IOSR Journal of Engineering (IOSRJEN)
e-ISSN: 2250-3021, p-ISSN: 2278-8719, www.iosrjen.org
Volume 2, Issue 9 (September 2012), PP 20-30

Optimization and analysis of some variants through Vogel's approximation method (VAM)

Shweta Singh[1] G.C. Dubey[2,] Rajesh Shrivastava[3]
[1]*Radharaman Institute of Technology and Science, Bhopal India*
[2]*Govt. M.G.M. College, Itarsi (M.P.), India*
[3]*Govt. Science and Commerce College, Benazir, Bhopal, India*

***Abstract*—** *VAM is one of the most efficient solution to the transportation problems. In VAM Variants are proposed using total opportunity cost and allocation cost. Through the survey of computational experiments the results are evaluated through VAM and improved version of VAM Performance if improved version of VAM over VAM is also discussed and analysis through iterations. Total opportunity cost in transportation has been studied through critical problem in industries, military etc. In Analysis we found that the quality of solution in basic version of VAM coupled with total opportunity cost yields a very efficient initial solution. In survey experiment on an average about 20% of the time, the VAMTOC yields the optimal solution and about 80% of the time it yields to a solution nearest to the optimal Value. A comparative study is also shown through various charts.*

Keywords*—Initial basic Feasible solution (IBFS), Transportation problem, Vogel's Approximation method, Total opportunity cost (TOC).*

I. INTRODUCTION

Transportation models deal with problems concerning as to at happens to the effectiveness function. When we associated each of origins (Sources) with each of a possibly different umber of destinations (jobs). The total movement from each origin and the total movement to each destination is given and it is desired to find how the associations be made subject to the limitations on totals.

The transportation problem deals with the transportation of a product manufactured at different plants or factories (supply of origin) to a number of different ware houses. (Demand, destinations). The objective is to satisfy the destination requirement within the plant's capacity constraints at the minimum transportation cost.

The transportation problem constitutes on important part of logistics management and also a special kind of the network optimization problem. Transportation models play an important role in logistics and supply chains. The easiest way to recognize a transportation problem is to consider a typical situation as shown in the following figure. Assume that a manufacturer has three factories F_1, F_2, F_3 producing the same product, from these factories, the product is transported to three warehouses W_1, W_2 and W_3, each factory has limited supply and each warehouse has specific demand. Each factory can transport to each warehouse but the transportation cost vary for different combinations the problem is to determine the quantity each factory. Should transport to each ware house in order to minimize total transportation Cost

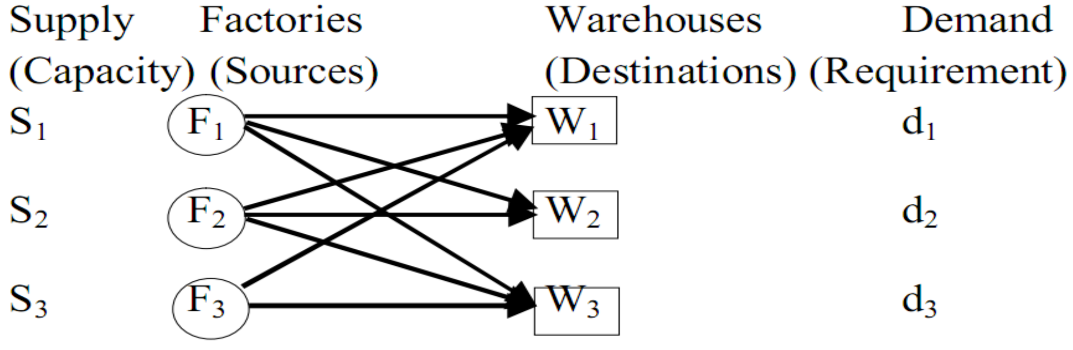

Fig.1 Example of a Transportation problem

1.1 Structure and formulation of the Transportation Problem

An important feature of the standard transportation problem is that it can be expressed in the form of a table, which displays of all values of data coefficients (s_i, d_j, c_{ij}) associated with the problem

Origins		**Destinations**					**Supply (Avai-lability)**
	d_1	d_2	d_3	d_4	-	dn	
O_1	C_{11} X_{11}	C_{12} X_{12}	C_{13} X_{13}	C_{14} X_{14}		C_{1n} X_{1n}	A_1
O_2	C_{21} X_{21}	C_{22} X_{22}	C_{23} X_{23}	C_{24} X_{24}		C_{2n} X_{2n}	A_2
O_3	C_{31} X_{31}	C_{32} X_{32}	C_{33} X_{33}	C_{34} X_{34}		C_{3n} X_{3n}	A_3
O_4	C_{41} X_{41}	C_{42} X_{42}	C_{43} X_{43}	C_{44} X_{44}		C_{4n} X_{4n}	A_4
O_5	C_{51} X_{51}	C_{52} X_{52}	C_{53} X_{53}	C_{54} X_{54}		C_{5n} X_{5n}	A_5
. . .							
O_m	C_{m1} X_{m1}	C_{m2} X_{m2}	C_{m3} X_{m3}	C_{m4} X_{m4}		C_{mn} X_{mn}	A_m
Demand (Requirement)	B_1	B_2	B_3	B_4		B_m	$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

Fig.2 General Transportation Table Structure

Formulation of the linear programming model

Minimize Z= $\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij}\, c_{ij}$

Subject

to $\sum_{i=1}^{m} x_{ij} = a_i$ i=1,2,3,-m

(1) (Capacity constraint)

$\sum_{j=1}^{n} x_{ij} = b_i$ j=1,2,3,-n

(2) (Requirement Constraint)

$x_{ij} > 0$

Where

The two sets of constraints will be consistent ie the system will be balance if

$$\sum_{i=1}^{m} a_j = \sum_{j=1}^{n} b_j \text{ ....(3)}$$

Thus eq (3) is necessary as well as a sufficient conditions for a transportation problem to have a feasible solution. Problems that satisfy this condition are called balanced. Transportation problem (BTP)

Techniques have been developed for solving BTP or STP (stand are transportation problem). The reader may refer to Wagner [1] and Taha [2] for detailed coverage of T.P.

The following terms are to be defined with reference to the transportation problem

[A] ***Feasible Solution (F.S.)***

A non-negative allocation xij zo which satisfy the row and column restrictions is known as F.S.

[B] ***Basic Feasible solution (B.F.S.)***

A feasible solution to m-origin and n-destination problem is said to be B.F.S. if the number of positive allocation are (m+n-1)

If the number of allocation in a basic feasible solution are less than (m+n-1) it is called degenerate B.F.S. (D.B.F.S.) otherwise non-degenerate

[C] ***Optimal Solution(O.S.):-***

A feasible solution (not necessarily basic) is said to be optimal if it minimizes the total transportation cost

The Basic steps to solve transportation problem are:

Step

1. To determine the IBFS (Initial basic feasible solution)
2. To determine the optimal solution using IBFS.

As IS (Initial solution) is a first attempt to match supply and demand along advantageous distribution routes. Also there are several methods available to obtain on initial solution (IS). Well-known heuristics methods are North West Corner [3], Lowest cost entry method VAM method [4], Shimshak et. Als version of VAM [5], Goyal's version of VAM [6], Ramkrishnan's version of VAM [7] etc. Kirca and Satir [8] developed a heuristics to obtain efficient IBFS is called TOM. Bala Krishnan [9] Proposed a modified version of VAM for unbalanced Transportation problem. Sharma and Prasad [10] proposed heuristics gives significantly better solutions than the well know VAM. Mathirajan and Meenakshi [11] were extended. TOM using the VAM procedure.

1.2 Variant's of Vogel's approximation method and the allocation:

VAM is a heuristics and usually provides a better starting solution than other methods. Application of VAM to a give problem does not guarantee that an optimal solution will result. However, a very good solution is invariably obtained with comparatively little effort [12].

This method also takes cost into account in allocation five steps are involved in applying this heuristics

Step:1. Determine the difference between the lowest two cells in all rows and columns, including dummies.

Step:2. Identify the row or column with the largest difference ties may be broken arbitrarily.

Step:3. Allocate as much as possible to the lowest-cost cell in the row or column with the highest differences. If two or more differences are equal, allocate as much as possible to the lowest-cost cell in these rows or columns.

Step:4. Stop the process if all row and go to next step column requirement are met, if not

Step:5. Recalculate the differences between the two lowest cells remaining in all rows and columns. Any row and column with zero supply demand should not be used in calculating further differences than go to step-2

The VAM usually produces an optimal or near-optimal starting solution. One study found that VAM yields an optimum solution in 80% of the sample problems tested.

Example-1

To find the Initial basic feasible solution (IBFS) by VAM method

Table- 1

	D_1	D_2	D_3	D_4	D_5	Supply source
O_1	68	35	4	74	15	18
O_2	57	88	91	3	8	17
O_3	91	60	75	45	60	19
O_4	52	53	24	7	82	13
O_5	51	18	82	13	7	15
Required	16	18	20	14	14	82/82

Table-2 The Initial Solution obtained by VAM method

	D_1	D_2	D_3	D_4	D_5	Supply Source
O_1	68	35	4 18	74	15	18
O_2	57	88	91	3 3	8 14	17
O_3	91 16	60 3	75	45	60	19
O_4	52	53	24 2	7 11	82	13
O_5	51	18 15	82	13	7	15
Required	16	18	20	14	14	82/82

Table-3 Second Basic Feasible solution task of VAM

	D_1	D_2	D_3	D_4	D_5	Supply Source
O_1	68	35	4 18	74	15	18
O_2	57	88	91	3 3	8 14	17
O_3	91 5	60 14	75	45	60	19
O_4	52 11	53	24 2	7	82	13
O_5	51	18 4	82	13 11	7	15
Required	16	18	20	14	14	

This problem is not feasible since number of allocations are 8 where as m+n-1=5+5-1=9. Then the given solution is made feasible by allocating □ □ units to the least cost, vacant independent cell (O_5, D_5) and then, optimality test is performed. The IBFS took mine Iteration to get the solution while the optimal solution was achieved using transportation simplex algorithm within three iterations and the final cost was found to be Zmin=2,202. Solution of the proposed method (IVAM) for the same problem is illustrated in the following section.

A detailed literature review on the basic solution methods is not presented. All the optimal an initial basic feasible solution to obtain the optimal solution.

1.3 Variants of Improved Vogel's Approximation Method (IVAM)

Since the basic idea of TOM is extended along with the VAM procedure, ToM is first briefly discussed here. The ToM is an effective application of the "best cell method" along with the tie breaking features on the total opportunity (cost) matrix. The TOC matrix is obtained by adding the row opportunity cost matrix (row opportunity cost matrix for each row, the smallest cost of that row is subtracted from each element of the same row) and the "Column opportunity cost matrix" (Column opportunity cost matrix for each column of the original transportation cost matrix the smallest cost of that column is subtracted from each \element of the same column) [5]. Proposed algorithm is applied to the TOC matrix that considers highest three penalty cost and calculated alternative cost in the VAM procedure. It than selects the minimum among them. Detailed processes are given below:

Step: 1. Balance the given transportation problem if either (total supply > total demand) or (total < total demand).

Step: 2. Obtain ToC matrix

Step: 3. Determine the penalty cost for each row and column by subtracting the lowest cell cost in the row or column from the next lowest cell in the same row or column.
Step: 4. Select the rows or columns with the highest three penalty cost (breaking ties arbitrarily or crossing the lowest – cost cell)
Step: 5. Compute three transportation cost for selected three rows or columns in Step 4 by allocating as much as possible to the feasible cell with the lowest transportation cost.
Step: 6. Select minimum transportation cost of three allocations instep 5 (breaking this arbitrarily or choosing the lowest-cost cell)
Step: 7. Repeat steps 3-6 until all requirements have been meet.
Step: 8. Compute total transportation cost for the feasible allocation using original balanced – transportation cost matrix.

Remarks:-

The Algorithm will be improved if we add the following two additional steps for breaking ties.

1. If there is a tie in penalty or minimum transportation cost, choose the largest penalty for allocation.
2. If there is a tie in penalty and minimum transportation cost, then calculate their corresponding row opportunity cost value/column opportunity cost value, and select the one with maximum.

Table -4 Initial solution table by IVAM

	D_1	D_2	D_3	D_4	D_5	Supply Source
O_1	68	35	4 [18]	74	15	18
O_2	57 [3]	88	91	3	8 [14]	17
O_3	91 [2]	60 [13]	75	45 [4]	60	19
O_4	52 [11]	53	24 [2]	7	82	13
O_5	51	18 [5]	24	7 [10]	82	15
Required	16	18	20	14	14	82/82

For the transportation problem given in table – 1 the initial solution of VAM requires five additional stages to reach to the optional solution. The problem was resolved using IVAM and IBFS for this problem is given in table-4 initial solution of IVAM is the optimal solution of the given example problem without additional iterations, initial cost from the table 4 is 2, 247 and also is the optimal value of the considered problem.

1.4 Computational experiments:-

For evaluating the performance of the VAM and its variants and TOM, computational experiments were carried out. The experiments and the analysis of the experimental data are presented in this section. The main goal of the experiment was to evaluate the quality of the solution obtained by VAM and its variants and TOM by comparing them with optimal solution also, the iteration numbers to the optimal solution and computation times of VAM and improved version of IVAM

SIMULATION EXPERIMENTS:-

For evaluating the performance of the VAM and its variant IVAM, simulation experiments were carried out on a 2.13 GHz Intel Core 2 Duo machine with 4096 MB RAM.

1.5 Measure of effectiveness:-

The performances of VAM and IVAM are compared using the following measures: The transportation problems were randomly generated with twelve different sizes (row x column) : 5x5, 10x10, 10x20, 10x30, 10x40, 20x20, 10x60, 30x30, 10x100, 40x40, 50x50, and 100x100, respectively. The experimental design was implemented using ANSI C.

a. Average Iteration (AI): Mean of iteration numbers to obtain optimal solutions using the initial solutions of VAM and IVAM over various sized problem instances.
b. Number of best solutions (NBS): A frequency which indicates the number of instances VAM and IVAM yielded optimal solution with lower iteration over the total of problem instances. NBS does not contain case of equal iteration between VAM and IVAM.
c. Computation Time: The CPU time is represented by three variables: T_a, T_b and T_c. T_a is the time to reach initial solution. T_b is the time to reach optimal solution from initial solution and T_c is the total time from the beginning that is sum of T_a and T_b

1.6 Comparison of VAM and IVAM:-

The experiments and the analysis of the experimental data are presented in the section. For each problem instance, the heuristic solution were obtained using VAM and IVAM. The performance of the VAM and IVAM in comparison with the optimal solution is presented below. AI and other statistics for the iteration numbers of VAM and IVAM over various sized problems are given in Table 5.

Table 5. Statistical indicators for the iteration numbers of VAM and IVAM

Problem Size	AI		Standard Error		Median		Range	
	VAM	IVAM	VAM	IVAM	VAM	IVAM	VAM	IVAM
5X5	2.198	2.676	0.034	0.35	2	3	5	5
10X10	6.155	6.359	0.069	0.066	6	6	14	14
10X20	10.063	9.913	0.094	0.092	10	10	18	18
10X30	13.264	12.951	0.120	0.116	13	13	24	27
10X40	17.761	17.495	0.141	0.144	17	17	28	30
20X20	17.007	16.337	0.146	0.135	17	16	31	31
10X60	22.869	22.118	0.173	0.166	23	22	36	32
30X30	29.912	28.363	0.210	0.200	30	28	56	39
10X100	32.561	31.139	0.221	0.220	32	31	40	43
40X40	44.801	42.603	0.291	0.269	44	42	55	59
50X50	60.505	57.651	0.316	0.328	60	57	74	60
100X100	158.890	149.53	0.657	0.610	158	149	137	112

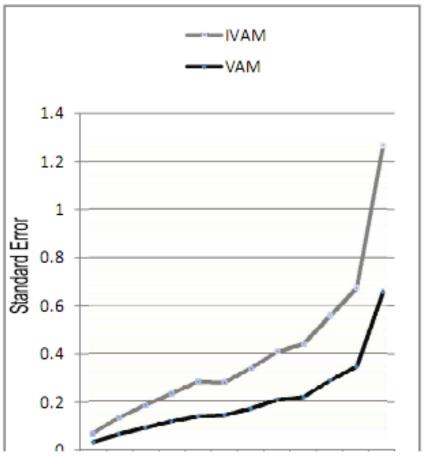

Fig 3- Graphical Representation of Standard Errora

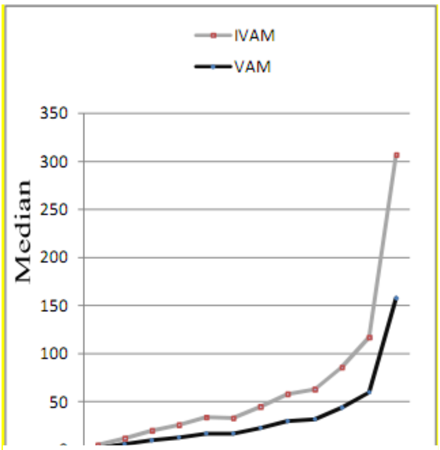

Fig 4- Graphical Representation of Median

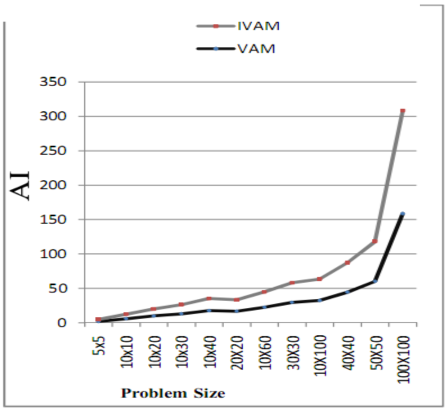

Fig 5- Graphical Representation of Average Iteration

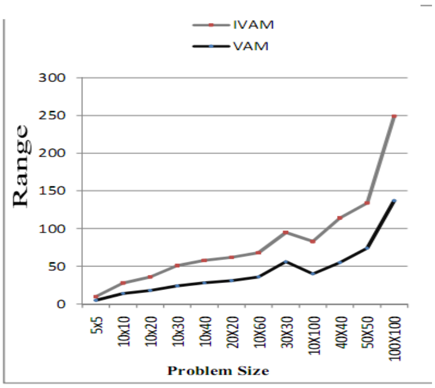

Fig. 6- Graphical Representation of Range

Both parametric and nonparametric statistical tests were performed using MINITAB-15 Statistical Package for comparing iteration numbers of VAM and IVAM. Firstly, Students T-test was used for testing the mean of differences of the iteration numbers between VAM and IVAM based on 1000 samples. Secondly Wilcoxon test was used for testing the median of differences of the iteration numbers between VAM and IVAM on the same samples. Table 6 gives a summary of the results of two-sided statistical tests.

Table6. Statistical tests for difference of iteration numbers between VAM and IVAM

Problem size	Studet's t-test				Wilcoxon text		
	Mean Standared Error	Confidence Interval%95	T	P	Estimated Median	Wilcoxon Statistic	P
5X5	-0.478±0.039	-0.555;-0.401	-12.190	0.000	-0.500	52276.00	0.000
10X10	-0.20±.074	-0.348;-0.060	-2.770	0.006	0.000	148409.00	0.008
10X20	0.150±0.12	-0051;0.351	1.470	0.143	0.000	20381.00	0.103
10X30	0.313±0.125	0.068;0.558	2.510	0.012	0.500	218149.500	0.022
10X40	0.226±0.159	-0.047;0.579	1.670	0.096	0.500	229297.500	0.104
20X20	0.670±0.157	0.362;0.978	4.270	0.000	0.500	245585.500	0.000
10X60	0.751±0.191	0.376;1.126	3.930	0.000	1.00	248683.500	0.000
30X30	1.549±0.229	1.099;1.999	6.760	0.000	1.500	281480.500	0.000
10X100	1.422±0.249	0.934;1.910	5.720	0.000	1.500	276839.00	0.000
40X40	2.198±0.313	1.584;2.812	7.030	0.000	2.500	290830.500	0.000
50X50	2.85±0.375	2.118;3.590	7.600	0.000	3.000	293811.500	0.000
100X100	9.360±0.725	7.937;10783	12.900	0.000	9.000	349416.00	0.000

It is seen from Table 6 that, VAM has better results at the 0.01 significance level for the cases 5x5 and 10x10 problem sizes regarding both mean and median test. There is no difference at the 0.05 significance level between VAM and IVAM in the cases 10x20 and 10x30 problem sizes, regarding both mean and median test. On the other hand, in the rest of all the cases 10x10, 20x20, 10x60, 30x30, 10x100, 40x40, 50x50, and 100x100, both Student's t-test and Wilcoxon test show the same result the method of IVAM is statistically significantly different from the method of VAM.

d. Performance Measure- NBS: VAM and IVAM yield optimal solutions with different iteration numbers for different sized 1000 problem instances. These values are given in Table 7. From Table 7, it is clear that the NBS of VAM and IVAM significantly vary for different sized problems. Graphical representation of these values is shown in Figure.

Table 7. Number of best solutions

Matrix size	NBS	
Mxn	VAM	IVAM
5x5	460	183
10x10	450	365
10x20	420	455
10x30	413	482
10x40	430	499
20x20	414	511

10x60	406	523
30x30	395	557
10x100	394	564
40x40	382	578
50x50	395	587
100x100	321	664

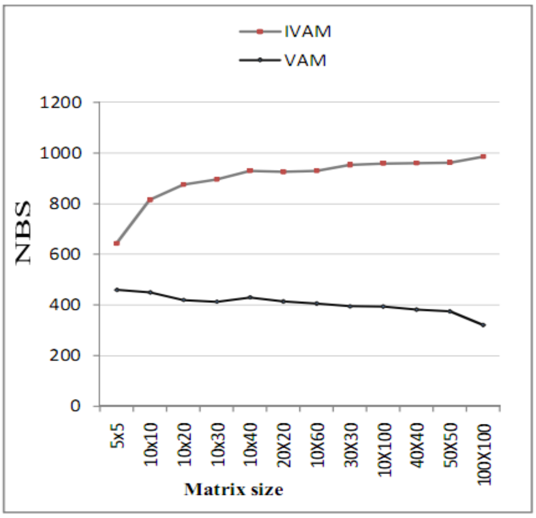

Fig. 7 Graphical Representation of Number of best solution

NBS does not include case of equal iteration between VAM and IVAM.

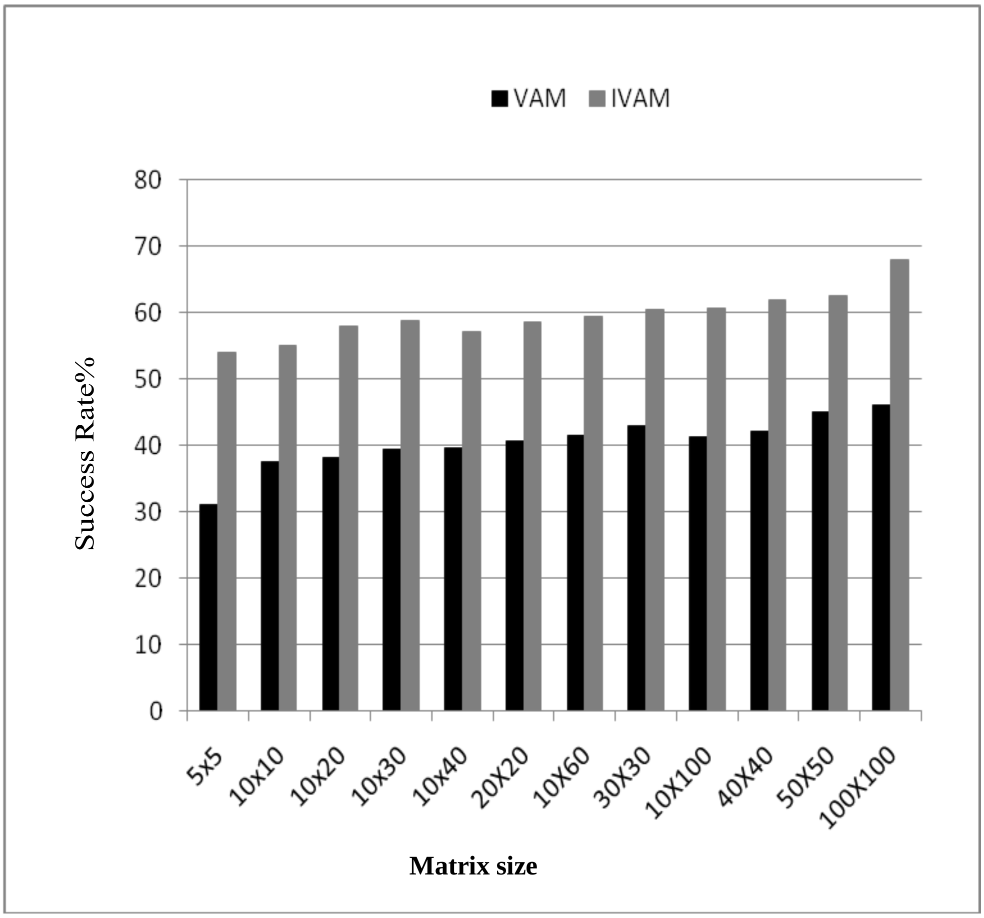

Fig. 8 Graphical Representation of Success rate

Performance measure-CPU time: T_a, T_b and T_c times for VAM and IVAM over various sized problem instances are given in Table 8. For each different sized problem; mean, standard error, coefficient of variation and range of times are calculated based on 1000 samples.

Table 8 T_a, T_b and T_c times for VAM and IVAM

Problem Size	Time	Mean ± Standard Error		Coefficient of Variation		Range	
		VAM	IVAM	VAM	IVAM	VAM	IVAM
5X5	Ta	0.23±0.005	1.335±0.003	20.060	6.720	0.969	2.456
	Tb	0.490±0.004	0.532±0.023	23.500	140.460	1.507	23.811
	Tc	1.413±0.007	1.867±0.024	15.960	41.230	1.795	24.410
10X10	Ta	3.914±0.007	4.395±0.255	5.83	183.730	1.004	255.404
	Tb	1.591±0.021	1.599±0.012	41.65	23.510	14.416	3.447
	Tc	5.505±0.022	5.995±0.256	13.05	134.910	12.805	255.960
10X20	Ta	8.188±0.017	9.115±0.059	6.85	20.540	13.777	47.398
	Tb	5.421±0.368	4.853±0.054	214.84	35.390	264.654	21.039
	Tc	13.610±0.371	13.969±0.090	86.26	20.450	267.549	68.193
10X30	Ta	14.427±0.023	14.700±0.046	5.15	9.860	16.248	22.308
	Tb	9.646±0.0778	9.5091±0.082	25.54	27.540	24.748	35.524
	Tc	24.074±0.815	24.209±0.094	10.71	12.270	26.586	35.934
10X40	Ta	23.599±0.027	23.908±0.046	3.46	6.110	16.223	17.899
	Tb	23.198±0.167	22.872±0.175	22.74	24.160	38.456	47.612
	Tc	46.798±0.170	46.780±0.185	11.49	12.480	47.656	48.769
20X20	Ta	20.604±0.618	20.790±0.049	2.34	7.460	12.533	24.137
	Tb	20.393±0.194	19.415±0.178	30.10	28.920	42.207	50.310
	Tc	41.997±0.196	40.205±0.185	15.11	14.540	44.453	50.462
10X60	Ta	37.521±0.272	37.134±0.044	2.290	3.750	20.643	15.402
	Tb	50.587±0.361	49.185±0.345	22.570	22.210	74.967	66.375
	Tc	88.108±0.363	86.319±0.347	13.020	12.730	75.154	66.355
30X30	Ta	64.438±0.098	60.469±0.147	4.830	7.690	51.111	73.508
	Tb	126.01±0.900	119.450±	22.830	21.860	271.920	165.460
	Tc	190.45±0.917	179.920±0.858	15.230	26.080	275.680	224.840
10X100	Ta	82.871±0.122	75.701±0.144	4.670	6.000	72.1770	49.895
	Tb	182.6301±230	175.180±1.720	21.290	22.080	275.440	243.270
	Tc	265.500±1.250	250.880±1.250	14.870	15.690	278.990	256.180
40X40	Ta	156.980±0.138	143.22±0.341	2.780	7.52	70.390	273.43
	Tb	552.460±3.560	526.340±3.290	20.400	19.76	678.010	717.07
	Tc	709.403±3.570	669.550±3.310	15.930	15.63	688.250	732.43
50X50	Ta	321.800±0.040	276.580±0.075	0.390	0.860	20.630	41.410
	Tb	1662.900±9.440	1585.700±8.960	17.950	17.870	2029.700	1622.700
	Tc	1984.700±9.440	1862.300±8.960	15.094	15.220	2029.300	1620.800
100X100	Ta	4737.7±1.440	2625.400±0.646	0.950	0.780	799.4	278.400
	Tb	5181±214.000	48760±199	13.060	12.900	44226	37299
	Tc	56549±214.000	51386±199	11.960	12.240	44226	37332

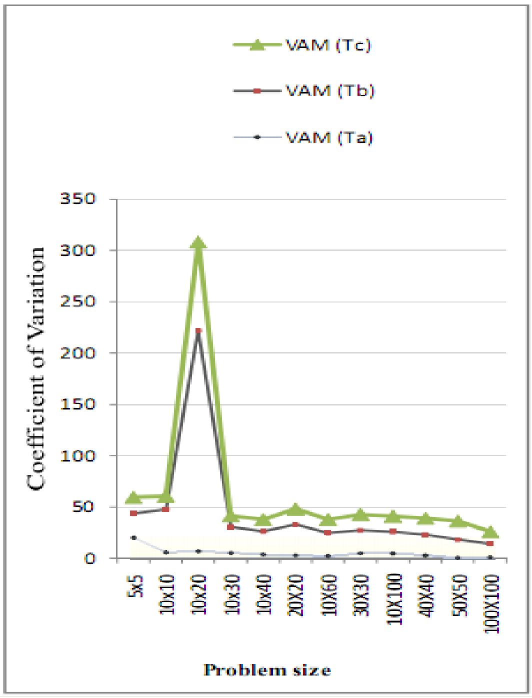

Fig. 9 Graphical Representation of Coefficient of Variation of VAM in (T_a, T_b, T_c) Time

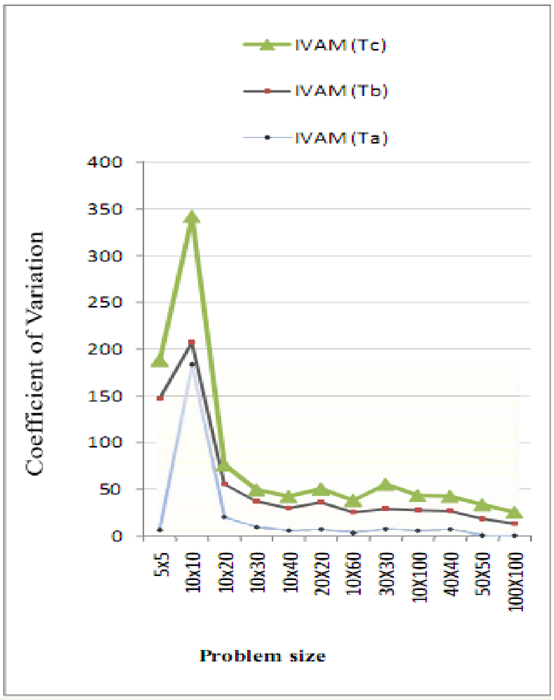

Fig. 10 Graphical Representation of Coefficient of Variation of IVAM in (T_a, T_b, T_c) Time

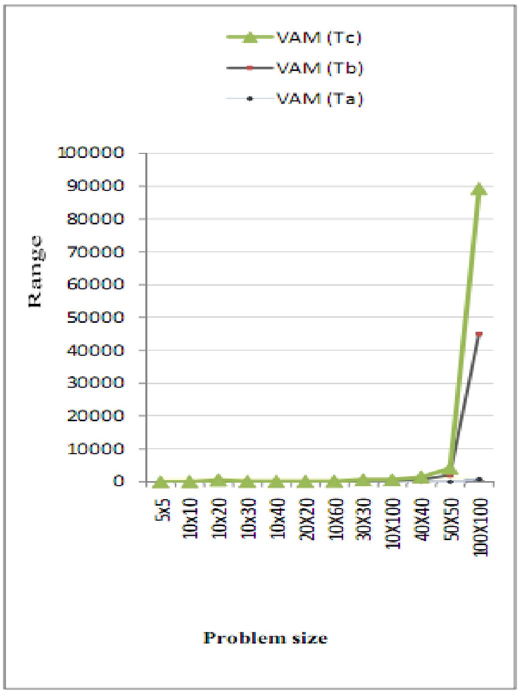

Fig 11- Graphical Representation of VAM in ($T_{a,}$ T_b T_c)

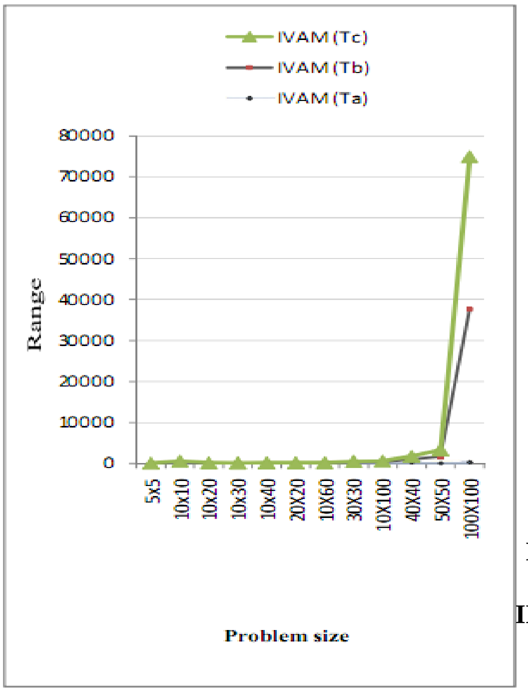

Fig 12- Graphical Representation of IVAM in ($T_{a,}$ T_b T_c)

II.

III.

IV. CONCLUSION

Vogel's approximation method (Penalty or regret method) is a heuristics method and is preferred always because it gives an initial solution which is wearer to an optimal solution or is the optimal solution itself. VAM was improved by using total opportunity cost and regarding alternative allocation costs. In analysis we found that the quality of solution in Basic version of VAM coupled with TOC yields a very efficient initial solution. Simulation experiments showed that VAM gets efficient initial solution for small sized transportation problems but it is insufficient for large sized transportation problem. IVAM conspicuously obtains more efficient initial solutions for large scale transportation times and computational difficulty for the optimal solution. A comparative study with the help of the graph is shown through various charts.

REFERENCES

[1] H. Wagner, *Principles of Operations Research, New Jersey: Prentice0Hall, Englewood Cliffs, 1969.*
[2] H.A. Taha, *Operations Research: An Introduction,* New York, : Macmillan Publishing Company, 1987.
[3] G.B. Dantzi, *Linear Pro*gramming *and Extensions,* New, Jersey: Princeton University Press, 1963.
[4] J.E. Reeb and S. Leavengood, Transportation problem: A Special case for linear programming, Oregon: Oregon State University Extension Service Publications EM 8779, 2002.
[5] D.G. Shimshank, J.A. Kaslik and T.D. Barclay, A modification of Vogel's approximation method through the use of heuristics, *infor* 19, 259-263, 1981,
[6] S.K. Goyal, Improving VAM for unbalanced transportation problems, *Journal of Operational Research Society* 35 (12), 1113-1114, 1984.
[7] G.S. Ramakrishnan, An improvement to Goyal's modified VAM for the unbalanced transportation problem, *Journal of Operational Research Society* 39(6), 609-610-, 1988.
[8] O. Kirca and A. Satir, A heuristic for obtaining an initial solution for the transportation problem, *Journal of Operational Research Society* 41(9), 865-871, 1990.
[9] N. Balakrishnan, Modified Vogel's Approximation method for unbalanced transportation problem. *Applied Mathematics Letters* 3 (2), 9-11, 1990
[10] R.R.K. Sharma and S. Prasad, Obtaining a good primal solution to the incapacitated transportation problem, *European Journal of Operational Research* 144, 560-564, 2003
[11] M. Mathirajan and B. Meenakshi, Experimental Analysis of some Variants of Vogel's Approximation Method, *Asia-Pacific Journal of Operational Research* 21(4), 447-462, 2004.
[12] H.H. Shore, The Transportation Problem and the Vogel Approximation Method, *Decision Sciences* 1 (3-4), 441-457, 1970.