# Squaring of circle and arbelos and the judgment of arbelos in choosing the real Pi value (Bhagavan Kaasi Visweswar method) 

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#### Abstract

It is a transcendental number. This number says firmly, that the squaring of a circle is impossible. New $\pi$ value was discovered in March 1998, and it is $\frac{14-\sqrt{2}}{4}=3.14644660942$. $\qquad$ It is an algebraic number. Squaring of a circle is done in this paper. With this new $\pi$ value, exact area of the arbelos is calculated and squaring of arbelos is also done. Arbelos of Archimedes chooses the real $\pi$ value.


Keywords: - Arbelos, area, circle, diameter, squaring, side

## I. INTRODUCTION

Circle and square are two important geometrical entities. Square is straight lined entity, and circle is a curvature. Perimeter and area of a square can be calculated easily with $a^{2}$ and $4 a$, where ' $a$ ' is the side of the square. A circle can be inscribed in a square. The diameter ' $d$ ' of the inscribed circle is equal to the side ' $a$ ' of the superscribed square. To find out the area and circumference of the circle, there are two formulae $\pi r^{2}$ and $2 \pi r$, where ' $r$ ' is radius and $\pi$ is a constant. $\pi$ constant is defined as "the ratio of circumference and diameter of its circle. So, to obtain the value for $\pi$, one must necessarily know the exact length of the circumference of the circle. As the circumference of the circle is a curvature it has become a very tough job to know the exact value of circumference. Hence, a regular polygon is inscribed in a circle. The sides of the inscribed polygon doubled many times, until, the inscribed polygon reaches, such that, no gap can be seen between the perimeter of the polygon and the circumference of the circle. The value of polygon is taken as the value of circumference of the circle. This value is $3.14159265358 \ldots$
In March 1998, it was discovered the exact $\pi$ value from Gayatri method. This new value is $\frac{14-\sqrt{2}}{4}=$ 3.14644660942.

In 1882, C.L.F. Lindemann and subsequently, Vow. K. Weirstrass and David Hilbert (1893) said that $3.14159265358 \ldots$ was a transcendental number. A transcendental number cannot square a circle. What is squaring of a circle ? One has to find a side of the square, geometrically, whose area is equal to the area of a circle. Even then, mathematicians have been trying, for many centuries, for the squaring of circle. No body could succeed except S. Ramanjan of India. He did it for some decimals of $3.14159265358 \ldots$. His diagram is shown below.


Then the square on BX is very nearly equal to the area of the circle, the error being less than a tenth of an inch when the diameter is 40 miles long.

With the discovery of $\frac{14-\sqrt{2}}{4}=\mathbf{3 . 1 4 6 4 4 6 6 0 9 4 2} \ldots$ squaring of circle has become very easy and is done here.
Archiemedes (240 BC) of Syracuse, Greece, has given us a geometrical entity called arbelos. The shaded area is called arbelos. It is present inside a larger semicircle but outside the two smaller semicircles having two different diameters.
In this paper squaring of circle and squaring of arbelos are done and are as follows.


## II. PROCEDURE

1. Draw a square and inscribe a circle.

Square $=\mathrm{ABCD}, \mathrm{AB}=\mathrm{a}=$ side $=1$
Circle. $\mathrm{EF}=$ diameter $=\mathrm{d}=$ side $=\mathrm{a}=1$
2. Semicircle on EF
$\mathrm{EF}=$ diameter $=\mathrm{d}=$ side $=\mathrm{a}=1$
Semicircle on EG
$\mathrm{EG}=$ diameter $=\frac{4 \mathrm{a}}{5}=\frac{4}{5}$
Semicircle on GF $=\mathrm{EF}-\mathrm{EG}=1-\frac{4}{5}=\frac{1}{5}$
$\mathrm{GF}=$ diameter $=\frac{\mathrm{a}}{5}=\frac{1}{5}$
3. Arbelos is the shaded region.

Draw a perpendicular line at G on EF diameter, which meets circumference at H. Apply Altitude theorem to obtain the length of GH.

$$
\mathrm{GH}=\sqrt{\mathrm{EG} \times \mathrm{GF}}=\sqrt{\frac{4}{5} \times \frac{1}{5}}=\frac{2}{5}
$$

4. Draw a circle with diameter $\mathrm{GH}=\frac{2}{5}=\mathrm{d}$

Area of the G.H. circle $=\frac{\pi \mathrm{d}^{2}}{4}=\left(\frac{\pi}{4}\right) \times \frac{2}{5} \times \frac{2}{5}=\frac{\pi}{25}$
5. $\quad$ Area of the G.H. circle $=$ Area of the arbelos

So, area of the arbelos $=\frac{\pi}{25}=\left(\frac{14-\sqrt{2}}{4}\right) \frac{1}{25}=\frac{14-\sqrt{2}}{100}$

## Part II: Squaring of circle present in the ABCD square

6. Diameter $=\mathrm{EF}=\mathrm{d}=\mathrm{a}=1$

Area of the circle $=\frac{\pi \mathrm{d}^{2}}{4}=\left(\frac{\pi}{4}\right) \times 1 \times 1=\frac{\pi}{4}$
7. To square the circle we have to obtain a length equal to $\frac{\pi}{4}$. It has been well established by many methods - more than one hundred different geometrical constructions - that $\pi$ value is $\frac{14-\sqrt{2}}{4}$. Let us find out a length equal to $\frac{\pi}{4}$.
8. Triangle KOL
$\mathrm{OK}=\mathrm{OL}=$ radius $=\frac{\mathrm{d}}{2}=\frac{\mathrm{a}}{2}=\frac{1}{2}$
$\mathrm{KL}=$ hypotenuse $=\frac{\sqrt{2} \mathrm{~d}}{2}=\frac{\sqrt{2} \mathrm{a}}{2}=\frac{\sqrt{2}}{2}$
$\mathrm{DJ}=\mathrm{JK}=\mathrm{LM}=\mathrm{MC}=\frac{\text { Side }- \text { hypotenuse }}{2}$
$=\left(\mathrm{a}-\frac{\sqrt{2} \mathrm{a}}{2}\right) \frac{1}{2}=\left(\frac{2-\sqrt{2}}{4}\right) \mathrm{a}=\frac{2-\sqrt{2}}{4}$
So, DJ $=\frac{2-\sqrt{2}}{4}$
9. $\mathrm{JA}=\mathrm{DA}-\mathrm{DJ}=\mathrm{a}-\left\{\left(\frac{2-\sqrt{2}}{4}\right) \mathrm{a}\right\}=\left(\frac{2+\sqrt{2}}{4}\right) \mathrm{a} . \quad$ So, JA $=\frac{2+\sqrt{2}}{4}$

Bisect JA twice
$\mathrm{JA} \rightarrow \mathrm{JN}+\mathrm{NA} \rightarrow \mathrm{NP}+\mathrm{PA}$
$=\frac{2+\sqrt{2}}{4} \rightarrow \frac{2+\sqrt{2}}{8} \rightarrow \frac{2+\sqrt{2}}{16}$
So, $\mathrm{PA}=\frac{2+\sqrt{2}}{16}$
10. $\mathrm{DP}=\mathrm{DA}$ side $-\mathrm{AP}=1-\left(\frac{2+\sqrt{2}}{16}\right)=\frac{14-\sqrt{2}}{16}$
11. $\mathrm{DP}=\frac{\pi}{4}=\frac{14-\sqrt{2}}{16} \quad$ (As per S.No. 7)
12. $\quad$ Draw a semicircle on $\mathbf{A D}=$ diameter $=1$
$\mathrm{AP}=\frac{2+\sqrt{2}}{16}$,
$\mathrm{DP}=\frac{14-\sqrt{2}}{16}$
13. Draw a perpendicular line on AD at P , which meets semicircle at Q . Apply Altitude theorem to obtain $P Q$ length
$\mathrm{PQ}=\sqrt{\mathrm{AP} \times \mathrm{DP}}=\sqrt{\left(\frac{2+\sqrt{2}}{16}\right) \times\left(\frac{14-\sqrt{2}}{16}\right)}=\frac{\sqrt{26+12 \sqrt{2}}}{16}$
14. Join QD

Now we have a triangle QPD

$$
\mathrm{PQ}=\frac{\sqrt{26+12 \sqrt{2}}}{16}, \quad \mathrm{PD}=\frac{14-\sqrt{2}}{16}
$$

Apply Pythagorean theorem to obtain QD length

$$
\mathrm{QD}=\sqrt{(\mathrm{PQ})^{2}+(\mathrm{PD})^{2}}=\sqrt{\left(\frac{\sqrt{26+12 \sqrt{2}}}{16}\right)^{2}+\left(\frac{14-\sqrt{2}}{16}\right)^{2}}=\frac{\sqrt{14-\sqrt{2}}}{4}
$$

15. $\frac{\sqrt{14-\sqrt{2}}}{4}$ is the length of the side of a square whose area is equal to the area of the inscribed circle

$$
\frac{\pi}{4} \text {, where } \pi=\frac{14-\sqrt{2}}{4}, \frac{\pi}{4}=\frac{14-\sqrt{2}}{16}
$$

$$
\text { Side }=\frac{\sqrt{14-\sqrt{2}}}{4}=\mathrm{a}
$$

Area of the square $=\mathrm{a}^{2}=\left(\frac{\sqrt{14-\sqrt{2}}}{4}\right)^{2}=\frac{14-\sqrt{2}}{16}$

## Thus squaring of circle is done.

## Part III: Squaring of arbelos

The procedure that has been adopted for squaring of circle is also adopted here. Here also the new $\pi$ value alone does the squaring of arbelos, because, the derivation of the new $\pi$ value $\frac{14-\sqrt{2}}{4}=3.14644660942 \ldots$ is based on the concerned line-segments of the geometrical constructions.
16. Arbelos $=$ EKHLFG shaded area. GH = Diameter (perpendicular line on EF diameter drawn from G upto H which meets the circumference of the circle.
Area of the arbelos $=$ Area of the circle with diameter $\mathbf{G H}=\frac{\pi}{25}$ of S.No. 4

$$
\text { So, } \frac{\pi}{25}=\left(\frac{14-\sqrt{2}}{4}\right) \frac{1}{25}=\frac{14-\sqrt{2}}{100}, \text { where } \pi=\frac{14-\sqrt{2}}{4}
$$

17. To square the arbelos, we have to obtain a length of the side of the square whose area is equal to area of the arbelos $\frac{14-\sqrt{2}}{100}$.
18. $\quad \mathrm{EG}=$ diameter $=\frac{4}{5} . I$ is the mid of EG .
$\mathrm{EI}+\mathrm{IG}=\mathrm{EG}=\frac{2}{5}+\frac{2}{5}=\frac{4}{5}$
So EI $=\frac{2}{5}$
19. $\quad$ Small square $=$ STBR

Side $=\mathrm{RB}=\mathrm{EI}=\frac{2}{5}$
Inscribe a circle with diameter $\frac{2}{5}=$ side, and with Centre Z. The circle intersects RT and SB diagonals at K' and L'. Draw a parallel line connecting RS side and BT side passing through K' and L'.
20. Triangle K' $\mathbf{Z L}^{\prime}$,
$\mathrm{ZK}^{\prime}=\mathrm{ZL},=$ radius $=\frac{1}{5}$
$\mathrm{K}^{\prime} \mathrm{L}^{\prime}=$ hypotenuse $=\frac{1}{5} \times \sqrt{2}=\frac{\sqrt{2}}{5}$
$\mathrm{RB}=\frac{2}{5}$
21. $L^{\prime} \mathrm{U}=\frac{\text { Side }- \text { hypotenuse }}{2}=\left(\frac{2}{5}-\frac{\sqrt{2}}{5}\right) \frac{1}{2}=\frac{2-\sqrt{2}}{10}$
22. $\operatorname{So}, L^{\prime} \mathrm{U}=\frac{2-\sqrt{2}}{10}=\mathrm{BU}$

BT $=$ Side of the square $=\frac{2}{5}$
$\mathrm{UT}=\mathrm{BT}-\mathrm{BU}=\frac{2}{5}-\left(\frac{2-\sqrt{2}}{10}\right)=\frac{2+\sqrt{2}}{10}$
So, UT $=\frac{2+\sqrt{2}}{10}$
23. Bisect UT twice
$\mathrm{UT} \rightarrow \mathrm{UV}+\mathrm{VT} \rightarrow \mathrm{VX}+\mathrm{XT}$
$\frac{2+\sqrt{2}}{10} \longrightarrow \frac{2+\sqrt{2}}{20} \longrightarrow \frac{2+\sqrt{2}}{40}$
So, $\mathrm{XT}=\frac{2+\sqrt{2}}{40}$
24. $\quad \mathrm{BT}=\frac{2}{5} ; \quad \mathrm{XT}=\frac{2+\sqrt{2}}{40}$
$\mathrm{BX}=\mathrm{BT}-\mathrm{XT}=\frac{2}{5}-\left(\frac{2+\sqrt{2}}{40}\right)$
$B X=\frac{14-\sqrt{2}}{40}$
25. Draw a semi circle on $B T$ with $\frac{2}{5}$ as its diameter.
26. Draw a perpendicular line on BT at X which meets semicircle at Y .

XY length can be obtained by applying Altitude theorem
$\mathrm{XY}=\sqrt{\mathrm{BX} \times \mathrm{XT}}=\sqrt{\left(\frac{14-\sqrt{2}}{40}\right) \times\left(\frac{2+\sqrt{2}}{40}\right)}$
$=\frac{\sqrt{26+12 \sqrt{2}}}{40}=\mathrm{XY}$
27. Triangle BXY
$B X=\frac{14-\sqrt{2}}{40}$,

$$
X Y=\frac{\sqrt{26+12 \sqrt{2}}}{40}
$$

BY can be obtained by applying Pythagorean Theorem
$B Y=\sqrt{B X^{2}+X Y^{2}}=\sqrt{\left(\frac{14-\sqrt{2}}{40}\right)^{2}+\left(\frac{\sqrt{26+12 \sqrt{2}}}{40}\right)^{2}}=\frac{\sqrt{14-\sqrt{2}}}{10}$
BY is the required side of the square whose area is equal to the area of the arbelos of Archimedes.
Side $=\frac{\sqrt{14-\sqrt{2}}}{10}=\mathrm{a}$
Area of the square on $B Y=a^{2}=\left(\frac{\sqrt{14-\sqrt{2}}}{10}\right)^{2}=\frac{14-\sqrt{2}}{100}$ of S.No. 16
= Area of arbelos

## Part-IV (The Judgment on the Real Pi value)

In this paper, the correctness of the area of the arbelos of Archimedes can be confirmed. How? Here are the following steps.
28. New $\pi$ value $\frac{14-\sqrt{2}}{4}$ gives area of the arbelos as $\frac{14-\sqrt{2}}{100}=0.12585786437$. Whereas the official $\pi$ value $3.14159265358 \ldots$ gives the area of the arbelos as $\frac{\pi d^{2}}{4}=3.14159265358 \times \mathrm{dx} \mathrm{dx} \frac{1}{4}$ $\mathrm{d}=\mathrm{GH}=\frac{2}{5}$ of S.No. 3
$3.14159265358 \times \frac{2}{5} \times \frac{2}{5} \times \frac{1}{4}=0.12566370614$
Thus, the following are the two different values for the same area of the arbelos.
Official $\boldsymbol{\pi}$ value gives $=\mathbf{0 . 1 2 5 6 6 3 7 0 6 1 4}$
New $\pi$ value gives $=\mathbf{0 . 1 2 5 8 5 7 8 6 4 3 7}$
29. Diameter of the arbelos circle $\mathrm{GH}=\mathrm{d}=\frac{2}{5}$

Square of the diameter $=\mathrm{d}^{2}=\frac{2}{5} \times \frac{2}{5}=\frac{4}{25}$
Reciprocal of the square of the diameter $=\frac{1}{\mathrm{~d}^{2}}=\frac{1}{\frac{4}{25}}=\frac{25}{4}$
30. Area of arbelos, if multiplied with $\frac{25}{4}$ we get the area of the inscribed circle in the ABCD square

Area of the circle $=\frac{\pi \mathrm{d}^{2}}{4}$
$\mathrm{d}=\mathrm{a}=1, \quad \pi=\frac{14-\sqrt{2}}{4}$
$=\frac{14-\sqrt{2}}{4} \times 1 \times 1 \times \frac{1}{4}=\frac{14-\sqrt{2}}{16}$
31. Area of the arbelos $\times$ reciprocal of the square of the arbelos circle's diameter $=$ Area of the inscribed circle in ABCD square

$$
\frac{14-\sqrt{2}}{100} \times \frac{25}{4}=\frac{14-\sqrt{2}}{16}
$$

S. No. 16 S.No. 29 S.No. 30
32. Let us derive the following formula from the dimensions of square ABCD

ABCD square, $\mathrm{AB}=$ side $=\mathrm{a}=1$
$\mathrm{AC}=\mathrm{BD}=$ diagonal $=\sqrt{2} \mathrm{a}=\sqrt{2}$, Perimeter of of ABCD square $=4 \mathrm{a}$
Perimeter of ABCD square
Half of 7 times of $A B$ side of square $-\frac{1}{4}$ th of diagonal
$=\frac{4 \mathrm{a}}{\frac{7 \mathrm{a}}{2}-\frac{\sqrt{2} \mathrm{a}}{4}}=\frac{4}{\frac{7}{2}-\frac{\sqrt{2}}{4}}=\frac{4}{\frac{14-\sqrt{2}}{4}}=\frac{16}{14-\sqrt{2}}$
33. In this step, above 2 steps (S.No. 29 and 32) are brought in.

Arbelos area $x \frac{25}{4} \times \frac{16}{14-\sqrt{2}}=$ Area of the ABCD square, equal to 1 .
As there are two values representing for the same area of the arbelos, let us verify, with the both the values, which is ultimately the correct one.
Arbelos area of official $\pi$ value 3.14159265358
$0.12566370614 \times \frac{25}{4} \times \frac{16}{14-\sqrt{2}}=0.99845732137$ and
Arbelos area of new $\pi$ value $\frac{14-\sqrt{2}}{4}$
$\frac{14-\sqrt{2}}{100} \times \frac{25}{4} \times \frac{16}{14-\sqrt{2}}=1$
This process is done by understanding the actual and exact interrelationship among, 1. area of the ABCD square, 2. area of the inscribed circle in ABCD square and, 3. area of the arbelos of Archimedes.
34. For questions "why", "what" and "how" of each step, the known mathematical principles are insufficient, unfortunately.
So, as the exact area of ABCD square equal to 1 is obtained finally with new $\pi$ value. The new $\pi$ value equal to $\frac{14-\sqrt{2}}{4}$ is confirmed as the real $\pi$ value. This is the Final Judgment of arbelos of Archimedes.

## III. CONCLUSION

This study, proves, that squaring of a circle is not impossible, and no more an unsolved geometrical problem. The belief in its (squaring of circle) impossibility is due to choosing the wrong number 3.14159265358... as $\pi$ value. The new $\pi$ value $\frac{14-\sqrt{2}}{4}$ has done it. The arbelos of Archimedes has also chosen the real $\pi$ value in association with the inscribed circle and the ABCD superscribed square.

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