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Finite Element Analysis of One Dimensional Bio-Heat Transfer in Human Tissue

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Abstract: - Different therapeutic treatment requires precise monitoring of bio-heat transfer of human tissue for preserving the healthy tissues near the skin surface, from burning or freezing during therapeutic application. Bio-heat transfer analysis of thermal medical problems usually has to simultaneously face the transient or spatial heating both on skin surface and in interior of the biological bodies. It is difficult to perform heat transfer analysis on complex shape of biological body by analytical solution. In this research Finite Element Method was used to analyze 1D bio-heat transfer in human tissue. Based upon the Finite Element Method a generalized computer program was developed. Solution obtained by using Finite Element Analysis was compared with analytical solution. Here it was shown that computer based approximate solution is best option to perform heat transfer analysis of complex shape.

***Index Terms*: -** *Analytical Solution, Bio-heat Transfer, Numerical Solution, Pennes Equation.*

I. INTRODUCTION

Bio-heat transfer is the study of heat transfer in biological systems [1]. It is the study of how heat moves within the body or external to the body. The thermal therapies are based on the heat transfer in biological tissues [2]. The purpose of therapeutic application on biological body is either raising or lowering of temperature at various points in human tissue. Heat transfer is fundamental and very important process in living tissues in order to maintain an almost constant temperature. In practical field, it is difficult to evaluate accurately the thermal response of the biological tissues during therapeutic applications, because of the complex mechanisms that maintain body temperature such as- blood flow and metabolic heat generation. Therefore, it is very important to provide therapist with useful data concerning the thermal analysis of biological tissues. For example, in thermal diagnostics [3] and thermal comfort analysis [4–6], thermal parameter estimation [7–12], or burn injury evaluation [13-14], complex heating was encountered. By using Finite Element Analysis (FEA) we obtained temperature at various points in human tissue during therapeutic applications. Heat transfer was calculated by using the temperature at different nodal points. A generalized computer program is developed by using Finite Element Method for the analysis of one dimensional (1D) heat transfer in Human Tissue. The solution provided by the developed model is compared with analytical solution for validation.

II. DESCRIPTION OF THE MODEL

Pennes equation [15] is widely used for the analysis of heat transfer in living tissue, which describes the influence of blood flow on the temperature distribution in the tissue in terms of volumetrically distributed heat sinks or sources. The generalized 1D Pennes equation [15] can be written as

$$\rho c\frac{\partial T}{\partial t} = K\frac{d^2T_0(x)}{dx^2} + \omega_b\ \rho_b C_b[T_a - T_0(x)] + \ Q_m \quad (1)$$

Where σ, c, k are respectively the density, the specific heat, and the thermal conductivity of the tissue; their unit are respectively kg/m^3, J/kg°C, W/m°C; ρ_b, C_b denote density and specific heat of blood; ω_b the blood perfusion and unit is ml/s/ml;T_a arterial temperature which is treated as constant and T the tissue temperature in °C; Q_m is the metabolic heat generation and unit is W/m^3. "$\rho c\frac{\partial T}{\partial t}$" is heat store by tissue, which is zero for steady-state condition."$K\frac{d^2T_0(x)}{dx^2}$" is heat loss by conduction and $\omega_b\ \rho_b C_b[T_a - T_0(x)]$ is heat loss by blood perfusion. "Q_m" is metabolic heat generation. The summation of heat loses is equal to metabolic heat generation for steady-state condition, where work done is negligible.

The initial steady-state temperature field for biological bodies can be obtained through solving the following equations:

$$\begin{cases} K\dfrac{d^2T_0(x)}{dx^2} + \omega_b \ \rho_b C_b \ [T_a - T_0(x)] + Q_m = 0 \\ T_0(x) = T_c \quad , \quad x = L \\ -K\dfrac{dT_0(x)}{dx} = ho[T_f - T_0(x)] \ , \ x = 0 \end{cases}$$

Where, $T(x.0) = T_0(x)$ is steady-state temperature fields prior to heating, T_c the body core temperature and often regarded as a constant, h_0 the apparent physiological heat convection coefficient between the skin surface and surrounding air and is an overall contribution from natural convection and radiation, and T_f the surrounding air temperature. The forced convection coefficient is applied as h_f =100W/m^2°C, while the surrounding air temperature was chosen as T_f = 25°C [3].

Here skin surface is defined at x=0 while the body core at x=L. Previous research works [12, 16] indicate that, the interior tissue temperature usually remain a constant within a short distance such 2-3 cm from skin surface, therefore we use the length of domain L=3 cm in our research. For our analysis we made some estimation regarding tissue properties [9] which shown in table 1.

Table 1: Typical tissue properties

Tissue properties	Symbol	Value	Unit
Thermal conductivity of tissue	K	0.5	W/m°C
Heat convection coefficient between skin & surrounding	h_0	10	W/m^2°C
Force convection co-efficient	h_f	100	W/m^2°C
Surrounding air temperature	T_f	25	°C
The arterial temperature	T_a	37	°C
Body core temperature	T_c	37	°C
Metabolic heat generation	Q_m	33800	W/m^3
Specific heat of tissue	c	4200	J/kg °C
Specific heat of blood	c_b	4200	J/kg °C
Density of tissue	ρ	1000	kg/m^3
Density of blood	ρ_b	1000	kg/m^3
The Blood perfusion	ω_b	0.0005	ml/s/ml

III. TISSUE TEMPERATURE CALCULATION

Pennes [15] equation of 1D *Bio-heat* transfer in human tissue was solved using Finite Element Method by providing necessary assumption. The steps are described below.

A. Boundary Value Problem

Pennes [15] equation is reformed as a boundary value problem.

$$\mathrm{K}\frac{\mathrm{d}^2\mathrm{T}_0(\mathrm{x})}{\mathrm{dx}^2} + \omega_b \ \rho_b \mathrm{C_b} \ [\mathrm{T}_a - \mathrm{T}_0(\mathrm{x})] + \mathrm{Q_m} = 0$$

$$\text{Or, } -\frac{\mathrm{d}}{\mathrm{dx}}(\mathrm{k}\frac{\mathrm{dT_0(x)}}{\mathrm{dx}} - \omega_\mathrm{b}\rho_\mathrm{b}\mathrm{C_b}[\mathrm{T}_a - \mathrm{T}_0(\mathrm{x})] - \mathrm{Q_m} = 0$$

$$\text{Or, } -\frac{\mathrm{d}}{\mathrm{dx}}\left(\mathrm{k}\frac{\mathrm{dT_0(x)}}{\mathrm{dx}}\right) + \omega_\mathrm{b}\rho_\mathrm{b}\mathrm{C_b}\mathrm{T}_0\,(\mathrm{x}) - (\omega_\mathrm{b}\rho_\mathrm{b}\mathrm{C_b}\mathrm{T}_a + \mathrm{Q_m}) = 0$$

Thus, the above equation can be rewritten as-

$$-\frac{\mathrm{d}}{\mathrm{dx}}\left(\mathrm{k}\frac{\mathrm{dT_0(x)}}{\mathrm{dx}}\right) + \mathrm{C}\,\mathrm{T}_0(\mathrm{x}) - \mathrm{q} = 0 \qquad (2)$$

Where, $\mathrm{C} = \omega_\mathrm{b}\rho_\mathrm{b}\mathrm{C_b}$, $\mathrm{q} = \mathrm{C}\,\mathrm{T}_a + \mathrm{Q_m}$

The boundary conditions of 1D *Bio-heat* transfer in human tissue are

1. $\mathrm{T}_0(\mathrm{x}) = \mathrm{T_c}$, Where x = L and 2. $-\mathrm{k}\frac{\mathrm{dT_0(x)}}{\mathrm{dx}} = \mathrm{h}_0[\mathrm{T_f} - \mathrm{T}_0(\mathrm{x})]$, where x= 0

B. Mesh Generation

Mesh is collection of elements [17]. To determine the precise monitoring of heat transfer in tissue we divide the 1D mesh into five elements between x = 0 and x = L as shown in figure 1.

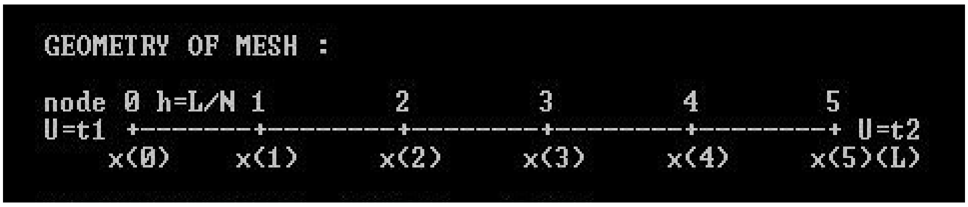

Figure 1: One-Dimensional (1D) Mesh

C. Derivation of Element Equations

Weak Formation

The weak from of boundary value problem is constructed by multiplying the equation (2) with weighted function w.

$$0 = \int_0^{h_e} w\left(-\left[\frac{d}{dx}\left(k\frac{dT_0(x)}{dx}\right)\right] + cT_0(x) - q\right)dx \qquad (3)$$

$$0 = \int_0^{h_e}\left(-\left[wk\frac{d^2T_0(x)}{dx^2}\right] + cT_0(x)w - wq\right)dx \qquad (4)$$

The integration limit, h_e represents the length of element. The equation (3) and equation (4) are simplified by using the method of integration by part.

$$0 = \int_0^{h_e}\left(k\frac{dw}{dx}\frac{dT_0(x)}{dx} + cw\,T_0(x) - wq\right)dx - \left[\left[wk\frac{du}{dx}\right]_0^{h_e}\right]$$

The weak form contains two types of expressions: those containing both w and $T_o(x)$ called bi-linear form, can be expressed in a single term,

$$B\big(w, T_0(x)\big) = \int_0^{h_e}\left(k\frac{dw}{dx}\frac{dT_0(x)}{dx} + cw\,T_0(x)\right)dx$$

Also those containing only w (but not $T_0(x)$) is denoted by l (w), called the linear form.

$$1(w) = \int_0^{h_e} wq\,dx + w(X_A)Q_A - w(X_B)Q_B$$

Then variation statement can be expressed as-

$$B[w, T_0(x)] = 1(w)$$

$$[K_{ij}(u_j = B\big(w_i, \textstyle\sum_{j=0}^{n} u_j w_i\big) = \textstyle\sum_{j=0}^{n} u_j \int_0^{h_e}\left(k\frac{dw_i}{dx}\frac{w_j}{dx} + cw_i\right)dx \quad \text{where, } u_j = T_0(x)$$

$$\left[K_{ij}\right] = \int_0^{h_e}\left(k\frac{dw_i}{dx}\frac{w_j}{dx} + cw_i\right)dx \qquad (5)$$

For linear element values of weight function (w) was found by replacing the element interpolation functions, Ψ_j

$$w_1 = \left(1 - \frac{x}{h_e}\right) \text{ for node 1 and } w_2 = \frac{x}{h_e} \text{ for node 2.}$$

D. Generation of Element Matrix

For getting the element matrix, value of w_i was putted in equation (5) and then by integration element matrix was found.

$$\left[K_{ij}\right] = \begin{bmatrix} 87.53 & -81.23 \\ -81.23 & 87.53 \end{bmatrix}$$

$$\{F_i\} = l(w_i) = \int_0^{h_e} w_i q\,dx + w_1Q_1 - w_2Q_2$$

Finally the Matrix form of the element equation from equation (5), can be expressed as-

$$[K]\{u\} = \{F\} \qquad (6)$$

E. Computer Program Generation

Element matrix [K] and {F} was imported to equation (6), to start program generation. The basic steps involved for computer program generation are given below.

Step 1: Global Matrix Formation

Element matrix K_{ij} was developed for a 2 node element. To approximate N element mesh the N^{th} element matrix need to sum. This summation only occurs in node point. By developing generalize C program, the summation for

N element mesh was obtained. After summing element matrix global matrix [G] was found for 5 elements. By putting the global matrix in (eq. 6) following output was found.

$$\begin{vmatrix} 87.53 & -81.23 & 0 & 0 & 0 & 0 \\ -81.23 & 175.07 & -81.23 & 0 & 0 & 0 \\ 0 & -81.23 & 175.07 & -81.23 & 0 & 0 \\ 0 & 0 & -81.23 & 175.07 & -81.23 & 0 \\ 0 & 0 & 0 & -81.23 & 175.07 & -81.23 \\ 0 & 0 & 0 & 0 & -81.23 & 87.53 \end{vmatrix} \mathrm{X} \begin{vmatrix} T_o \\ T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{vmatrix} = \begin{vmatrix} 334.5+Q_0 \\ 669.00 \\ 669.00 \\ 669.00 \\ 669.00 \\ 334.5+Q_5 \end{vmatrix}$$

From boundary condition
at x=L (eq. 2), at $T_5=37$
at x=0 (eq. 2), at $Q_0 = h_0[T_f - T_0] = h_0 T_f - h_0 T_0 = 250-10T_0$
By putting the boundary condition at equation (6) following output was found.

$$\begin{vmatrix} 97.53 & -81.23 & 0 & 0 & 0 & 0 \\ -81.23 & 175.07 & -81.23 & 0 & 0 & 0 \\ 0 & -81.23 & 175.07 & -81.23 & 0 & 0 \\ 0 & 0 & -81.23 & 175.07 & -81.23 & 0 \\ 0 & 0 & 0 & -81.23 & 175.07 & -81.23 \\ 0 & 0 & 0 & 0 & -81.23 & 87.53 \end{vmatrix} \mathrm{X} \begin{vmatrix} T_o \\ T_1 \\ T_2 \\ T_3 \\ T_4 \\ 37 \end{vmatrix} = \begin{vmatrix} 334.5+250 \\ 669.00 \\ 669.00 \\ 669.00 \\ 669.00 \\ 334.5+Q_5 \end{vmatrix}$$

Step 2: Gauss Elimination

An algorithm in C++ format was developed to determine, the tissue temperature at different nodal point based upon the gauss elimination of numerical solution. The result of gauss elimination is shown below.

$$\begin{vmatrix} 1 & -0.83 & 0 & 0 & 0 & 0 \\ 0 & 1 & -0.76 & 0 & 0 & 0 \\ 0 & 0 & 1 & -0.71 & 0 & 0 \\ 0 & 0 & 0 & 1 & -0.69 & 0 \\ 0 & 0 & 0 & 0 & 1 & -0.68 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} \mathrm{X} \begin{vmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \\ T_4 \\ 37 \end{vmatrix} = \begin{vmatrix} 5.99 \\ 10.75 \\ 13.58 \\ 15.15 \\ 16.01 \\ 51.21 + Q_5/31.92 \end{vmatrix}$$

Step 3: Finite Element Analysis

The value of temperature at various nodes as shown in table 1 was calculated by using the developed software and the results are shown in table 2.

Table 2: Temperature at corresponding co-ordinate of the nodal point for five elements

U_i	Node	Temperature(^{o}C)
T_0	1	43.26
T_1	2	44.74
T_2	3	44.93
T_3	4	43.85
T_4	5	41.34
T_5	6	37

IV. ANALYTICAL SOLUTION

The analytical solution is performed based on [18] and shown in equation 7.

$$T_0(x) = T_a + \frac{Q_m}{\omega_b \rho_b C_b} + \frac{\left(T_c - T_a - \frac{Q_m}{\omega_b \rho_b C_b}\right)[\sqrt{A}\cos h(\sqrt{A}x) + \frac{h_0}{k}\sin h(\sqrt{A}x)]}{\sqrt{A}\cos h(\sqrt{A}L) + \frac{h_0}{k}\sin h(\sqrt{A}L)} + \frac{\frac{h_0}{k}\left(T_f - T_a - \frac{Q_m}{\omega_b \rho_b C_b}\right)\sin h(\sqrt{A}(L-x))}{\sqrt{A}ch(\sqrt{A}L) + \frac{h_0}{k}\sin h(\sqrt{A}L)} \quad (7)$$

Where, $A = \frac{\omega_b \rho_b C_b}{k}$

By putting the value of tissue properties in equation (7) temperature at corresponding coordinate of the nodal point for five elements are shown in table 3.

Table3: The temperature at various co-ordinates for Analytical Solution

U_i	Co-ordinate (m)	Temperature (°C)
T_0	0	43.19
T_1	0.0006	44.67
T_2	0.0012	44.86
T_3	0.0018	43.79
T_4	0.0024	41.3
T_5	0.003	37

V. RESULTS AND DISCUSSION

In this study three cases are taken for showing the significance of Finite Element Analysis and the effects of element number on finite element analysis. In case 1, two elements are taken. In case 2, five elements are taken. And case 3 twenty elements are taken, as shown in figure2, 3 and 4 respectively.

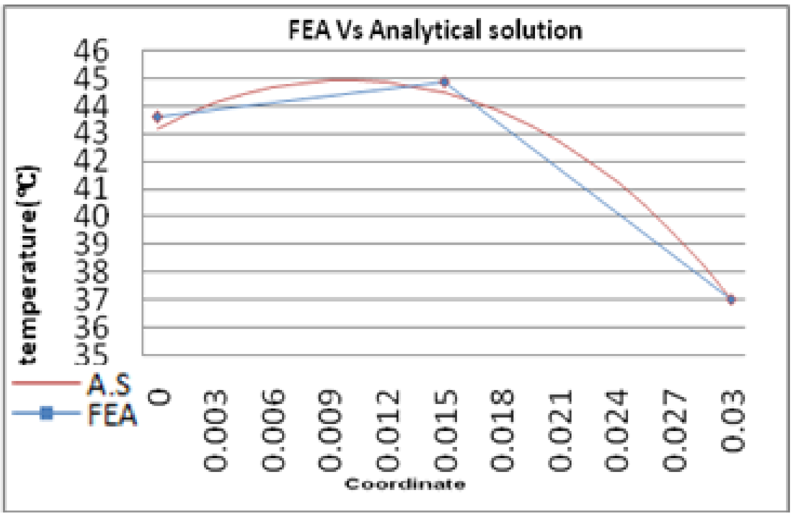

Figure 2: Temperature distribution for FEA in comparison with Analytical solution for case 1.

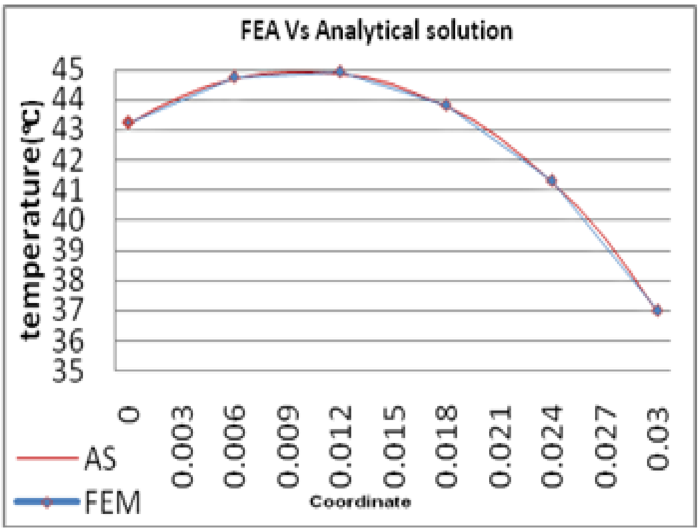

Figure 3: Temperature distribution for FEA in comparison with Analytical solution for case 2.

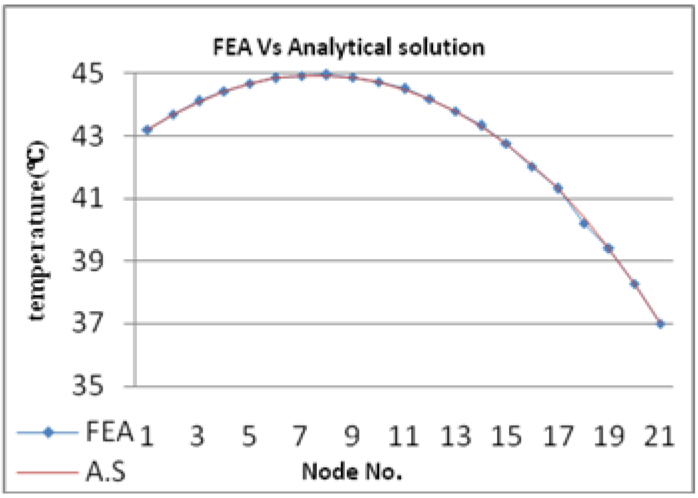

Figure 4: Temperature distribution for FEA in comparison with Analytical solution for case 3.

Figure 2, 3 and 4 shows that the precision of the result for two elements is less than the result for five elements. Also the accuracy of the result for five elements is less than the result for twenty elements. The perfection of the result of Finite Element Analysis increases with the increases of element number. By using temperature at different nodal point heat flux also can be calculated for various nodes as shown in table 4.

Table 4: The Heat Flux at various nodes for five elements

Node number	Co-ordinate (m)	Heat flux (W/m^2)
1	0	-182.56
2	0.0006	-123.68
3	0.0012	-15.68
4	0.0018	89.89
5	0.0024	209.4
6	0.003	361.38

VI. CONCLUSION

In the therapeutic treatment the nature of Bio-heat transfer is required to protect transient healthy tissue. It is difficult to find out analytical solution of complex shape like human tissue. In such case finite element method is appropriate to determine the *Bio-heat* transfer in human tissue. In this study a brief introduction to finite element method and finite element formulation of heat transfer in human tissue has been described. To demonstrate the situation three cases are taken for same length domain. In the case 1 two elements, case 2 five elements, case 3 twenty are taken. Result of case 3 is almost exact which validates the generated computer program. Similar procedure can be employed for the FEM formulation of other problems.

REFERENCES

[1] Website: http://en.wikipedia.org/wiki/Bioheat_transfer (last modified on 9 April 2011 at 03:33, Retrieved: July,24,2011)

[2] F. Florin, B. I. Andrei, C. M. Iulia, **"**Computer-aided analysis of the heat transfer in skin tissue", 3rd WSEAS Int. Conference on finite differences - finite elements - finite volumes - boundary elements, Vol. 53, pp.53-59.

[3] J. Liu, L.X. Xu, ''Boundary Information Based Diagnostics on the Thermal States of Biological Bodies,'' Int. J. Heat Mass Transf., Vol. 43, pp. 2827–2839, 2000.

[4] N.L. Markee, K.L. Hatch, H.I. Maibach, R.L. Barker, P. Radhakrishnaiah, S.S. Woo, ''In Vivo Cutaneous and Perceived Comfort Response to Fabric. Part IV. Perceived Sensations to Three Experimental Garments Worn by Subjects Exercising in a Hot, Humid Environment,'' Textile Research Journal, Vol.60, pp. 561–568, 1990.

[5] S.D. Burch, S. Ramadhyani, J.T. Pearson,''Analysis of Passenger Thermal Comfort in an Automobile under Severe Winter Conditions,'' ASHRAE Trans., pt. 1, ASHRAE Winter Meeting, Atlanta, pp. 247–257, 1991.

[6] E. Arens, P. Bosselmann, ''Wind, Sun and Temperature. Predicting the Thermal Comfort of People in Outdoor Spaces,'' Building and Environ., Vol.24, pp. 315–320, 1989.

[7] J.C. Chato, ''Measurement of Thermal Properties of Growing Tumors,'' Ann. N.Y. Acad. Sci., Vol.335, pp. 67–85, 1980.

[8] M.M. Chen, K.R. Holmes, V. Rupinskas,''Pulse-Decay Method for Measuring the Thermal Conductivity of Living Tissues,'' ASME J. Bio-mech. Eng., Vol.103, pp. 253–260, 1981.

[9] A.T. Patera, B.B. Mikic, G. Eden, H.F. Bowman, ''Prediction of Tissue Perfusion from Measurement of the Phase Shift Between Heat Flux and Temperature,'' Winter Annual Meeting of ASME, Advances in Bioengineering, pp. 187–191, 1979.

[10] P.A. Patel, J.W. Valvano, J.A. Pearce, S.A. Prahl, C.R. Denham, ''A Self-Heated Thermistor Technique to Measure Effective Thermal Properties from the Tissue Surface,'' ASME J. Biomech. Eng., Vol.109, pp. 300–315,1987.

[11] G.T. Anderson, J.W. Valvano, R.R. Santos, ''Self-heated Thermistor Measurements of Perfusion,'' IEEE Trans. Biomed. Eng., Vol.39, pp. 877–885,1992.

[12] J. Liu, L.X. Xu, ''Estimation of Blood Perfusion Using Phase Shift in Temperature Response to Sinusoidal Heating at the Skin Surface,'' IEEE Trans. Biomed. Eng., Vol.46, pp. 1037–1043, 1999.

[13] D.A. Torvi, J.D. Dale, ''A Finite Element Model of Skin Subjected to a Flash Fire,'' ASME J. Biomech. Eng., Vol. 116, pp. 250–255, 1994.

[14] K.R. Diller, ''Modeling of Bioheat Transfer Processes at High and Low Temperatures,'' Adv. Heat Transfer, Vol. 22, pp. 157–357, 1992.

[15] H.H. Pennes, "Analysis of Tissue and Arterial Blood Temperatures in the Resting Human Forearm", J. Appl. Physiol., Vol. 1, pp. 93–122, 1948.

[16] S. Weinbaum, L.M. Jiji, D.E. Lemons, ''Theory and Experiment for the Effect of Vascular Microstructure on Surface Tissue Heat Transfer-Part I: Anatomical Foundation and Model Conceptualization,'' ASME J. Biomech. Eng., Vol. 106, pp. 321–330,1984.

[17] K.R. Holmes, K. R., "Biological Structures and Heat Transfer", Allerton Workshop on the Future of Biothermal Engineering, 1997.

[18] D. Zhong-Shan, L. Jing, "Analytical study on bio-heat transfer problems with spatial or transient heating on skin surface or inside biological bodies". Journal of Biomechanical Engineering, [DOI: 10.1115/1.1516810], 2002.

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