_0\left(a\alpha_n r_A^*\right)}{J_0(a\alpha_n)+J_0(ka\alpha_n)} \exp\left[-(a\alpha_n)^2\tau\right] \qquad (20)$$

where

$$U_0\left(a\alpha_n r_A^*\right) = J_0\left(a\alpha_n r_A^*\right)Y_0(ka\alpha_n) - Y_0\left(a\alpha_n r_A^*\right)J_0(ka\alpha_n) \qquad (21)$$

$$r_A^* = \frac{r}{a} \quad K = \frac{b}{a} \quad \tau = \frac{Dt}{a^2}$$

Table 1

Roots of $J_0(a\alpha_n)Y_0(b\alpha_n) - J_0(b\alpha_n)Y_0(a\alpha_n)$.

| $\frac{b}{a}$ | $a\alpha_1$ | $a\alpha_2$ | $a\alpha_3$ | $a\alpha_4$ | $a\alpha_5$ |
|---|---|---|---|---|---|
| 1.2 | 15.7014 | 31.4126 | 47.1217 | 62.8302 | 78.5385 |
| 1.5 | 6.2702 | 12.5598 | 18.8451 | 25.1294 | 31.4133 |
| 2.0 | 3.1230 | 6.2734 | 9.4182 | 12.5614 | 15.7040 |
| 2.5 | 2.0732 | 4.1773 | 6.2754 | 8.3717 | 10.4672 |
| 3.0 | 1.5485 | 3.1291 | 4.7038 | 6.2767 | 7.8487 |
| 3.5 | 1.2339 | 2.5002 | 3.7608 | 5.0196 | 6.2776 |
| 4.0 | 1.0244 | 2.0809 | 3.1322 | 4.1816 | 5.2301 |

***2.3.2. Slow decay: Case B***

Özişik [12] had given the solution to the problem of diffusion into a hollow cylinder by the method of separation of variables with initial concentration *F(r)* , the two surfaces r = a with zero flux and kept at zero concentration. The general solution is

$$C = \sum_{n=1}^{\infty} \frac{1}{N(\alpha_n)} e^{-D\alpha_n^2 t} R_0(\alpha_n r)\int_a^b rR_0(\alpha_n r)F(r)dr \qquad (22)$$

Where

$$R_0(\alpha_n r) = J_0(\alpha_n r)Y_0(\alpha_n b) - J_0(\alpha_n b)Y_0(\alpha_n r) \tag{23}$$

$$\frac{1}{N(\alpha_n)} = \frac{\pi^2}{2} \times \frac{{\alpha_n}^2 J_1^2(\alpha_n a)}{J_1^2(\alpha_n a) - J_0^2(\alpha_n b)} \tag{24}$$

And

$$J_1(a\alpha)Y_0(b\alpha) - J_0(b\alpha)Y_1(a\alpha) = 0 \tag{25}$$

Here $\alpha_n$ are the positive roots of Eq. (25) which were determined as given in Table 2.

Table 2

Roots of $J_1(a\alpha_n)Y_0(b\alpha_n) - J_0(b\alpha_n)Y_1(a\alpha_n)$.

| $\frac{b}{a}$ | $a\alpha_1$ | $a\alpha_2$ | $a\alpha_3$ | $a\alpha_4$ | $a\alpha_5$ |
|---|---|---|---|---|---|
| 1.2 | 8.14628 | 23.661 | 39.3308 | 55.0214 | 70.7197 |
| 1.5 | 3.4029 | 9.52064 | 15.766 | 22.0327 | 28.3067 |
| 2.0 | 1.79401 | 4.80206 | 7.90896 | 11.0351 | 14.168 |
| 2.5 | 1.24267 | 3.22655 | 5.28885 | 7.36856 | 9.45462 |
| 3.0 | 0.959569 | 2.43717 | 3.97818 | 5.53497 | 7.09771 |
| 3.5 | 0.78554 | 1.96238 | 3.19134 | 4.43458 | 5.68343 |
| 4.0 | 0.666971 | 1.645 | 2.66644 | 3.70083 | 4.74048 |

In this case, consider, Eq.(12) , the concentration of the steady state, as the initial condition i.e. $C=C_0$ . The boundary conditions are

$$t \le 0, a < r < b, C = C_0$$

$$t > 0, \quad r = a, \frac{\partial C}{\partial r} = 0$$

$$t > 0, \quad r = b, C = 0 \tag{26}$$

Substituting Eq. (23) and (26) into the integral

$$\int_a^b r R_0(r\alpha_n)F(r)dr$$

in Eq. (22), one obtains

$$\int_a^b r R_0(r\alpha_n)F(r)dr = C_0 \int_a^b r\{J_0(\alpha_n r)Y_0(\alpha_n b) - J_0(\alpha_n b)Y_0(\alpha_n r)\}dr =$$

$$C_0\left\{\frac{2 - a\pi\alpha_n[J_1(\alpha_n a)Y_0(\alpha_n b) - J_0(\alpha_n b)Y_1(\alpha_n a)]}{\pi{\alpha_n}^2}\right\} \tag{27}$$

Substituting Eq. (27) into Eq. (22), one obtains

$$C = \sum_{n=1}^{\infty} \frac{\pi C_0}{2} \times \frac{\{2 - a\pi\alpha_n[J_1(\alpha_n a)Y_0(\alpha_n b) - J_0(\alpha_n b)Y_1(\alpha_n a)]\}J_1^2(\alpha_n a)R_0(\alpha_n r)}{J_1^2(\alpha_n a) - J_0^2(\alpha_n b)}$$

$$\times \exp(-D{\alpha_n}^2 t) \tag{28}$$

The normalized concentration of this case is

$$\frac{C}{C_0} = \sum_{n=1}^{\infty} \frac{\pi}{2} \times \frac{\{2-\pi\alpha_n a[J_1(\alpha_n a)Y_0(k\alpha_n a) - J_0(k\alpha_n a)Y_1(\alpha_n a)]\}J_1^2(\alpha_n a)R_0(\alpha_n a r_B^*)}{J_1^2(\alpha_n a) - J_0^2(k\alpha_n a)}$$

$$\times \exp(-\alpha_n{}^2 a^2 \tau) \qquad (29)$$

where

$$R_0\left(a\alpha_n r_B^*\right) = J_0\left(a\alpha_n r_B^*\right)Y_0\left(ka\alpha_n\right) - J_0\left(ka\alpha_n\right)Y_0\left(a\alpha_n r_B^*\right) \qquad (30)$$

$$r_B^* = \frac{r}{a} \quad K = \frac{b}{a} \quad \tau = \frac{Dt}{a^2}$$

### *2.3.3. Slow decay: Case C*

Özişik [12] had given the solution to the problem of diffusion into a hollow cylinder by the method of separation of variables with initial concentration *F(r)* , the two surfaces kept at zero concentration and r=a with zero flux. The general solution is

$$C = \sum_{n=1}^{\infty} \frac{1}{N(\alpha_n)} e^{-D\alpha_n{}^2 t} R_0(\alpha_n r) \int_a^b r R_0(\alpha_n r) F(r) dr \qquad (31)$$

where

$$R_0(\alpha_n r) = J_1(\alpha_n b)Y_0(\alpha_n r) - J_0(\alpha_n r)Y_1(\alpha_n b) \qquad (32)$$

$$\frac{1}{N(\alpha_n)} = \frac{\pi^2}{2} \times \frac{\alpha_n{}^2 J_0^2(\alpha_n a)}{J_0^2(\alpha_n a) - J_1^2(\alpha_n b)} \qquad (33)$$

$$J_0(a\alpha)Y_1(b\alpha) - J_1(b\alpha)Y_0(a\alpha) = 0 \qquad (34)$$

Here $\alpha_n$ are the positive roots of Eq. (34) which were determined as given in Table 3.

Table 3

Roots of $J_0(a\alpha_n)Y_1(b\alpha_n) - J_1(b\alpha_n)Y_0(a\alpha_n)$.

| $\frac{b}{a}$ | $a\alpha_1$ | $a\alpha_2$ | $a\alpha_3$ | $a\alpha_4$ | $a\alpha_5$ |
|---|---|---|---|---|---|
| 1.2 | 7.56667 | 23.4688 | 39.2141 | 54.9381 | 70.6549 |
| 1.5 | 2.88989 | 9.34479 | 15.6601 | 21.957 | 28.2478 |
| 2.0 | 1.36078 | 4.6459 | 7.81416 | 10.9671 | 14.1151 |
| 2.5 | 0.866058 | 3.08354 | 5.20107 | 7.30541 | 9.40534 |
| 3.0 | 0.625598 | 2.30404 | 3.89542 | 5.47516 | 7.05095 |
| 3.5 | 0.485007 | 1.83725 | 3.11249 | 4.37731 | 5.63854 |
| 4.0 | 0.393456 | 1.52661 | 2.59082 | 3.64558 | 4.69706 |

In this case, consider the concentration of the steady state ( C=Co), as the initial condition. The boundary conditions are

$$t \le 0, a < r < b, C = C_0$$

$$t > 0, \quad r = a, C = 0$$

$$t > 0, \quad r = b, \frac{\partial C}{\partial r} = 0$$

(35)

Substituting Eqs. (32) and (35) into the integral

$$\int_a^b rR_0(r\alpha_n)F(r)dr$$

in Eq. (31), one obtains

$$\int_a^b rR_0(r\alpha_n)F(r)dr = C_0\int_a^b r\{J_1(\alpha_n b)Y_0(\alpha_n r) - J_0(\alpha_n r)Y_1(\alpha_n b)\}dr =$$

$$C_0\{\frac{a[J_1(\alpha_n a)Y_1(\alpha_n b) - J_1(\alpha_n b)Y_1(\alpha_n a)]}{\alpha_n}\} \quad (36)$$

Substituting Eq. (36) into Eq. (31), one obtains

$$C = \sum_{n=1}^{\infty}\frac{\pi^2 C_0}{2}\times\frac{\alpha_n a J_0^2(\alpha_n a)[J_1(\alpha_n a)Y_1(\alpha_n b) - J_1(\alpha_n b)Y_1(\alpha_n a)]\}R_0(\alpha_n r)}{J_0^2(\alpha_n a) - J_1^2(\alpha_n b)}$$

$$\times\exp(-D\alpha_n{}^2 t) \quad (37)$$

The normalized concentration of this case is

$$\frac{C}{C_0} = \sum_{n=1}^{\infty}\frac{\pi^2}{2}\times\frac{\alpha_n a J_0^2(\alpha_n a)[J_1(\alpha_n a)Y_1(k\alpha_n a) - J_1(k\alpha_n a)Y_1(\alpha_n a)]\}R_0(\alpha_n a r_C^*)}{J_0^2(\alpha_n a) - J_1^2(k\alpha_n a)}$$

$$\times\exp(-\alpha_n{}^2 a^2\tau) \quad (38)$$

$$R_0\left(a\alpha_n r_C^*\right) = J_1(ka\alpha_n)Y_0\left(a\alpha_n r_C^*\right) - J_0\left(a\alpha_n r_C^*\right)Y_1(ka\alpha_n) \quad (39)$$

$$r_C^* = \frac{r}{a} \quad K = \frac{b}{a} \quad \tau = \frac{Dt}{a^2}$$

## III. RESULTS AND DISCUSSION

### *3.1. Set up transient state*

The concentration distribution of $C/C_O$ predicted by Eq. (12) are plotted against $r^*$ for K = 1.2, 1.5, 2, 2.5, 3, 3.5 and 4, at , 0.4 and 1 as shown in Figs. 1(a)-(c), respectively. These curves are obtained by using the first five roots,$\alpha_n$ 's, of the Eq. (10) which are listed in Table 1.The concentration profile increases with increasing ㄟ and decreasing K. Generally, the concentration profiles of this case are symmetrical. All curves reveal similar concave shapes. For K = 1.2 and 1.5, there is no difference between Figs. 1(a) and (b) since steady state has already been reached at ㄟ ≧0.2. Also, these curves indicate that the greater the value of K, the more ㄟ is needed to reach a steady state.

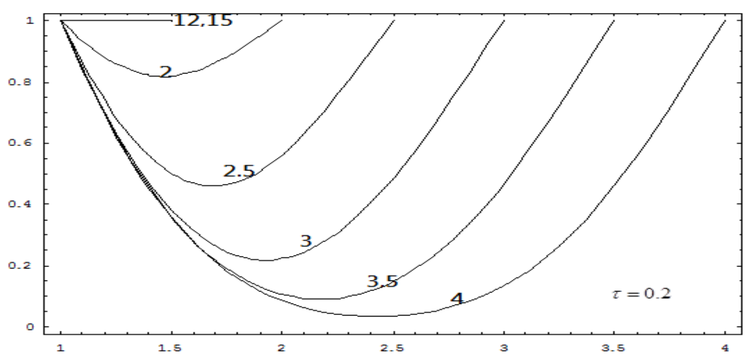


Fig.1(a)

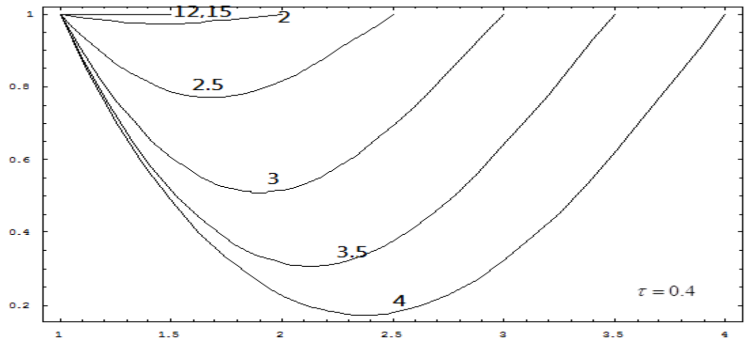


Fig.1(b)

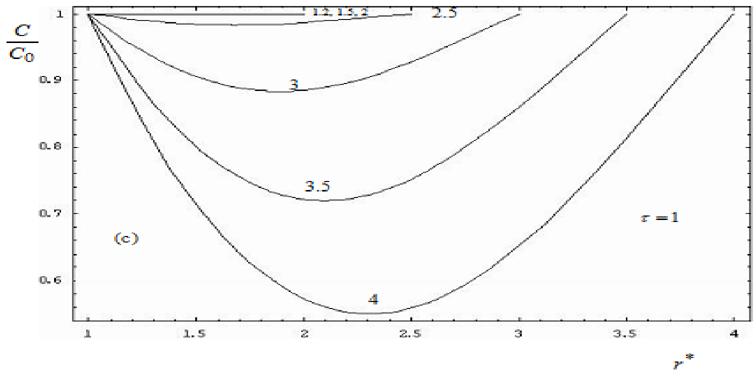


Fig.1(c)

Fig. 1. Set up transient state concentration distribution through cylinder wall at (a) $\tau$=0.2 (b) $\tau$=0.4 (c) $\tau$=1 . Numbers on curves are values of K.

### *3.2. Decay transient state*

#### *3.2.1. Fast decay: Case A*

The concentration distribution of $C/C_O$ calculated by Eq. (20) are plotted against $r_A^*$ at various $\tau$ and K, as shown in Figs. 2(a)-(c), respectively. These curves are also plotted by using the first five roots, $\alpha_n$ 's, of the Eq. (10) which are listed in Table 1. The concentration profile decreases with increasing $\tau$ and decreasing K. The concentration profiles are symmetrical. Especially for K≦1.2, it is almost as symmetrical as the sheet case [13], as shown in Fig. 2(a). All curves reveal similar convex shapes.

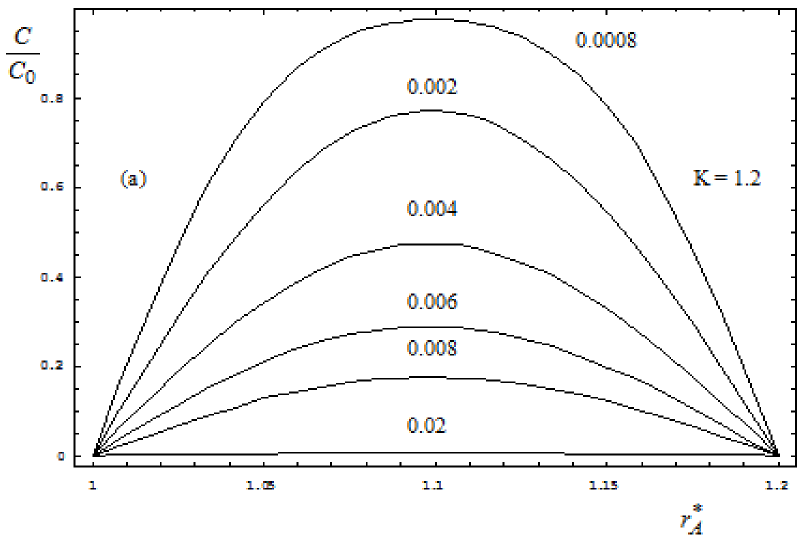


Fig.2(a)

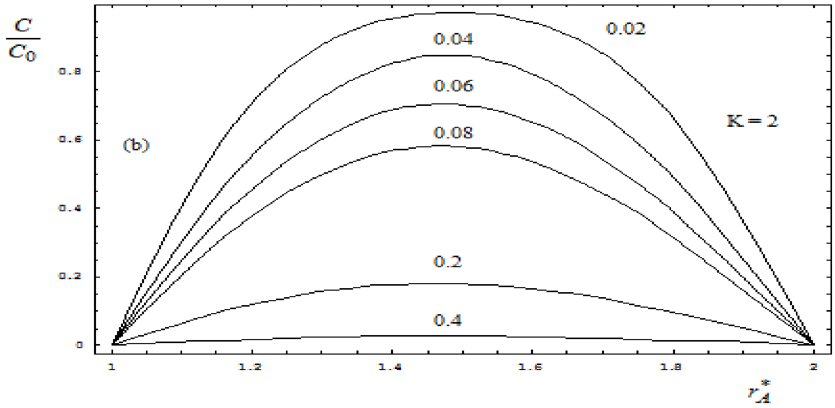


Fig.2(b)

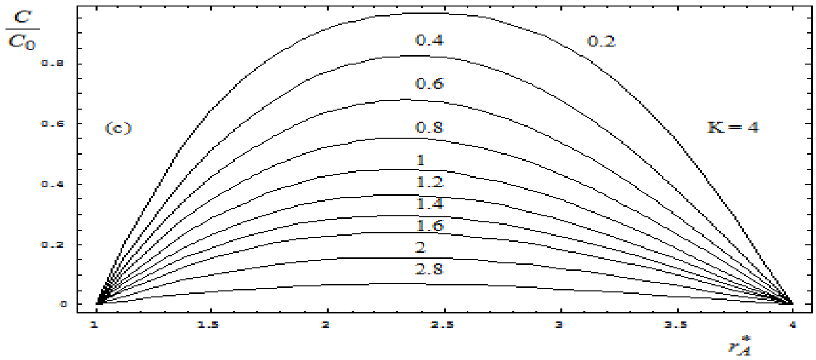


Fig.2(c)

Fig. 2. Decay transient state concentration distribution through cylinder wall for (a) K = 1.2(b) K = 2 (c) K = 4. Numbers on curves are values of   for case A.

### *3.2.2. Slow decay: Case B*

The concentration distribution of $C/C_O$ calculated by Eq. (29) are plotted against $r_B^*$ at various $\tau$ and K, as shown in Figs. 3(a)-(c), respectively. These curves are also plotted by using the first five roots, $\alpha_n$ 's, of the Eq. (25) which are listed in Table 2. The concentration profile decreases with increasing $\tau$ and decreasing K. All curves reveal similar convex shapes.

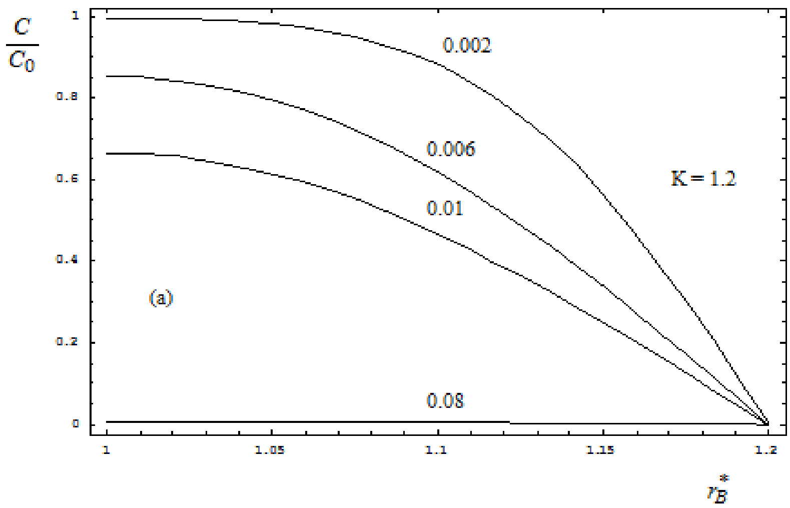


Fig.3(a)

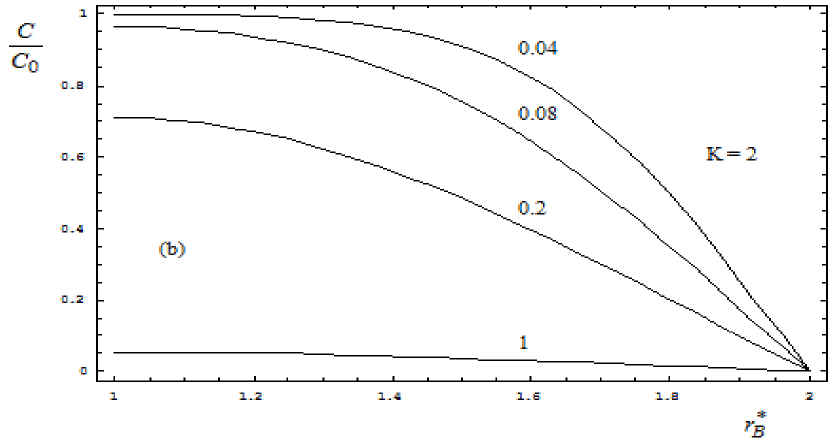


Fig.3(b)

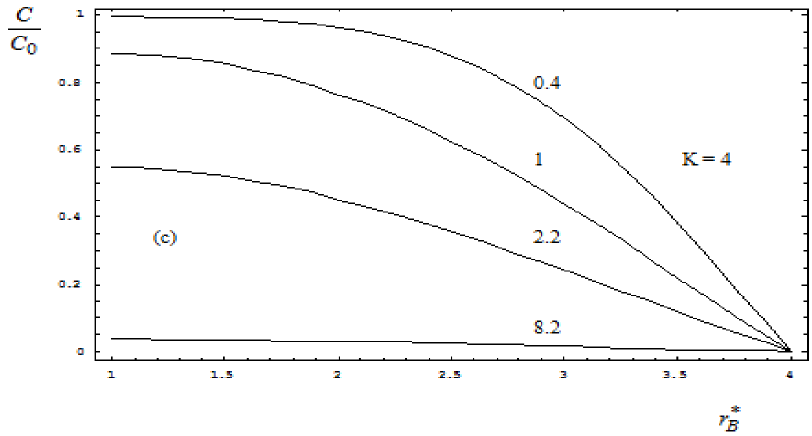


Fig.3(c)

Fig. 3. Decay transient state concentration distribution through cylinder wall for (a) K = 1.2(b) K = 2 (c) K = 4. Numbers on curves are values of for case B.

### *3.2.3. Slow decay: Case C*

The concentration distribution of $C/C_O$ calculated by Eq. (38) are plotted against $r_c^*$ at various $\tau$ and K, as shown in Figs. 4(a)-(c), respectively. These curves are also plotted by using the first five roots, $\alpha_n$ 's, of the Eq. (34) which are listed in Table 3. The concentration profile decreases with increasing $\tau$ and decreasing K. All curves reveal similar convex shapes. Comparing the decreasing rate of concentration profile for cases A, B and C, case A is the fastest, case C is the slowest for K≧2. However, for K≦1.2, the decreasing rate of concentration profile for cases B and C is the same.

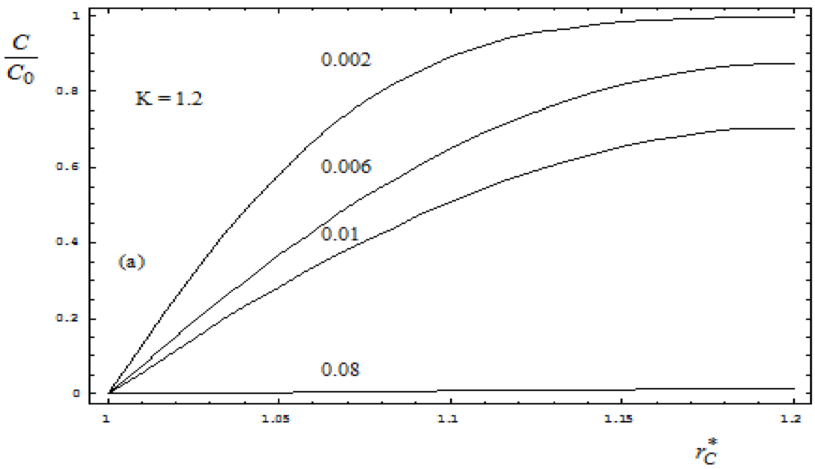


Fig.4 (a)

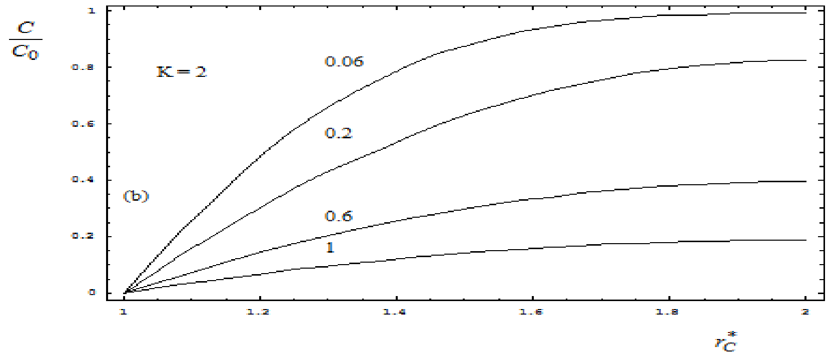


Fig.4 (b)

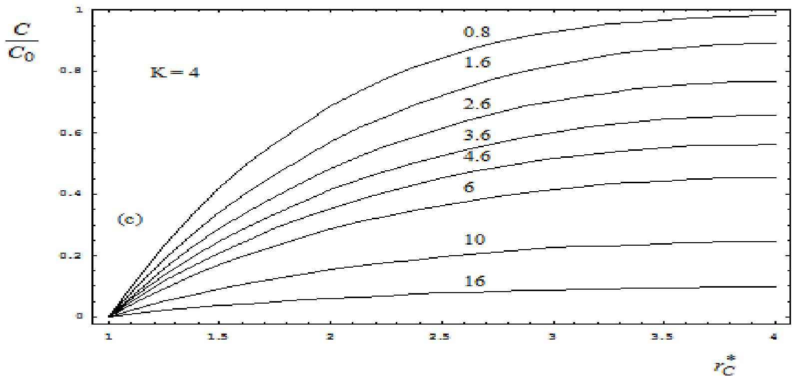


Fig.4(c)

Fig. 4. Decay transient state concentration distribution through cylinder wall for (a) K = 1.2(b) K = 2 (c) K = 4. Numbers on curves are values of for case C.

## IV. Conclusion

The mathematical solutions of diffusion in hollow cylinder with different K for the set up transient and decay transient states are given in Eqs. (12), (20), (29), and (38), respectively. Through the mathematical analysis and figure plotting, a few conclusions are drawn:

1. In the set up transient state and decay transient state for case A, the concentration profiles are reverse and symmetrical.
2. In the decay transient state for cases B and C, the normalized concentration profiles are reverse and reveal the similar convex shapes.
3. In decay transient states for cases A, B and C, comparing the decreasing rate of the concentration profile, case A is the fastest, for the same K.

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