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Double-Sampling Control Charts for Attributes

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Abstract: *Atypical controlchartis a time-graphofa sequence of valuesof a givenstatistic.It is used tostatisticallycontrol a process and distinguishes betweencausesof variationpresentedover time;the variability due tocommon causes (which may be described by random models) or thevariability due tospecial causes (which maybe explained in terms ofanoperating parameterof the process).This work presentsthe use ofa double-sampling control chartforattributesin processes withlowproductionvolume.*

Keywords: Attributes, Control charts, Double-Sampling.

I. Introduction

During the fabrication of a product there areprocessesinvolved that have to be monitored to prevent or eliminate failures that create products outside the expected specifications of the client. A control chart or graph is used to monitor a process as well as detect when there is a failure,ensuring that it can be rectified in atimely fashion.

A process is a group of interlocking activities that transform determined input (entries) in a final result (exit) or product. A process can be comprised of several stages while inputsare, for example, machinery, equipment, raw material and labor(Gutierrez Pulido & de la Vara Salazar, 2009).

A finished product must meet certain specifications.If this fails to occur then the product is declared “defective” and must be reworked or discarded. This incurs cost for manufacturing companies; therefore, it is vital that the number of defects are prevented or kept to a minimum.

Defects occur due to the variations that exist during the processes; for example, raw material is not homogenous, machinery and equipment wear out with use, etc. Usually, the variation is divided in two types; 1) the natural variability or that due to common causes and 2) the variability due to special causes. The first type of variability surfaces when we do not know the process thus it can not be attributed to a specific cause and is considered acceptable, that is, a process operating under this type of variability alone should not cause problems. On the contrary, the variability due to special causes can be assigned to a particular cause, for example, the wear or fracture of a cutting tool, an abnormality in the raw material, etc. A process operating under this type of variation can generate largequantities of defective products.

A control chart (CC) is designed to detect when the variability due to special causes is present. Basically, a CC is a graphical representation of traditional hypotheses that are carried out in sequence respect to time concerning the status of the process; for that purpose, in instants of time,pieces from the process that must conform to a random sampleare extracted. This sample, also called sub-group, is inspected carrying out a certain measurement or observation in each piece. The result of the inspection depends on the quality characteristic being monitored. For example, during a screw manufacturing process perhaps the length of the screws is what requires monitoring, that is, in this case, we are talking about a continuous variable. Or, perhaps the variable of interest determines if the screw “passes” or “fails” through a certain mechanism; so, in this case we are talking about a discrete variable. Consequently, there are control charts for both continuous and discrete variables. It is widely known that a variable of interest is continuous if its measurement can be expressed in some type of continuous scale; otherwise, if the object under study is classified according to the presence or absence of any characteristic or attribute of interest, we have a discrete variable.

Usually, a CC consists of a central line that represents the average value of the quality characteristic, corresponding to the in-control status. Two lines represent the lower and upper control limit. The control limits, the sampling intervaland the size of the sample are design parameters of the CC.

From the timeW. Shewhart developed the first control article in 1931up to today, a large amount of CC’s have been proposed. In fact, control charts based on the information of each sample, only and exclusively, are known as Shewhart type charts. Subsequently, CC’s that include other methods were developed, such as the cumulative sums (CUSUM), looking to improve in a sense the efficiency of the CC’s (Pepio and Polo, 1988). Charts of the Shewhart type and CUSUM can be found both for variables and for attributes.

For example, charts of the Shewhart type for variables under different approaches have been developed, some examples are the works proposed by Lee (2011), Yang and Chen (2009) and De Magalhães et al. (2009).

Most of the works on CC's are for variables. However, for many situations found in industry, medicine, economy, environment, etc., the characteristics of interest are categorical, that is, the items inspected are classified as conforming, which meet certain conditions, ornonconforming, which do not meet certain conditions (Jozani and Mirkamali, 2011). Thus, this work is focused in CC's for attributes, attempting to disseminate them as useful tools not only for big industries but also for small and medium manufacturing and service companies.

The best-known graphs to monitor categorical processes are pandnpfor binomial processes; the candugraphs for Poisson variables. Some are considered adaptive;this type of graph allowsfor variation, of at least one of its parameters (sample size, sampling interval and control limits) as it is monitored, depending on the status of the process. It has been demonstrated that these types of charts are superior to control charts respecting the detection speed of changes during the process (Wang and Ma, 2003 and Wu and Lu, 2004). Additionally, double-sampling has been fully utilized to improve the performance of control charts, without increases in sampling. During the first phase, a process sample is extracted then inspected and depending on the result is considered "in-control".Otherwise,immediately following, we proceed with the next phase where a second sample is taken from the process, the results of which are vital to determine if the process is in-control or not.

Some control charts developed using the adaptive approach to variables were proposed by De Magalhães et al. (2009), Yang and Chen (2009), Lee (2011),and Seif et al. (2011). Some examples of adaptive control charts for attributes are those proposed by Eppretch et al. (2010), Zhou and Lian (2011),and Hariday et al. (2012, 2013).

Control graphs for attributes under the double-sampling approach were proposed by De Araújo Rodrigues et al.(2011); namely, double-samplingnpgraphs to monitor binomial processes:an "alert" interval whereif the statistical value of the test is within the same interval, it is impossible to decide on the status of the process which then leads to a second sample extraction in order to conclude. The efficiency of the np graph is increased without increasing the expected sampling.

Additionally, Pérez, et al. (2010) proposed an improvement to the control chart u, for Poisson discrete variables, also under the double sampling approach.

This work is centered ondouble-sampling control charts for attributes; in particular, the use of the chart proposed by De AraújoRodríguez (2011)is studied during processes where there is the need to monitor binomial variables and,additionally, where the production standards are lower. The rest of the document is organized as follows: first, details of the monitoring method proposed by De AraújoRodríguez (2011); next, a description of the work including examples of application and conclusions.

II. Double sampling npchart

The double samplingnp chart (DS np) developed by De AraújoRodríguez et al. (2011) is adequate for situations where a process, having its quality characteristics modeled,must be monitored independently with a binomial distribution using bothnandp parameters. If a quality characteristic is represented with d, the number of nonconforming units in a sample of n size, then d ~ Binomial (n, p), where ppoints out the proportion of defective pieces in the process.

Under the double-sampling approach, $d_1$and$d_2$point out the number of defective pieces in the first and second sampling of sizes n_1 andn_2 respectively.

Double-sampling npcharts define the lower limits of control (the lower limit is zero), since the objective is to detect assignable causes that result in increments of p (that is, $p = p_0$when the process is in control and$p = p_1$when the process is out of control, where$p_1>p_0$).

The parameters of design of the chart are the size of the first sample (n_1), the alert limit for the first sample (AL), the upper control limit for the first sample (LSC_1), the size of the second sample (n_2), and the upper control limit for the second stage (LSC_2).

III. Method

In his work, De Araújo Rodríguez et al. (2011) proposed an optimization model to locate optimal designs for different scenarios. However, in order to implement this method in a particular process, none of the scenarios explored by De AraújoRodríguez et al. (2011) envisaged a similar situation to the process at hand.

This work was promptedby a process that consists of welding two metallic pieces that are part of torque converters for automobiles where defects are associated to the weld, such as porous welding. Historical production data reveals that the standard production level consists of 512 pieces per shift. Experience indicates that this process must be monitored in order to detect, as early in the process as possible, when the capacity to obtain articles within quality specifications associated to the process are lost; these quality specifications are measured in a nominal scale, thus it is proper to use a control chart for attributes for which we propose the application of the double-samplingnpchart proposed by De Araújo Rodríguez et al. (2011) as an alternative to take account of the aforementioned situation.

Usually, in these types of studies, the size of the sample is an important deciding factor, since large sample sizes implicate higher cost and longer inspection time. However, in the CEP, large sample sizes implicate more precision in detecting causes of special variation.

In this particular case, some of the designs presented by De AraújoRodríguez et al. (2011) could imply the inspection of more than half of the production or nearly all of it. Other designs are impossible to apply during processes with production volumes approaching512 pieces per shift, since the required sample sizes exceed this amount.

Although some designs that require small sample sizes are presented(these are applicable when the objective value of p_0 = 0.02) the percentage of defective pieceswill be 2%. These designs were considered among the possible applicable designs.

Accordingly, it is necessary to find the best designs from the Double-Samplingnp Chart applicable to processes with low production volume. To this effect, the formal approach of the optimization problem proposed by De AraújoRodríguez et al. (2011) was considered. This approach was made to find optimal designs for different scenarios. To use the Double-Sampling npChartduring the process under study an additional restriction to the original approach was added which is represented by means of equation (1).

$$ASS_1 \leq 150 \qquad (1)$$

WhereASS_1 which is the average sample size (ASS). It was restricted to 150 based on a preliminary search of designs that could be used in processes such as the one in question.
The ASS of the Double-SamplingnpChart is estimated in function of the proportion observed of defective units or pieces p, and it is given by:

$$ASS = n_1 + n_2\,[\,\Pr(\lfloor LA \rfloor < d_1 < \lceil LSC_1 \rceil)|p] \qquad (2)$$

In this case,$[\Pr(\lfloor LA \rfloor < d_1 < \lceil LSC_1 \rceil)|p]$is the probability to take a second sample. The $ASS_1$is estimated with p = p_1. $\lfloor \cdot \rfloor$andpoints out the maximum integer value which is less than or equal to the argument while$\lceil \cdot \rceil$points out the minimum integer value which is bigger than or equal to the argument.
The optimization problem with the additional restriction is the following:
Minimize ARL_1

Subject to

$$ASS_1 \leq 150 \qquad (1)$$
$$ASS_0 \leq n \qquad (3)$$
$$ARL_0 \geq ARL_{0\,min} \qquad (4)$$
$$0.5n \leq n_1 \leq 0.8n \qquad (5)$$
$$n_1 \leq n_2 \leq 5n_1 \qquad (6)$$
$$0 \leq \lfloor WL \rfloor \leq \lfloor UCL \rfloor \qquad (7)$$
$$\lfloor WL \rfloor + 2 \leq \lceil UCL_1 \rceil \leq \lfloor WL \rfloor + \lfloor UCL \rfloor \qquad (8)$$
$$\lceil UCL_1 \rceil + 1 \leq \lfloor UCL_2 \rfloor \leq \left(\left| 0.8 \left(\lceil UCL_1 \rceil \sqrt{\frac{n_1 + n_2}{n_1}} \right) \right| - 1 \right) \qquad (9)$$

Where n and $ARL_{0\,min}$ are specific values, UCL is the upper limit of control of the classic np chart. The expressions to estimate ARL_0, $ARL_1$and $ASS_0$are given in equations (10), (11), (2), respectively.$ASS_0$is estimated with p = p_0. The symbol$\lfloor \cdot \rfloor$ represents the largest integer which is less than or equal to the argument and the symbol $\lceil \cdot \rceil$the largest integer which is bigger than or equal to the argument.

$$ARL_0 = \frac{1}{(1 - P)} \qquad (10)$$

Where P is estimated with p = p_0(Assuming that the process is in-control at the beginning).

$$ARL_1 = \frac{1}{(1 - P)} \qquad (11)$$

The expression to estimate ARL_1 is similar to equation (10). However, ARL_1 is the average number of samples taken until an alarm occurs signaling the process is out of control; therefore,p = p_1 indicates an increase in the proportion of defective pieces. It is assumed that the shifting in p does not occur during the removal of a sample but between sampling points.

After each sampling, the probability that the chart indicates that the process is in-control is given by:

$$P = P_1 + P_2 \tag{12}$$

Where P_1points out the probability that the number of defective pieces in the sample is less than the alert limit in the first stage of the sampling, and $P_2$is the probability that a second sample is needed and that the number of defective pieces in the two samples is less than the limit of control of the second phase.

P_1andP_2probabilities are given by:

$$P_1 = \Pr[(d_1] \leq \lfloor WL \rfloor) = \sum_{d_1=0}^{\lfloor WL \rfloor} \frac{n_1!}{d_1!\,(n_1-d_1)!} p^{d_1}(1-p)^{n_1-d_1} \tag{11}$$

$$P_2 = \Pr(\lfloor WL \rfloor < d_1 < \lceil UCL_1 \rceil, d_1 + d_2 \leq \lfloor UCL_2 \rfloor) = \sum_{d_1=\lfloor WL \rfloor+1}^{\lceil UCL_1 \rceil-1} \left[\frac{n_1!}{d_1!\,(n_1-d_1)!} p^{d_1}(1-p)^{n_1-d_1} \left(\sum_{d_2=0}^{\lfloor UCL_2 \rfloor-d_1} \frac{n_2!}{d_2!\,(n_2-d_2)!} p^{d_2}(1-p)^{n_2-d_2} \right) \right] \tag{12}$$

Whered_1 andd_2 are the number of defective units in the first and second samples respectively.

To solve the optimization problem with the new restriction, a program in R Gui® version 2.11.1 was developed. The values of the entry parameters were p_0 = 0.008, 0.01; γ = 2, 3 ($\gamma = \frac{p_1}{p_0}$); n = 35, 50, 80 and $ARL_{0mín}$=200. To elect the values of the parameters, the volume of production of the process under study as well as the target proportion of non-defective pieces was considered. Table 1 shows the optimal designs found for the parameters used.

Table 1. Optimal design for the Double Sampling npChart.

	Entry Parameters			Optimal Design Parameters						
Design	γ	p_0	n	n_1	n_2	WL	UCL_1	UCL_2	ARL0	ARL_1
1	2	0.01	35	26	115	1.5	2.5	4.5	201.17	21.20
2	2	0.01	50	41	141	1.5	3.5	5.5	201.34	13.02
3	3	0.01	35	28	99	1.5	2.5	4.5	201.03	7.19
4	2	0.008	50	40	200	1.5	3.5	5.5	251.20	5.27
5	2	0.008	80	54	167	1.5	3.5	5.5	200.37	12.89
6	3	0.008	80	63	139	1.5	3.5	5.5	202.28	3.88

These results allow for deciding which is the most recommendable design, considering the performance of the chart in terms of the ARL.

It should be noted that: ifp_0 decreases andnincreases, the Double-Sampling npChart is more efficient (in terms of the ARL)to detect large shifts in $p_0$and that some parameter combinations of entry influence obtain very large values of ARL_1, which is, in practice, not recommended. However, they can provide useful information about the sensitivity of the Double-Sampling np Chart.

IV. Example of application

In order to illustrate the use of the Double-Sampling npControl Chart, an example of applicationis presented in this section,assuming that the process began within control and that after an instant of time the status of the process changed to an out of control status. The size of the sample was set according to design 6 of Table 1.

Assume that you wish to monitor a production process with a quality characteristic that has anallowable proportion of defective piecesp$_0$=0.008. This process is monitored with design 6, because it is the one that shows the best performance. This design has an ARL_1= 3.88 to detect increments of 200% in the proportion of defective pieces. Design parameters of the Double-Sampling npChart are set in sample sizes for the first and second phase of $n_1 = 63$ and$n_2 = 139$, respectively, and the control limits LA = 1.5, LSC_1= 3.5,and LSC_2 =5.5.

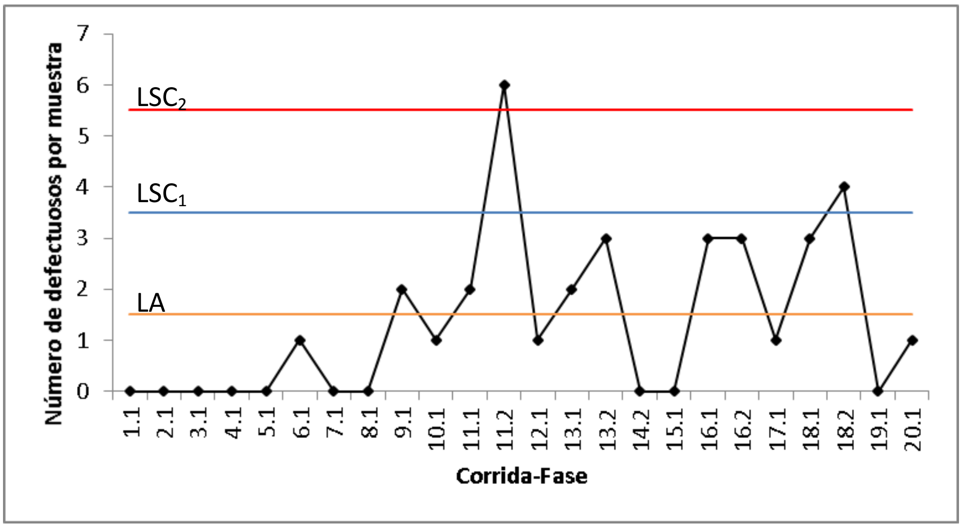

Figure 1: Example of application with p_0=0.008 andp_1 = 0.016.

To monitor the process we extract, in intervals of fixed samples, a sample of the process n_1 size,d_1, the number of defective pieces in the sample is obtained and compared to the control limits and an alert is established to indicate the transition to the second phase of the sampling, each time it is required.

Figure 1 shows, graphically, the results of 20 runs. In the first place we observe that the first 10 runs are generated with $p = p_0 = 0.008$; in run 11 a shift in the process was induced, thus the last 10 runs were generated with $p = p_1 = 0.016$;this with the purpose to simulate the change in behavior of the process. As shown, the Double-SamplingnpChart does not show any signal out of control in the first 10 runs, where a control status was assumed. Also, it was observed that the detection of the shift occurs in the same run where the p shifting was induced. However, the rest of the points fall within the control status when it would be expected that these points were out of control; this situation is due to the fact that the sensitivity, although useful, it is not so strong.

V. Conclusions

In this work we provide the designs of the Double-Sampling npControl Chart which minimized the ARL_1against increments of 100% and 200% in the fraction of defective pieces for processes with a fraction of defective pieces$p_0$equal to 0.008 and 0.01,and low volume of production for $ARL_0 \geq 200$ and sample sizes of n = 35, 50 and 80.

Although designs with acceptable sample sizes were found from the practical point of view with minimum acceptable ARL_1values (values close to 1); for smaller values of p_0 (0.008 orless) and minimal shifting ofp, the minimum ARL_1 for small values of nis too large. This defines a limit of the Double-Sampling np Chart for processes with very small $p_0$and low volume of production.

Taken into consideration was the production volume of the process studied. The designs of the Double-Sampling npChart were obtained which minimized the ARL_1 restricting the sample size.

As a result of comparing the designs in terms of ARL, Design 6 is recommended to monitor the process under study, due to the fact that it shows the best performance. Though, eventually, the use of this design will require a second sample of 139 pieces, an amount that is still very large with respect to the production standard as it is proper to monitor processes with similar characteristics.

Designs of the Double-Sampling npChart for tiny$p_0$samples require very large sample sizes. Otherwise, the efficiency of detection will be compromised, which can lead to wrong conclusions concerning the status of the process.

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