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Determination of The Winding Inductances Of A Two-Phase Machine.

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Abstract: - An accurate method for determining the winding inductances of a 2-phase Stator winding is herein presented. A 2-phase stator winding is considered. The process begins with understanding the winding configuration, obtaining the winding function from that, and considering the air-gap function. Comparison is made for the inductances of the same stator windings considering saliency and non-saliency. It is observed that in the case of salient pole rotors, the inductances of the stator winding depends on the rotor position. On the other hand, the inductances of the stator winding in the case of non-saliency is constant. Non-linear effects on the magnetizing inductances were not considered.

***Keywords**: Magnetizing Inductance, Winding function, Harmonics, Air-gap function.*

I. INTRODUCTION

Electric machines have assumed an enviable place in the drive of development and civilization, world over. More than 95% of world electricity are produced from electric machines. Again, electric machines are the work horse of industries, and find extensive applications in several domestic, transport, health, and agricultural facilities. One property of immense importance to the electric machine is the winding inductances. Often times though, the stator windings develop fault, maybe due to high temperature, power system transients, or some other factors [1]. Because these machines are expensive apparatus, it is often the practice to re-coil the windings and restore the machine to use. Moreover, for the purpose of research, or to achieve some other machine performance, it may be required to replace the windings of a stator to achieve certain design objectives. Hence, it is important that at such times, the engineer or the designer must be able to determine the new inductances of the machine with respect to the new stator winding.

Whether for design or maintenance purposes, determination of winding inductances is of immense importance, as this affects next to the entire behavior of the machine, hence, its accurate computation is of huge importance.While the major interest in inductance determination would be to determine the self and mutual inductances of the windings involved, a unique determination of the inductances associated with the leakage flux can only be calculated or approximated from design considerations [2]. Hence, the leakage inductance is not dealt with in this work. However, it is the magnetizing inductances that is the interest of this work.

II. WINDING DESCRIPTION

A double-layer, integral-slot winding with chorded coils will be used. Taking 12 slots per pole, the stator number of slots is 24. Each slot holds 18 conductors, which could be of either or both phases. Windings were distributed as near sinusoidal as possible. References here include [2, 3, 4]. Furthermore, the windings of both phases are identical but displaced from each other by an angle of 90^o. The winding clock diagram appears in figure 1. The following can easily be observed from the winding clock diagram:
Number of slots = 24; Slot angular pitch = 15^o; Phase belt = 10 slots; Phase spread = 150^o; phase shift = 90^o; Number of poles =2; Pole-pitch = 12 slots per pole.

III. ASSUMPTION

Each winding section is aligned axially within the air gap. This is to say that the wire is neither slanted in the circumferential direction nor tilted in the radial direction as it passes through the air gap. Also, the MMF in the iron is neglected.

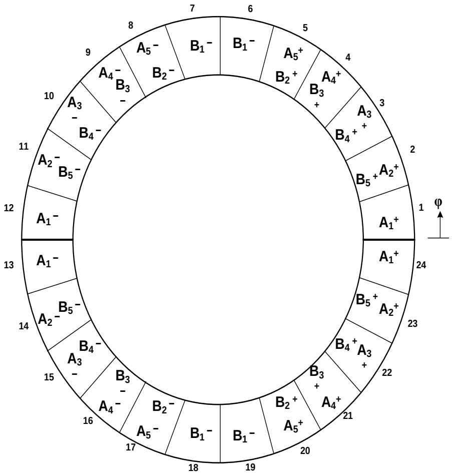

Figure 1: *Winding Clock Diagram*

IV. WINDING FUNCTION

The Winding function method of analysis of the Stator winding is presented here. This is because of the well-known fact that the Winding Function is the basis of calculating machine inductances [5]. Again, the power quality of the machine is a function of the shape of the air-gap MMF due to stator windings. Equation (1) represents the relationship between the air gap MMF, F_s, the winding function, $N_s(\phi_s)$, and the stator current, i_s. Subscript "s" represents stator quantity.

$$F_s = N_s(\phi_s)i_s \tag{1}$$

Where ϕ_s is an angular displacement along the stator inner circumference. The winding function (WF) methodology developed in [6] was employed in this analysis. The turns function, $n(\phi_s)$ and the average turns function, n_{ave} are given by:

$$n(\phi_s) = No. of turns in integration path \tag{2}$$

$$n_{ave} = \frac{1}{2\pi}\int_0^{2\pi} n(\phi_s)\, d\phi_s \tag{3}$$

The winding function is then given by:

$$N_s(\phi_s) = n(\phi_s) - n_{ave} \tag{4}$$

Figure 2 and 3 shows the actual winding function, in stair-case form.

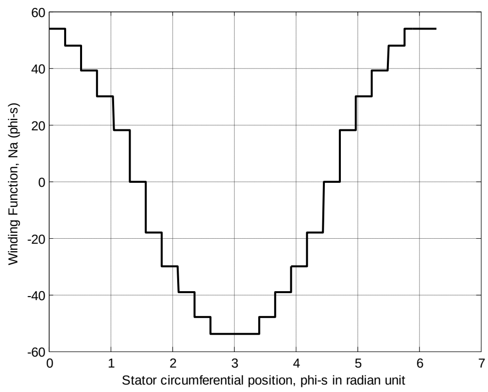

Figure 2: *Actual Winding function of the a-phase winding*

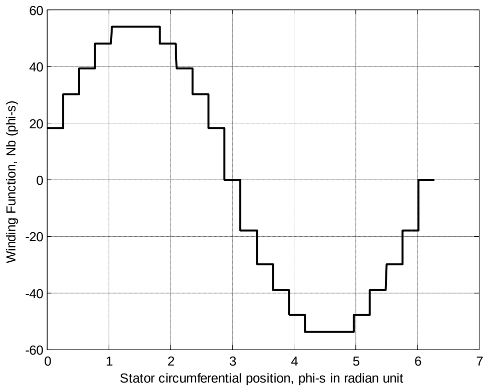

Figure 3:*Actual Winding Function of the b-phase winding*

V. HARMONICS ANALYSIS

The method of Fourier series [7] was applied to obtain the various harmonics present in the winding functions expressed in Figures 2 and 3. The Fourier series of the winding function was performed and various plots obtained using the MATLAB tool [8]. The fundamental components of the winding functions for the two stator windings are:

$$N_{as} = N_s cos(\phi_s - \delta) \tag{5}$$

$$N_{bs} = N_s sin(\phi_s - \delta) \tag{6}$$

Where, δ is the phase shift, and subscript s refers to stator variables.

Figures 4 and 5 give a picture of the harmonic contents in the winding function, up to the 50th non-zero harmonic (odd harmonics only). This is a reflection of the distortion that will be present in the stator voltage and current waveform. It is put in better perspective by figure 6.

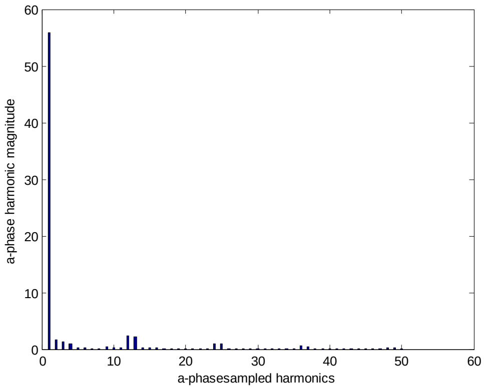

Figure 4:*Harmonics of the a-phase winding function*

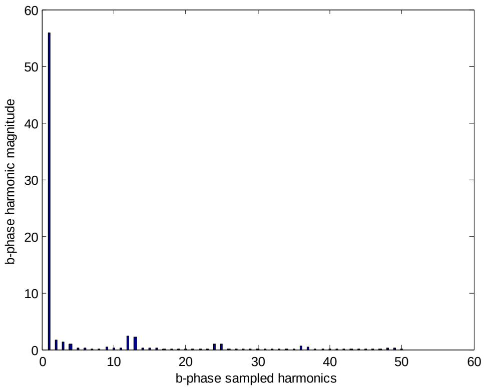

Figure 5:*Harmonics of the b-phase winding function*

Figure 6 shows a close approximation of the fundamental component of the Winding Function to the resultant Winding Function. Thus, the value of the fundamental component of the Winding Function can be used for inductance calculations, with a little error tolerance. These values are given for the two phases as:

$$N_{as} = 55.8254\angle - 0.1309rad \tag{7}$$

$$N_{bs} = 55.8254\angle 1.4399rad \tag{8}$$

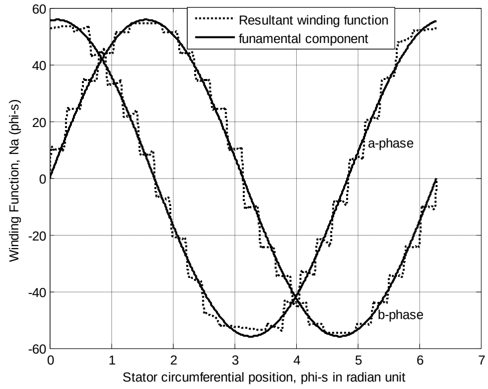

Figure 6:*Comparison between the resultant winding function and the fundamental component of the function*.

Hence, the effective number of stator turns will be taken as $N_s = 55.8254$. It is clear that the influence of winding design features which have not been considered in the analysis are accounted for in N_s.

Air-gap Function

The air gap function requires definition. It will be helpful to describe the air gap of the machine over an angular displacement of 2π, as shown in Figure 7a,

where g_1= the minimum air gap length

g_2= the maximum air gap length

β = the pole arc/pole pitch ratio

The inverse air-gap function is also shown in figure 7b. See Appendix A for parameter values. For the purpose of inductance computation, it is beneficial to obtain the inverse air gap function, rather than the air gap function itself. The Fourier series approach can be employed to determine the inverse air-gap function more accurately.

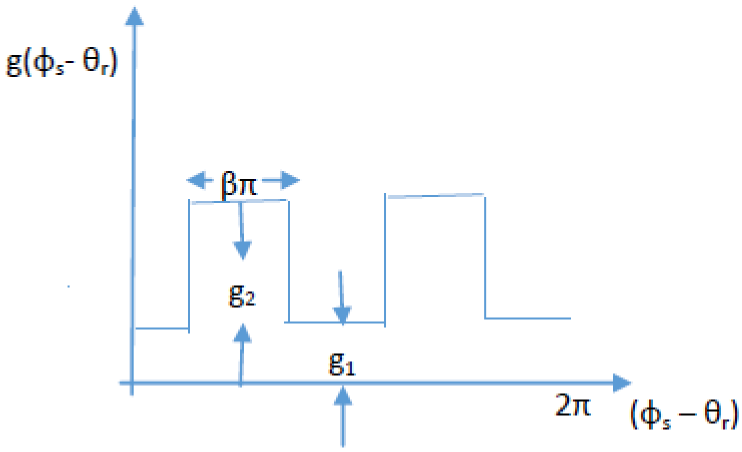

***Figure 7a**: Approximate Air-Gap function*

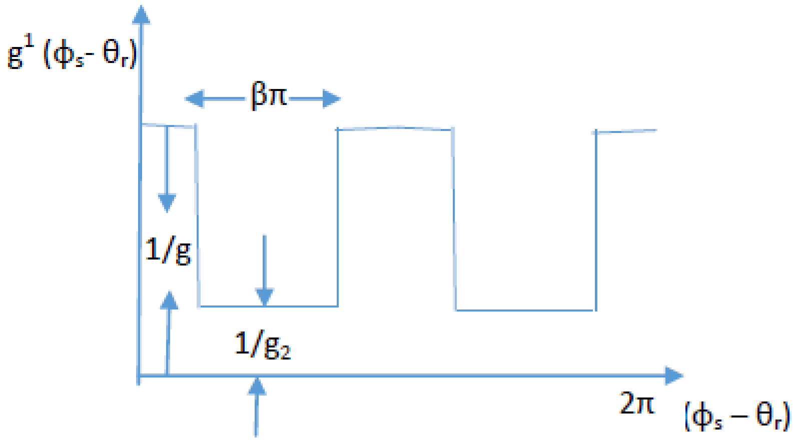

Figure 7b: *Approximate inverse Air Gap Function*

Another accurate approach to determine the inverse air-gap function is as follows:

$$\alpha_1 = \frac{(1-\beta)}{g_1} + \frac{\beta}{g_2} \tag{9}$$

$$\alpha_2 = \frac{2}{\pi} sin\beta\pi \left(\frac{1}{g_1} - \frac{1}{g_2}\right) \tag{10}$$

Then $g(\phi_s) = \frac{1}{\alpha_1 + \alpha_2 cos2(\phi_s - \theta_r)}$ (11)

Therefore

$$g^{-1}(\phi_s) = \alpha_1 + \alpha_2 cos2(\phi_s - \theta_r) \tag{12}$$

Where θ_r describes the rotor position. Obviously, both figure 7 and equations (9) through (12) accounted for machine saliency. However, if a round rotor is considered, then:

$$g_1 = g_2 \tag{13}$$

Substituting the parameter values of Appendix A into equations (9) through (12), the inverse air gap function will be obtained as follows:

With saliency:

$$g^{-1}(\phi_s) = 1373.3 + 1977.8 \cos 2(\phi_s - \theta_r) \tag{14}$$

Without Saliency:

$$g^{-1}(\phi_s) = 3333.3333 \tag{15}$$

VI. MAGNETIZING INDUCTANCE (NON-LINEAR EFFECTS NEGLECTED)

The determination of self-inductance requires the flux linking a winding due to its own current to be computed. In the case of mutual inductance, the flux linking one winding due to current flowing in another winding is required [2].Generally, the differential flux passing through a differential volume of cross-sectional area $(rd\phi)l$ and length g is given by [6],

$$d\Phi = FdP = \frac{\mu_0 rl}{g} Fd\phi_s \tag{16}$$

Integrating (16), bearing in mind that F is a function of ϕ_s:

$$\Phi = \frac{\mu_0 rl}{g} \int_0^{2\pi} F(\phi_s)\, d\phi_s \tag{17}$$

Where P = Permeance, r = inner stator radius,l = axial length of the air gap, and μ_0=Permittivity of free space.

It is common knowledge that flux linkage is expressed as:

$$\lambda = N\Phi = \text{iL} \tag{18}$$

Where L is inductance and I is current.

Equation (17) is the flux linking one turn of an N_s turn winding. If it is recalled that N_s is now a function of ϕ_s, and the inverse of the air-gap length a function of $(\phi_s - \theta_r)$, then for a winding with $N_s(\phi_s)$ effective number of turns, the flux linkage of winding A due to current in winding B will be given as:

$$\lambda_{AB} = \mu_0 rl \int_0^{2\pi} N_{as}(\phi_s) F_B(\phi_s) g^{-1}(\phi_s - \theta_r)\, d\phi_s \tag{19}$$

If equation (1) and (18) are kept in mind, then (19) can be solved for the mutual inductance between windings A and B:

$$L_{AB} = \mu_0 rl \int_0^{2\pi} N_{as}(\phi_s) N_{bs}(\phi_s) g^{-1}(\phi_s - \theta_r)\, d\phi_s \tag{20}$$

From equations (18) through (20), it is clear that reciprocity holds since the order of the two windings may be interchanged. Hence,

$$L_{AB} = L_{BA} \tag{21}$$

The above equations are also valid for cases where windings A and B are one and the same. Hence the magnetizing inductances of windings A and B, respectively, are given by:

$$L_{AA} = \mu_0 rl \int_0^{2\pi} N_{as}^2(\phi_s) g^{-1}(\phi_s - \theta_r)\, d\phi_s \tag{22}$$

$$L_{BB} = \mu_0 rl \int_0^{2\pi} N_{bs}^2(\phi_s) g^{-1}(\phi_s - \theta_r)\, d\phi_s \tag{23}$$

With saliency:

Substituting equations (5), (6) (neglecting δ), and (12) into equations (20), (22), and (23), and solving the same, the following results will be obtained:

$$L_{AA} = \mu_0 rl\pi N_{as}^2 \left(\alpha_1 + \frac{\alpha_2}{2} cos2\theta_r\right) \tag{24}$$

$$L_{BB} = \mu_0 rl\pi N_{bs}^2 \left(\alpha_1 + \frac{\alpha_2}{2} cos2\theta_r\right) \tag{25}$$

$$L_{AB} = \mu_0 rl\pi N_{as} N_{bs} \frac{\alpha_2}{2} sin2\theta_r \tag{26}$$

Without Saliency:

Considering equations (9) and (10) when equation (13) is true, it will be observed that

$$\alpha_1 = \frac{1}{g} \tag{27}$$

$$\alpha_2 = 0 \tag{28}$$

Substituting equations (27) and (28) into equations (24), (25), and (26) results to:

$$\mathrm{L_{AB}} = 0 \tag{27}$$

$$\mathrm{L_{AA}} = \frac{\mu_0 \mathrm{rl}\pi}{\mathrm{g}} N_{as}^2 \tag{28}$$

$$\mathrm{L_{BB}} = \frac{\mu_0 \mathrm{rl}\pi}{\mathrm{g}} N_{bs}^2 \tag{29}$$

Using the parameter values of Appendix A, Table 1 and figure 8 were obtained.

The Q- and D-axes Magnetizing Inductances

Often times in the modeling and simulation of electric machines, it is convenient to eliminate all time-varying inductances by a change of variables from the stationary machine variables to the q-d variables of a convenient reference frame. If the rotor reference frame is assumed, then the q-d magnetizing inductances can be estimated as follows:

The transformation matrix to be used is:

$$T_m = \begin{bmatrix} cos\theta_r & sin\theta_r \\ sin\theta_r & -cos\theta_r \end{bmatrix} \tag{30}$$

Let $L_s = \begin{bmatrix} L_{AA} & L_{AB} \\ L_{BA} & L_{BB} \end{bmatrix}$ (31)

Then $L_{mdq} = T_m L_s (T_m)^{-1}$ (32)

$$L_{mdq} = diag[L_{md} \quad L_{mq}] \tag{33}$$

Equation (32) yields equation (33), where L_{mq} and L_{md} are respectively the q- and d-axes inductances required, θ_r is an angular displacement of the rotor.

Discussion and Conclusion

The leakage inductances were not considered in this work. Nevertheless, the self-inductances of the windings will be the sum of their leakage inductances and their magnetizing inductances. Of course, non-linear effects on the inductances are not considered at this stage.

Table 1: *The magnetizing inductances*

Magnetizing inductance	With Saliency	Without Saliency
L_{AA}	$0.0761 + 0.0548\cos 2\theta_r$	0.1846
L_{BB}	$0.0761 - 0.0548\cos 2\theta_r$	0.1846
L_{AB}	$0.0548\sin 2\theta_r$	0
L_{BA}	$0.0548\sin 2\theta_r$	0

Stator and Air-gap Dimensions
r = 0.06m
l = 0.075m
g_1 = 0.0003m
$g_2 = 50g_1$
$\mu_o = 4\pi e\text{-}7$

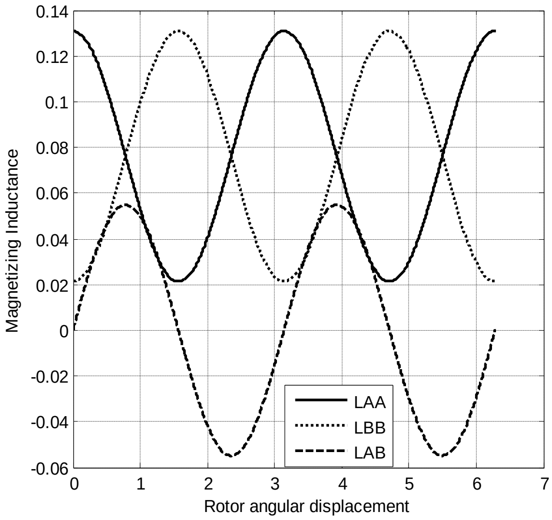

Figure 8: *The variation of Stator inductances with rotor angular positionin the case of saliency.*

It will be observed that the magnetizing inductances are a function of the rotor position in the case of salient pole machines, while they are constants in the case of round rotor machines. Moreover, considering that the windings are displaced in space 90^o from each other, it is clear that their mutual inductance will be zero in the case of round rotor machines, as also was obtained. However, the saliency of the rotor, in the case of salient pole rotors, leads to some flux linkage between the windings, resulting to a non-zero mutual inductance.
The process applied in this work can be applied as well to unbalanced windings and to 3-phase windings to obtain accurate values of the magnetizing inductances of any stator winding.

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