Unsteady MHD flow of a visco-elastic fluid along vertical porous surface with fluctuating temperature and concentration

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Abstract: - An unsteady magneto-hydrodynamics flow of an incompressible visco-elastic fluid   [Walters' fluid (Model B′)] along an infinite hot vertical porous surface with fluctuating temperature and concentration has been studied. The governing equations of motion, energy and concentration are solved by the successive perturbation techniques. The expressions for the skin friction, Nusselt number and Sherwood number are also derived. The variations in the fluid velocity, temperature and concentration are shown graphically whereas numerical values of skin friction, Nusselt number and Sherwood number are presented in a tabular form for various values of pertinent flow parameters.

Keywords*: - MHD flow; porous medium; chemical reaction/heat source*

I. INTRODUCTION

Flow between porous boundaries is of practical, as well as theoretical interest. The practical interest includes problems of gaseous diffusion, transpiration cooling, lubrication of porous bearing etc. Flow through porous medium under the influence of temperature and concentration differences find great applications in Geothermy, Geophysics and Technology.

Several researches [1-6] have studied the two dimensional free convection MHD flow with heat and mass transfer. Walters [7] have analyzed second order effects in elasticity plasticity and fluid dynamics. Sharma and Pareek [8] later examined the unsteady flow and heat transfer through an elastico-viscous liquid along an infinite hot vertical porous moving plate with variable free stream and suction. Flow and heat transfer of an electrically conducting visco-elastic fluid between two horizontal squeezing/stretching plates have been studied by Rath *et al.*[9].

Alhabi *et al.*[10] have analyzed heat and mass transfer in MHD visco-elastic fluid flow through a porous medium over a stretching sheet with chemical reaction. Kumar and Sivraj [11] have studied MHD mixed convective visco-elastic fluid flow in a permeable vertical channel with Dufour effect and chemical reaction parameter. Damseh and Sharnaik [12] have also studied visco-elastic fluid flow past an infinite vertical porous plate in the presence of first order chemical reaction. MHD flow through a porous medium past a stretched vertical permeable surface in the presence of heat source/sink and a chemical reaction has been studied by Dash et al.[13]. Rath et al. [14] have studied the three dimensional free convection flow through porous medium in a vertical channel with heat source and chemical reaction. Sharma and Sharma [15] have studied the unsteady two dimensional flow and heat transfer through an elastico-viscous liquid along an infinite hot vertical porous surface bounded by porous medium.

The main objective of the present study is to investigate the combined effect of magnetic field, heat source and chemical reaction on two-dimensional visco-elastic flow through porous media in a vertical surface with fluctuating temperature and concentration. The governing equations of flow under the Boussinesq approximation are solved analytically using successive perturbation method.

II. FORMULATION OF THE PROBLEM

An unsteady two dimensional MHD flow past a vertical infinite surface embedded in a porous medium has been considered with the visco-elastic fluid model (Walters' fluid (Model B′) [7]). A uniform transverse magnetic field B_0 is applied normal to the direction of the fluid flow. The x^*-axis is taken along the surface in upwards direction i.e., opposite to the direction of gravity and y^*-axis is taken normal to the surface. The governing equations of continuity, motion, energy and species concentration for flow of a visco-elastic fluid through porous medium bounded by an infinite, vertical porous surface with oscillatory suction velocity are given by :

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$\left(\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial x^*} - g + v\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{v}{k^*}u^*$$

$$-\frac{K_0^*}{\rho}\left(\frac{\partial^3 u^*}{\partial t^* \partial y^{*2}} + v^*\frac{\partial^3 u^*}{\partial y^{*3}}\right) - \frac{\sigma B_0^2 u^*}{\rho} \tag{2}$$

$$\frac{\partial T^*}{\partial t^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{k}{\rho C_p}\frac{\partial T^{*2}}{\partial y^{*2}} + S^*(T^* - T_\infty) \tag{3}$$

$$\frac{\partial C^*}{\partial t^*} + v^*\frac{\partial C^*}{\partial y^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} - K_c^*(C^* - C_\infty) \tag{4}$$

Where u^*, v^* denote the components of velocity along x^* and y^* directions respectively, ρ the density of the fluid, g the acceleration due to gravity, v the Kinematic viscosity, k^* the permeability parameter K_0^* the non-Newtonian parameter, T^* the temperature of the fluid, k the thermal conductivity of fluid, C_p the specific heat of the fluid at constant pressure, t^* the time, C^* the concentration of fluid, K_c^* the chemical reaction parameter, D the mass diffusion coefficient and σ the electrical conductivity.

The boundary conditions are given by :

$$t>0: y^* = 0; u^* = 0, T^* = T_w + \varepsilon(T_w - T_\infty)e^{i\omega t}, C^* = C_w + \varepsilon(C_w - C_\infty)e^{i\omega t}$$

$$y^* \to \infty; u^* \to U^*(t^*),\ T^* \to T_\infty,\ C^* \to C_\infty \tag{5}$$

where T_w and C_w are the surface temperature and concentration respectively, $U^*(t^*)$ the free stream velocity, T_∞ the free stream temperature and C_∞ is the concentration of the fluid for away from the wall.

In view of Bousinesq approximation, the equation (2) is reduced to

$$\rho\left(\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*}\right) = \rho\frac{\partial U^*}{\partial t^*} + \frac{\mu}{k^*}(U^* - u^*) + \mu\frac{\partial^2 u^*}{\partial y^{*2}}$$

$$-K_0^*\left(\frac{\partial^3 u^*}{\partial t^* \partial y^{*2}} + v^*\frac{\partial^3 u^*}{\partial y^{*3}}\right) + \sigma B_0^2(U^* - u^*) + g\beta\rho(T^* - T_\infty)$$

$$+g\beta^*\rho(C^* - C_\infty) \tag{6}$$

where β and β^* are the volumetric coefficient of thermal and concentration expansion respectively and μ the coefficient of dynamic viscosity.

From equation of community (1), it is clear that the suction velocity normal to the plate is either a constant or a function of time. Here it is assumed in the form of

$$v^* = -v_0(1 + \varepsilon e^{i\omega^* t^*}) \tag{7}$$

where , ω^* the frequency of vibration, ε is a small parameter i.e., $0 < \varepsilon < 1$ and v_0 is a non-zero positive suction velocity. Here the negative sign indicates that the suction is towards the plate.

The governing equations of the flow are non-dimentionalised using the following transformation.

$$y = \frac{y^* v_0}{v},\ t = \frac{t^* v_0^2}{4v},\ u = \frac{u^*}{U},\ \omega = \frac{4v\omega^*}{v_0^2},$$

$$U(t) = \frac{U^*(t^*)}{U},\ T = \frac{T^* - T_\infty}{T_w - T_\infty},\ P_r = \frac{\mu C_p}{k}$$

$$k_p = \frac{k^* v_0^2}{v^2},\ K_0 = \frac{K_0^* v_0^2}{\rho v^2},\ G_r = \frac{vg\beta(T_w - T_\infty)}{Uv_0^2},$$

$$M = \frac{\sigma B_0^2 v}{\rho v_0^2},\ K_c = \frac{K_c^* v}{v_0^2}$$

$$C = \frac{C^* - C_\infty}{C_w - C_\infty}, S = \frac{S^* \nu}{v_0^2}, G_c = \frac{\nu g \beta^* (C_w - C_\infty)}{U v_0^2} \tag{8}$$

In view of equations (7) and (8), equations (3) – (4) and (6) become :

$$\frac{\partial^2 u}{\partial y^2} + \left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial u}{\partial y} = \frac{1}{4}\frac{\partial u}{\partial t} - G_r T - G_c C + \left(\frac{1}{k_p} + M\right)u - \left(\frac{1}{k_p} + M\right)\left(1 + \varepsilon e^{i\omega t}\right)$$

$$-\frac{1}{4}\varepsilon i\omega e^{i\omega t} + k_0\left[\frac{1}{4}\frac{\partial^3 u}{\partial t \partial y^2} - \left(1 + \varepsilon e^{iwt}\right)\frac{\partial^3 u}{\partial y^3}\right] \tag{9}$$

$$\frac{P_r}{4}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial y^2} + P_r\left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial T}{\partial y} + P_r S T \tag{10}$$

$$\frac{S_c}{4}\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} + S_c\left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial C}{\partial y} - S_c K_c C \tag{11}$$

where G_r the Grashof number for heat transfer, P_r the Prandtl number, K_0 the non-Newtonian parameter, G_c the Grashof number for mass transfer, S_c the Schmidt number, M the magnetic parameter and k_p the porosity parameter.

The corresponding boundary conditions in non-dimensional form:

$y = 0: u = 0, T = 1 + \varepsilon e^{i\omega t}, C = 1 + \varepsilon e^{i\omega t}$

$y \to \infty: u \to U(t) = 1 + \varepsilon e^{i\omega t}, T \to 0, C \to 0$ \hfill (12)

Solution procedure

In view of the above assumption it is justified to assume that :

$u(y, t) = u_0(y) + \varepsilon u_1(y)e^{i\omega t}$ \hfill (13)

$T(y, t) = T_0(y) + \varepsilon T_1(y) e^{i\omega t}$ \hfill (14)

$C(y, t) = C_0(y) + \varepsilon C_1(y) e^{i\omega t}$ \hfill (15)

Substituting equations(13) – (15) into equations (9) – (11) and equating the coefficients of ε^0 and ε, we get

Zeroth order :

$$K_0 u_0''' + u_0'' + u_0' - \left(M + \frac{1}{k_p}\right)u_0 = -G_r T_0 - \left(M + \frac{1}{k_p}\right) - G_c C_0 \tag{16}$$

$$T_0'' + P_r T_0' + P_r S T_0 = 0 \tag{17}$$

$$C_0'' + S_c C_0' - K_c S_c C_0 = 0 \tag{18}$$

First order:

$$K_0 u_1''' + \left(1 - \frac{k_0 i\omega}{4}\right)u_1'' + u_1' - \left(M + \frac{1}{k_p} + \frac{i\omega}{4}\right)u_1$$

$$= -G_r T_1 - G_c C_1 - K_0 u_0''' - u_0' - \left(M + \frac{1}{k_p} + \frac{i\omega}{4}\right) \tag{19}$$

$$T_1'' + P_r T_1' + P_r\left(S - \frac{i\omega}{4}\right)T_1 = -P_r T_0' \tag{20}$$

$$C_1'' + S_c C_1' - \left(\frac{i\omega}{4} + k_c\right)S_c C_1 = -S_c C_0' \tag{21}$$

where prime denote differentiation with respect to y.

The corresponding boundary conditions (12) are reduced to :

$y = 0: u_0 = 0, u_1 = 0, T_0 = 1, T_1 = 1, C_0 = 1, C_1 = 1,$

$y \to \infty : u_0 \to 1, u_1 \to 1, T_0 \to 0, T_1 \to 0, C_0 \to 0, C_1 \to 0$ \hfill (22)

Solving equations (17), (18), (20) and (21) under the boundary conditions (22), we get

$$T_0 = e^{m_1 y} \tag{23}$$

$$C_0 = e^{m_3 y} \tag{24}$$

$$T_1 = \left(1 + \frac{4i}{\omega} m_1\right) e^{m_2 y} - \frac{4i}{\omega} m_1 e^{m_1 y} \tag{25}$$

$$C_1 = \left(1 + \frac{4i}{\omega} m_3\right) e^{m_4 y} - \frac{4i}{\omega} m_3 e^{m_3 y} \tag{26}$$

The equations (16) and (19) are still of third order where $K_0 \neq 0$ and reduce to second order differential equation when $K_0 = 0$ i.e., for a Newtonian fluid case. Hence, the presence of a non-Newtonian parameter increases the order of differential equation. While from physical consideration only two boundary conditions are available. Since the non-Newtonian parameter (K_0) is very small for incompressible fluid (Walters[7]), therefore u_0 and u_1 can be expanded in powers of K_0 which is given by

$$u_0 (y) = u_{00}(y) + K_0 u_{01}(y) + O (K_0^2)$$

$$u_1 (y) = u_{10} (y) + K_0 u_{11} (y) + O (K_0^2) \tag{27}$$

introducing equation (27) into equations (16) and (19), we obtain the following systems of equations.

$$u_{00}'' + u_{00}' - \left(M + \frac{1}{k_p}\right) u_{00} = -\left(M + \frac{1}{k_p}\right) - G_r T_0 - G_c C_0 \tag{28}$$

$$u_{10}'' + u_{10}' - \left(M + \frac{1}{k_p} + \frac{i\omega}{4}\right) u_{10} = -G_r T_1 - G_c C_1 - u_{00}' - \left(M + \frac{1}{k_p} + \frac{i\omega}{4}\right) \tag{29}$$

$$u_{01}'' + u_{01}' - \left(M + \frac{1}{k_p}\right) u_{01} = -u_{00}''' \tag{30}$$

$$u_{11}'' + u_{11}' - \left(M + \frac{1}{k_p} + \frac{i\omega}{4}\right) u_{11} = -u_{00}''' - u_{01}' - u_{10}''' + \frac{i\omega}{4} u_{10}'' \tag{31}$$

With the corresponding boundary conditions:
$$\left.\begin{array}{l} y = 0, \ u_{00} = 0, \ u_{01} = 0, \ u_{10} = 0, \ u_{11} = 0 \\ y \rightarrow \infty : u_{00} \rightarrow 1, \ u_{01} \rightarrow 0, \ u_{10} \rightarrow 1, u_{11} \rightarrow 0 \end{array}\right\} \tag{32}$$

Solving equations (28) – (31) under the boundary condition (32), we get

$$u_{00} = (A_3 + A_4 - 1) \ e^{m_5 y} - A_3 e^{m_1 y} - A_4 e^{m_3 y} + 1 \tag{33}$$

$$u_{01} = A_9 e^{m_5 y} + A_7 e^{m_1 y} + A_8 e^{m_3 y} \tag{34}$$

$$u_{10} = A_{19} e^{m_6 y} + A_{17} e^{m_1 y} + A_{18} e^{m_3 y} + A_{10} e^{m_2 y} + A_{14} e^{m_5 y} + A_{12} e^{m_4 y} + 1 \tag{35}$$

$$u_{11} = A_{38} e^{m_6 y} + A_{35} e^{m_5 y} + A_{28} e^{m_4 y} + A_{36} e^{m_3 y} + A_{26} e^{m_2 y} + A_{37} e^{m_1 y} \tag{36}$$

Finally, the velocity u(y, t) is given by

$$u(y, t) = (A_3 + A_4 - 1) \ e^{m_5 y} - A_3 e^{m_1 y} - A_4 e^{m_3 y} + 1 + K_0 \left(A_9 e^{m_5 y} + A_7 e^{m_1 y} + A_8 e^{m_3 y}\right)$$

$$+ \ \varepsilon \left[\left\{A_{19} e^{m_6 y} + A_{17} e^{m_1 y} + A_{18} e^{m_3 y} + A_{10} e^{m_2 y} + A_{14} e^{m_5 y} + A_{12} e^{m_4 y} + 1\right\}\right.$$

$$\left.+ K_0 \left\{A_{38} e^{m_6 y} + A_{35} e^{m_5 y} + A_{28} e^{m_4 y} + A_{36} e^{m_3 y} + A_{26} e^{m_2 y} + A_{37} e^{m_1 y}\right\}\right] e^{i\omega t} \tag{37}$$

where $A_1 - A_{38}$, $m_1 - m_6$ are given in the Appendix.

Skin friction (C_f)

$$C_f = \frac{\tau_w^*}{\rho U_0 \nu} = \left[\frac{\partial u}{\partial y} - K_0 \left(\frac{1}{4} \frac{\partial^2 u}{\partial t \partial y} - \left(1 + \varepsilon e^{i\omega t} \right) \frac{\partial^2 u}{\partial y^2} \right) \right]_{y=0} \tag{38}$$

Nusselt Number (N_u)

$$\mathrm{Nu} = \frac{q\nu}{\nu_0 k (T_w - T_\infty)} = \left(\frac{\partial T}{\partial y} \right)_{y=0}$$

$$\therefore N_u = - \left[m_1 + \varepsilon e^{i\omega t} \left\{ \left(1 + \frac{4i}{\omega} m_1 \right) m_2 - \frac{4i}{\omega} m_1^2 \right\} \right] \tag{39}$$

Sherwood Number (S_h)

$$S_h = \frac{m\nu}{\nu_0 D (C_w - C_\infty)} = - \left(\frac{\partial c}{\partial y} \right)_{y=0}$$

$$= - \left[m_3 + \varepsilon e^{i\omega t} \left\{ \left(1 + \frac{4i}{\omega} m_3 \right) m_4 - \frac{4i}{\omega} m_3^2 \right\} \right] \tag{40}$$

Here τ_w^* is the dimensionless shear stress component of the elastico-viscous fluid and the symbols q and m represent heat and mass flux and they are given by

$$q = -k \left(\frac{\partial T^*}{\partial y^*} \right)_{y^*=0} \quad \text{and m} = -D \left(\frac{\partial C^*}{\partial y^*} \right)_{y^*=0}$$

III.   RESULTS AND DISCUSSION

The effects of physical parameters such as magnetic parameter (M), Elastic parameter (K_0), permeability parameter (k_p), chemical reaction parameter (K_c), heat source parameter (S), Grashoff number for heat transfer (G_r), Grashoff number for mass transfer (G_c), Prandtl number (P_r), Schmidt number (S_c) and Phase angle (wt) on the velocity, temperature, concentration profile have been presented with the help of graphs and Tables.

Fig. 1 exhibits the velocity distribution for various values of G_r, G_c, K_0, k_p, K_c, S_c, ω, ωt and M. It is seen that an increase in G_r and G_c increases the velocity near the plate (curves I, II and IX), due to free convection current which accelerates the velocity. The increase is insignificant in case of thermal buoyancy (G_r), whereas it is quite significant in case of mass buoyancy (G_c). Further, it is seen that the presence of transverse magnetic field reduces the velocity due to magnetic interaction, producing Lorentz force which opposes the motion (curves I and X). From the curves XI, XII and I it is observed that the velocity increases in case of generating reaction ($K_c < 0$) but is reversed in case of destructive reaction ($K_c > 0$). It is also observed that the fluid velocity increases due to increase in the elastic parameter (K_0) and reverse effect is observed in case of phase angle (ωt) and Schmidt number (S_c).

Fig. 2 shows the velocity distribution for various values of P_r and S. it is seen that higher Prandtl number fluid contributes very insignificantly in increasing the velocity (curves I, III). It is further to note that the heat source parameter reduces the velocity at all points (curve I, II).

Fig.3. depicts the temperature distribution in the flow domain. An increase in frequency parameter (ω) and heat source parameter (S) contribute to increase the thickness of thermal boundary layer (curves I, IV and VI) but with an increase in the value of Prandtl number (P_r) and Phase angle (ωt) the temperature decreases (curves I, II, III and V). This may be attributed to the fact that smaller values of P_r lead to increase the thermal conductivity of the fluid and thereby more amount of heat is diffused which causes the decrease in temperature.

Fig. 4 exhibits the concentration distribution for various values of Schmidt number (S_c), chemical reaction parameter (K_c), frequency parameter (ω) and phase angle (ωt). For heavier species i.e., for higher values of (S_c) concentration decreases at all the layers in the flow domain. Similar effects are also observed in case of chemical reaction parameter (K_c), frequency parameter (ω) and phase angle (ωt).

Table 1 shows that the skin friction (C_f) decreases due to higher value of Prandtl number and other parameters such as permeability parameter, magnetic parameter, elastic parameter, chemical reaction parameter, Schmidt number, frequency of oscillation but reverse effect is observed in case of Grashoff numbers, heat source parameter and phase angle (ωt).

From Table 2 it is observed that the rate of heat transfer (N_u) at the surface increases in case of higher Prandtl number (P_r) and frequency of oscillation (ω) but reverse effect is observed in case of heat source parameter (S) and phase angle (ωt).

From Table 3 it is observed that the rate of mass transfer (S_h) increases due to increase in Schmidt number, chemical reaction parameters and frequency parameter but decreases with an increase in phase angle.

IV.   CONCLUSION

i)   Flow characteristic in the present study is more dependent on mass buoyancy effect (G_c) rather than thermal buoyancy effect (G_r).

ii) Magnetic field, chemical reaction parameter and heat source parameter with high Prandtl number flow have a retarding effect on the velocity.

iii) Thinning of thermal boundary layer occurs for higher Prandtl number fluid.

iv) Higher rate of mass transfer is experienced in case of heavier species and in the presence of destructive reaction.

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APPENDIX

$$m_1 - \left(\frac{P_r + \sqrt{P_r^2 - 4P_r S}}{2} \right), \quad m_2 = -\left(\frac{P_r + \sqrt{P_r^2 - 4P_r\left(S - \frac{i\omega}{4} \right)}}{2} \right)$$

$$m_3 = -\left(\frac{S_c + \sqrt{S_c^2 + 4K_c S_c}}{2} \right), \quad \alpha_1 = -\left(\frac{iw}{4} + K_c \right) S_c,$$

$$m_4 = -\left(\frac{S_c + \sqrt{S_c^2 + 4\alpha_1}}{2} \right), \alpha_2 = M + \frac{1}{k_p}, \quad m_5 = -\left(\frac{1 + \sqrt{1 + 4\alpha_2}}{2} \right), \quad m_6 = -\left(\frac{1 + \sqrt{1 + 4\alpha_2 + \frac{i\omega}{4}}}{2} \right)$$

$A_1 = m_1^2 + m_1 - \alpha_2, \quad A_2 = m_3^2 + m_3 - \alpha_2, \quad A_3 = \frac{G_r}{A_1}, \quad A_4 = \frac{G_c}{A_2}, \quad A_5 = m_5^2 + m_5 - \alpha_2$

$$A_6 = \frac{(1 - A_3 - A_4)m_5^3}{A_1}, \quad A_7 = \frac{A_3 m_1^3}{A_1}, \quad A_8 = \frac{A_4 m_3^3}{A_2}$$

$$A_9 = -(A_7 + A_8), \quad A_{10} = \frac{-G_r\left(1 + \frac{4i}{\omega}m_1\right)}{m_2^2 + m_2 - \left(\alpha_2 + \frac{i\omega}{4}\right)}, \quad A_{11} = \frac{G_r \frac{4i}{\omega}m_1}{A_1 - \frac{i\omega}{4}},$$

$$A_{12} = \frac{-G_c\left(1 + \frac{4i}{\omega}m_3\right)}{m_4^2 + m_4 - \left(\alpha_2 + \frac{i\omega}{4}\right)}, A_{13} = \frac{G_c \frac{4i}{\omega}m_3}{A_2 - \frac{i\omega}{4}}, A_{14} = -\frac{(A_3 + A_4 - 1)m_5}{A_5 - \frac{i\omega}{4}}$$

$$A_{15} = \frac{A_3 m_1}{A_1 - \frac{i\omega}{4}}, A_{16} = \frac{A_4 m_3}{A_2 - \frac{i\omega}{4}},$$

$A_{17} = A_{11} + A_{15}, A_{18} = A_{13} + A_{16}, A_{19} = (-1 - A_{17} - A_{18} - A_{10} - A_{14} - A_{12})$

$A_{20} = m_6^2 + m_6 - \alpha_2, \quad A_{21} = m_2^2 + m_2 - \alpha_2, A_{22} = m_4^2 + m_4 - \alpha_2$

$$A_{23} = \frac{\left(\frac{i\omega}{4}m_6^2 - m_6^3\right)A_{19}}{A_{20} - \frac{i\omega}{4}}, A_{24} = \frac{\left(\frac{i\omega}{4}m_1^2 - m_1^3\right)A_{17}}{A_1 - \frac{i\omega}{4}}$$

$$A_{25} = \frac{\left(\frac{i\omega}{4}m_3^2 - m_3^3\right)A_{18}}{A_2 - \frac{i\omega}{4}}, A_{26} = \frac{\left(\frac{i\omega}{4}m_2^2 - m_2^3\right)A_{10}}{A_{21} - \frac{i\omega}{4}},$$

$$A_{27} = \frac{\left(\dfrac{i\omega}{4} m_5^2 - m_5^3\right) A_{14}}{A_5 - \dfrac{i\omega}{4}}, A_{28} = \frac{\left(\dfrac{i\omega}{4} m_4^2 - m_4^3\right) A_{12}}{A_{22} - \dfrac{i\omega}{4}}, \qquad A_{29} = \frac{-\left(A_3 + A_4 - 1\right) m_5^3}{A_5 - \dfrac{i\omega}{4}}, A_{30} = \frac{A_3 m_1^3}{A_1 - \dfrac{i\omega}{4}},$$

$$A_{31} = \frac{A_4 m_3^3}{A_2 - \dfrac{i\omega}{4}}, A_{32} = \frac{-A_9 m_5}{A_5 - \dfrac{i\omega}{4}}, \quad A_{33} = -\frac{A_7 m_1}{A_1 - \dfrac{i\omega}{4}}, A_{34} = -\frac{A_8 m_3}{A_2 - \dfrac{i\omega}{4}}$$

$A_{35} = A_{27} + A_{29} + A_{32}, A_{36} = A_{25} + A_{31} + A_{34}, A_{37} = A_{22} + A_{30} + A_{33}, A_{38} = -(A_{35} + A_{28} + A_{36} + A_{26} + A_{37})$

TABLE - 1   Skin friction (C_f)

G_r	P_r	K_p	ω	ωt	K_0	G_c	M	K_c	S_c	S	C_f
5	5	0.4	5	$\pi/4$	0.2	2	2	1	0.22	1	5.906
7	5	0.4	5	$\pi/4$	0.2	2	2	1	0.22	1	6.236
-5	5	0.4	5	$\pi/4$	0.2	2	2	1	0.22	1	4.249
-7	5	0.4	5	$\pi/4$	0.2	2	2	1	0.22	1	3.918
5	7	0.4	5	$\pi/4$	0.2	2	2	1	0.22	1	-442.203
5	5	100	5	$\pi/4$	0.2	2	2	1	0.22	1	-1.971
5	5	0.4	15	$\pi/4$	0.2	2	2	1	0.22	1	-1.328
5	5	0.4	5	$\pi/3$	0.2	2	2	1	0.22	1	11.626
5	5	0.4	5	$\pi/4$	0	2	2	1	0.22	1	22.009
5	5	0.4	5	$\pi/4$	0.2	5	2	1	0.22	1	10.885
5	5	0.4	5	$\pi/4$	0.2	2	5	1	0.22	1	2.269
5	5	0.4	5	$\pi/4$	0.2	2	2	-0.04	0.22	1	9.202
5	5	0.4	5	$\pi/4$	0.2	2	2	0	0.22	1	8.780
5	5	0.4	5	$\pi/4$	0.2	2	2	1	0.78	1	2.577
5	5	0.4	5	$\pi/4$	0.2	2	2	1	0.22	0.5	-76.803

TABLE - 2 Nusselt number (N_u)

P_r	ω	ωt	S	N_u
5	5	$\pi/4$	1	6.04581
7	5	$\pi/4$	1	8.81121
11.4	5	$\pi/4$	1	14.68664
5	15	$\pi/4$	1	6.31914
5	5	$\pi/3$	1	5.77632
5	5	$\pi/4$	0	7.20342
5	5	$\pi/4$	-1	8.16158

TABLE - 3 Sherwood number (S_h)

S_c	ω	ωt	K_c	S_h
1.002	5	$\pi/4$	1	1.69103
0.600	5	$\pi/4$	1	1.17297
0.780	5	$\pi/4$	1	1.41045
1.002	5	$\pi/4$	0	1.35615
1.002	5	$\pi/4$	1.5	1.75429
1.002	5	$\pi/4$	2	1.78982
1.002	15	$\pi/4$	1	1.88002
1.002	5	$\pi/3$	1	1.52351

Curve	G_r	P_r	K_p	ω	ωt	K_0	G_c	M	K_c	S_c	S
I	5	5	0.4	5	$\pi/4$	0.2	2	2	1	0.22	1
II	7	5	0.4	5	$\pi/4$	0.2	2	2	1	0.22	1
III	-5	5	0.4	5	$\pi/4$	0.2	2	2	1	0.22	1
IV	-7	5	0.4	5	$\pi/4$	0.2	2	2	1	0.22	1
V	5	5	100	5	$\pi/4$	0.2	2	2	1	0.22	1
VI	5	5	0.4	15	$\pi/4$	0.2	2	2	1	0.22	1
VII	5	5	0.4	5	$\pi/3$	0.2	2	2	1	0.22	1
VIII	5	5	0.4	5	$\pi/4$	0.0	2	2	1	0.22	1
IX	5	5	0.4	5	$\pi/4$	0.2	5	2	1	0.22	1
X	5	5	0.4	5	$\pi/4$	0.2	2	5	1	0.22	1
XI	5	5	0.4	5	$\pi/4$	0.2	2	2	-.04	0.22	1
XII	5	5	0.4	5	$\pi/4$	0.2	2	2	0	0.22	1
XIII	5	5	0.4	5	$\pi/4$	0.2	2	2	1	0.78	1

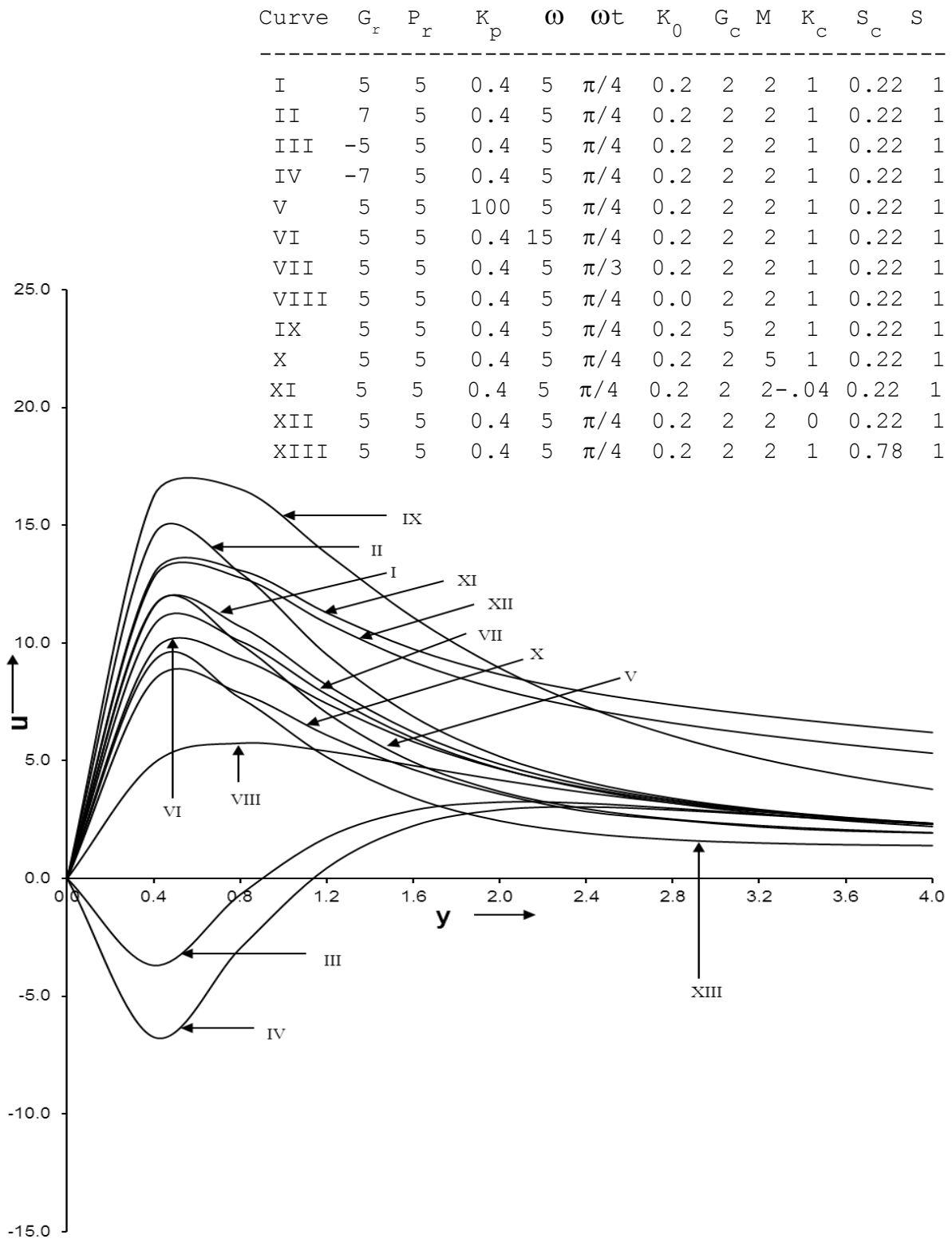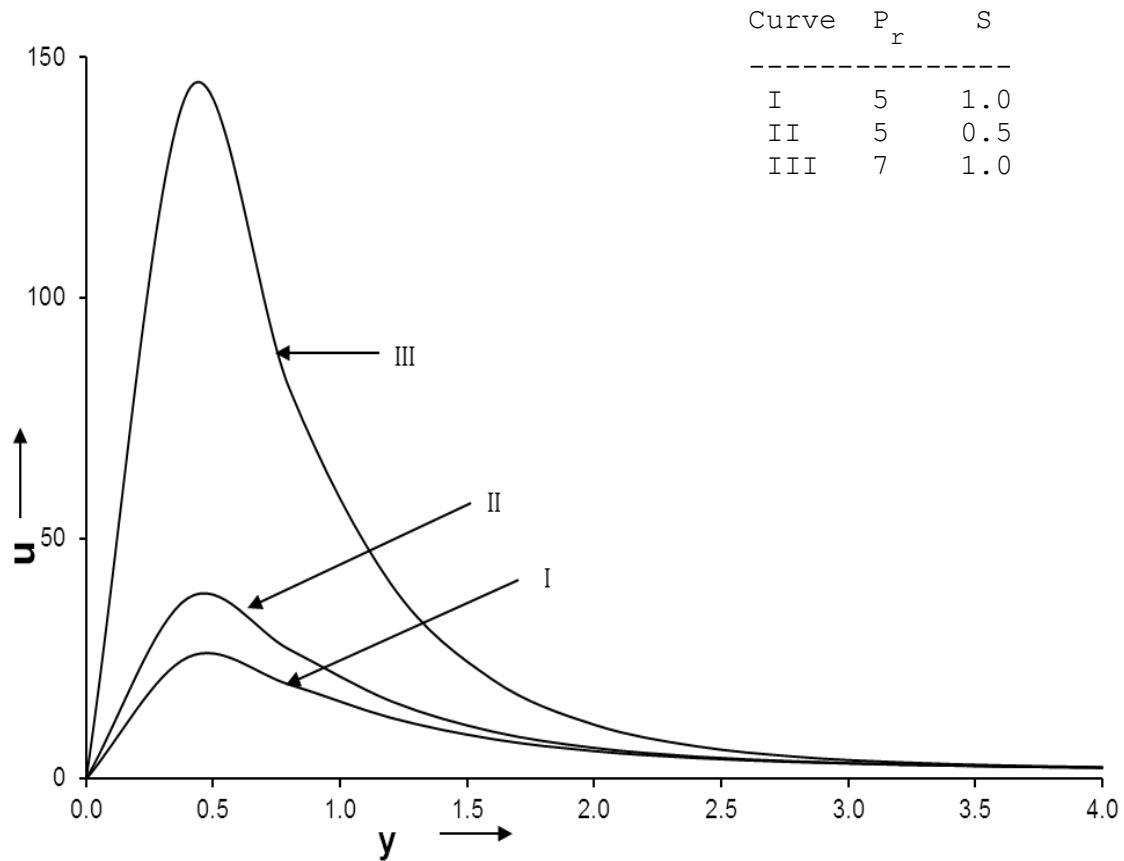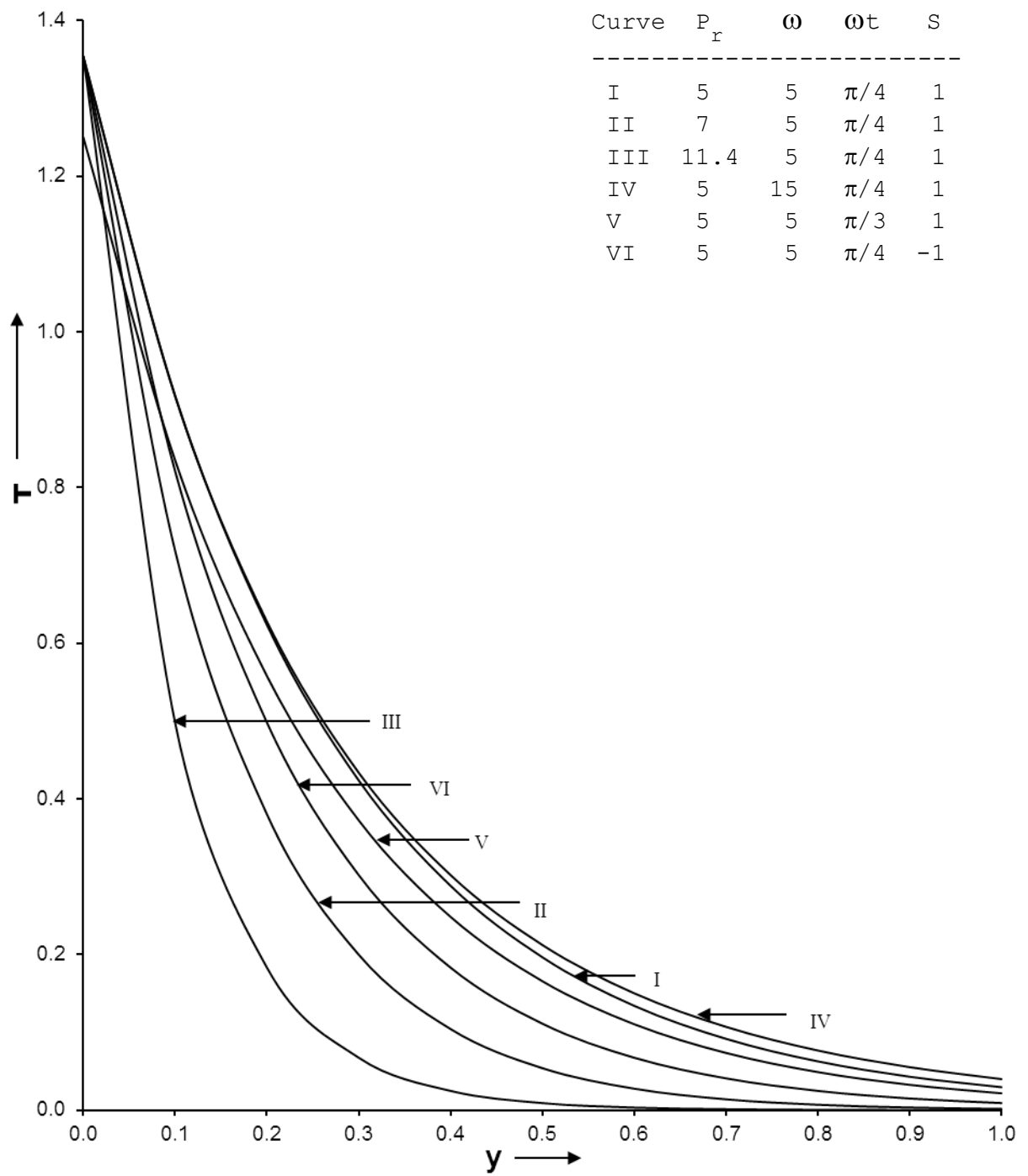**Fig- 1   Velocity distribution for various values of G_r, k_p, ω, ωt, K_0, G_c, M, S_c and K_c when ε = 0.5, P_r = 5 and S = 1**

Fig- 2   **Velocity distribution for various values of P_r and S when ε = 0.5, G_r = 5, ωt = π/4 ,K_p = 0.4, ω = 5, K_0 = 0.2, G_c = 2, M = 2, K_c = 1 & S_c = 0.22.**

Curve	P_r	ω	ωt	S
I	5	5	$\pi/4$	1
II	7	5	$\pi/4$	1
III	11.4	5	$\pi/4$	1
IV	5	15	$\pi/4$	1
V	5	5	$\pi/3$	1
VI	5	5	$\pi/4$	-1

Fig- 3 Temperature distribution for $\varepsilon = 0.5$

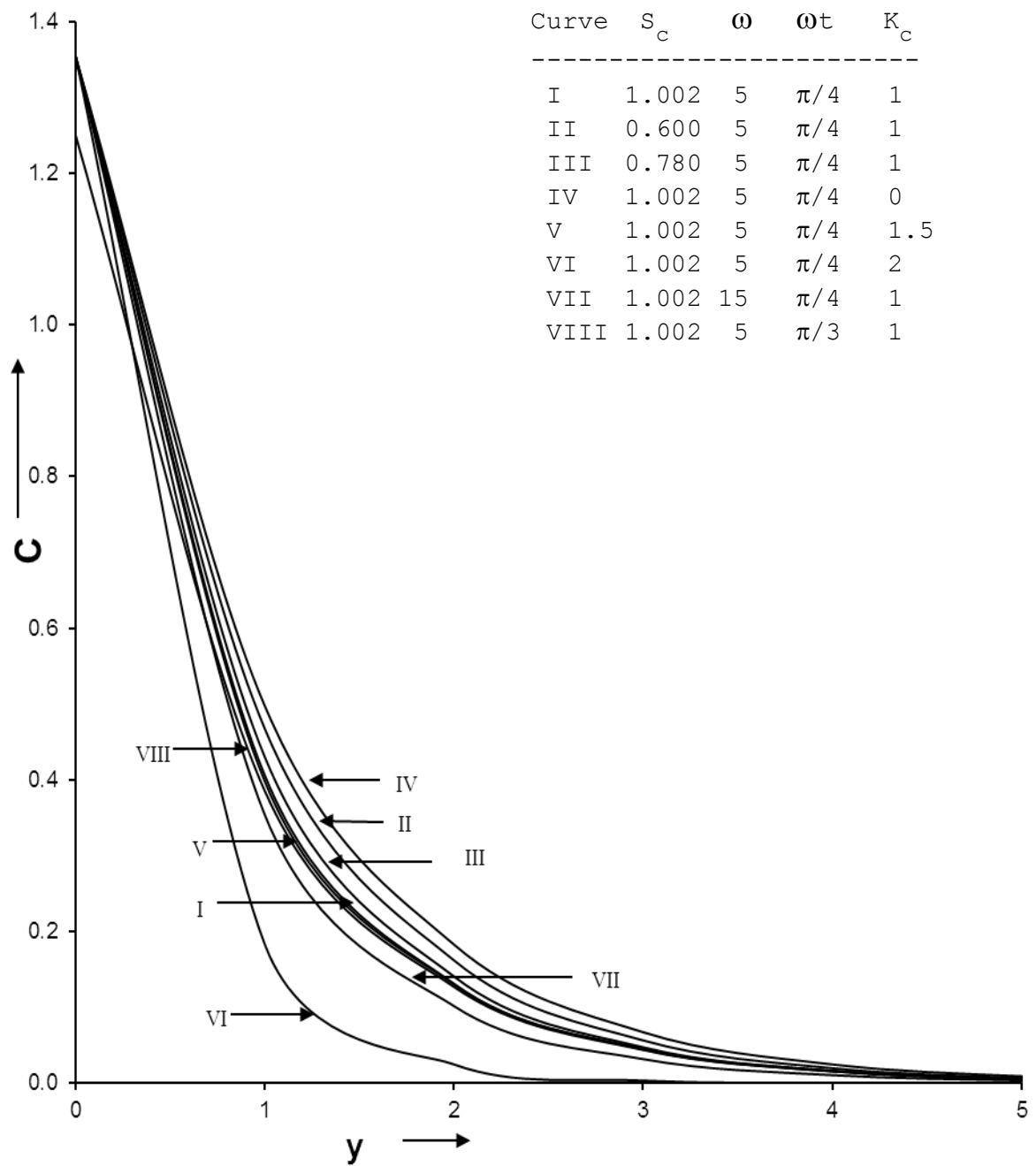| Curve	S_c	ω	ωt	K_c
I	1.002	5	$\pi/4$	1
II	0.600	5	$\pi/4$	1
III	0.780	5	$\pi/4$	1
IV	1.002	5	$\pi/4$	0
V	1.002	5	$\pi/4$	1.5
VI	1.002	5	$\pi/4$	2
VII	1.002	15	$\pi/4$	1
VIII	1.002	5	$\pi/3$	1

Fig- 4 Concentration distribution for $\varepsilon = 0.5$