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Random Fixed Point TheoremsIn Metric Space

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Abstract: - The present paper deals with some fixed point theorem for Random operator in metric spaces. We find unique Random fixed point operator in closed subsets of metric spaces by considering a sequence of measurable functions.
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I. INTRODUCTION

Fixed point theory is one of the most dynamic research subjects in nonlinear sciences. Regarding the feasibility of application of it to the various disciplines, a number of authors have contributed to this theory with a number of publications. The most impressing result in this direction was given by Banach, called the Banach contraction mapping principle: Every contraction in a complete metric space has a unique fixed point. In fact, Banach demonstrated how to find the desired fixed point by offering a smart and plain technique. This elementary technique leads to increasing of the possibility of solving various problems in different research fields. This celebrated result has been generalized in many abstract spaces for distinct operators.

Random fixed point theory is playing an important role in mathematics and applied sciences. At present it received considerable attentation due to enormous applications in many important areas such as nonlinear analysis, probability theory and for thestudy of Random equations arising in various applied areas.

In recent years, the study of random fixed point has attracted much attention some of the recent literature in random fixed point may be noted in [1, 5, 6, 8]. The aimof this paper is to prove some random fixed point theorem.Before presenting our results we need some preliminaries that include relevant definition.

II. PRELIMINARIES

Throughout this paper, (Ω,Σ)denotes a measurable space, X be a metric space andC is non-empty subset of X.
Definition 2.1: A function $f{:}\Omega\to C$ is said to be measurable if $f\,'(B\cap C)\in\Sigma$for every Borelsubset B of X.
Definition 2.2: A function $f{:}\Omega\times C\to C$ is said to be random operator, iff $(.,X){:}\Omega\to C$ is measurable for every $X\in C$.
Definition 2.3: A random operator $f{:}\Omega\times C\to C$ is said to be continuous if for fixed$t\in\Omega$,$f(t,.){:}C\times C$ is continuous.
Definition 2.4: A measurable function $g{:}\Omega\to C$ is said to be random fixed pointof the random operator $f{:}\Omega\times C\to C$, if $f(t, g(t))=g(t)$, $\forall\, t\in\Omega$.

III. Main Results

Theorem (3.1): Let (X, d) be a complete metric space and E be a continuous self-mappings such that

$$d(E\{\xi,g(\xi)\},\ E\{\xi,h(\xi)\}) \le \alpha\left[\frac{d\big(g(\xi),E\{\xi,g(\xi)\}\big)d\big(h(\xi),E\{\xi,h(\xi)\}\big)d\big(g(\xi),E\{\xi,h(\xi)\}\big) + d\big(g(\xi),h(\xi)\big)d\big(h(\xi),E\{\xi,g(\xi)\}\big)d\big(h(\xi),E\{\xi,h(\xi)\}\big)}{[d\big(g(\xi),h(\xi)\big)]^2 + d\big(g(\xi),E\{\xi,h(\xi)\}\big)d\big(h(\xi),E\{\xi,h(\xi)\}\big)}\right]$$

$$+\beta\left[d\big(g(\xi),E\{\xi,g(\xi)\}\big)+d\big(h(\xi),E\{\xi,h(\xi)\}\big)\right]$$

$$+\gamma\left[d\big(g(\xi),E\{\xi,h(\xi)\}\big)+d\big(h(\xi),E\{\xi,g(\xi)\}\big)\right]$$

$$+\eta\frac{d\left(g(\xi),E\{\xi,g(\xi)\}\right)d\left(h(\xi),E\{\xi,h(\xi)\}\right)}{d\left(g(\xi),h(\xi)\right)} \qquad (3.1.1)$$

$$+\delta[d\left(g(\xi),h(\xi)\right)]$$

Forall $g(\xi)$) and $h(\xi)\in X$ with $g(\xi)\neq h(\xi)$, where $\alpha,\beta,\gamma,\eta,\delta:R^{+}\rightarrow[0,1)$ are such that $\alpha+2\beta+2\gamma+\delta+\eta\prec 1$, then E has a unique fixed point in X.

Proof: Let $\{g_n\}$be a sequence, define d as follows

$$g_n\{\xi\}=E\{\xi,g_{n-1}(\xi)\}\ ,\ n=1,2,3,4\ldots$$

If $g_n\{\xi\}=g_{n+1}\{\xi\}$ for some n then the result follows immediately.

So let $g_n(\xi)\neq g_{n+1}(\xi)$ for all n then

$$d\left(g_n(\xi),g_{n+1}(\xi)\right)=d\left(E\{\xi,g_{n-1}(\xi)\},\ E\{\xi,g_n(\xi)\}\right)$$

$$\leq\alpha\left[\frac{d\left(g_{n-1}(\xi),E\{\xi,g_{n-1}(\xi)\}\right)d\left(g_n(\xi),E\{\xi,g_n(\xi)\}\right)d\left(g_{n-1}(\xi),E\{\xi,g_n(\xi)\}\right)+d\left(g_{n-1}(\xi),g_n(\xi)\right)d\left(g_n(\xi),E\{\xi,g_{n-1}(\xi)\}\right)d\left(g_n(\xi),E\{\xi,g_n(\xi)\}\right)}{\left[d\left(g_{n-1}(\xi),g_n(\xi)\right)\right]^2+d\left(g_{n-1}(\xi),E\{\xi,g_n(\xi)\}\right)d\left(g_n(\xi),E\{\xi,g_n(\xi)\}\right)}\right]$$

$$+\beta\left[d\left(g_{n-1}(\xi),E\{\xi,g_{n-1}(\xi)\}\right)+d\left(g_n(\xi),E\{\xi,g_n(\xi)\}\right)\right]$$

$$+\gamma\left[d\left(g_{n-1}(\xi),E\{\xi,g_n(\xi)\}\right)+d\left(g_n(\xi),E\{\xi,g_{n-1}(\xi)\}\right)\right]$$

$$+\eta\frac{d\left(g_{n-1}(\xi),E\{\xi,g_{n-1}(\xi)\}\right)d\left(g_n(\xi),E\{\xi,g_n(\xi)\}\right)}{d\left(g_{n-1}(\xi),g_n(\xi)\right)}$$

$$+\delta.d\left(g_{n-1}(\xi),g_n(\xi)\right)$$

$$\leq\alpha\left[\frac{d\left(g_{n-1}(\xi),g_n(\xi)\right)d\left(g_n(\xi),g_{n+1}(\xi)\right)d\left(g_{n-1}(\xi),g_{n+1}(\xi)\right)+d\left(g_{n-1}(\xi),g_n(\xi)\right)d\left(g_n(\xi),g_n(\xi)\right)d\left(g_n(\xi),g_{n+1}(\xi)\right)}{\left[d\left(g_{n-1}(\xi),g_n(\xi)\right)\right]^2+d\left(g_{n-1}(\xi),g_{n+1}(\xi)\right)d\left(g_n(\xi),g_{n+1}(\xi)\right)}\right]$$

$$+\beta\left[d\left(g_{n-1}(\xi),g_n(\xi)\right)+d\left(g_n(\xi),g_{n+1}(\xi)\right)\right]$$

$$+\gamma\left[d\left(g_{n-1}(\xi),g_{n+1}(\xi)\right)+d\left(g_n(\xi),g_n(\xi)\right)\right]$$

$$+\eta\frac{d\left(g_{n-1}(\xi),g_n(\xi)\right)d\left(g_n(\xi),g_{n+1}(\xi)\right)}{d\left(g_{n-1}(\xi),g_n(\xi)\right)}$$

$$+\delta.d\left(g_{n-1}(\xi),g_n(\xi)\right)$$

$$\leq\alpha\left[\frac{d\left(g_{n-1}(\xi),g_n(\xi)\right)d\left(g_n(\xi),g_{n+1}(\xi)\right)d\left(g_{n-1}(\xi),g_{n+1}(\xi)\right)}{d\left(g_{n-1}(\xi),g_{n+1}(\xi)\right)d\left(g_n(\xi),g_{n+1}(\xi)\right)}\right]$$

$$+\beta\left[d\left(g_{n-1}(\xi),g_n(\xi)\right)+d\left(g_n(\xi),g_{n+1}(\xi)\right)\right]$$

$$+\gamma\left[d\left(g_{n-1}(\xi),g_n(\xi)\right)+d\left(g_n(\xi),g_{n+1}(\xi)\right)\right]$$

$$+\eta.d\left(g_n(\xi),g_{n+1}(\xi)\right)$$

$$+\delta.d\left(g_{n-1}(\xi),g_n(\xi)\right)$$

$$\leq \alpha\left[d\left(g_{n-1}(\xi), g_n(\xi)\right)\right] + \beta\left[d\left(g_{n-1}(\xi), g_n(\xi)\right) + d\left(g_n(\xi), g_{n+1}(\xi)\right)\right]$$
$$+ \gamma\left[d\left(g_{n-1}(\xi), g_n(\xi)\right) + d\left(g_n(\xi), g_{n+1}(\xi)\right)\right] + \eta . d\left(g_n(\xi), g_{n+1}(\xi)\right)$$
$$+ \delta . d\left(g_{n-1}(\xi), g_n(\xi)\right)$$
$$= (\alpha + \beta + \gamma + \delta) . d\left(g_{n-1}(\xi), g_n(\xi)\right) + (\beta + \gamma + \eta) d\left(g_n(\xi), g_{n+1}(\xi)\right)$$

⇨ $$(1 - \beta - \gamma - \eta) d\left(g_n(\xi), g_{n+1}(\xi)\right) \leq (\alpha + \beta + \gamma + \delta) . d\left(g_{n-1}(\xi), g_n(\xi)\right)$$

⇨ $$d\left(g_n(\xi), g_{n+1}(\xi)\right) \leq \frac{(\alpha + \beta + \gamma + \delta).}{(1 - \beta - \gamma - \eta)} d\left(g_{n-1}(\xi), g_n(\xi)\right)$$

....

$$\leq \frac{(\alpha + \beta + \gamma + \delta).}{(1 - \beta - \gamma - \eta)} d\left(g_0(\xi), g_1(\xi)\right)$$

Thus by triangle inequality, we have for $m > n$

$$d\left(g_n(\xi), g_m(\xi)\right) \leq d\left(g_n(\xi), g_{n+1}(\xi)\right) + d\left(g_{n+1}(\xi), g_{n+2}(\xi)\right) + \ldots\ldots + d\left(g_{m-1}(\xi), g_m(\xi)\right)$$
$$= \left(s^n + s^{n+1} + s^{n+2} + \ldots . + s^{m-1}\right) d\left(g_0(\xi), g_1(\xi)\right)$$

Where $s = \frac{(\alpha + \beta + \gamma + \delta).}{(1 - \beta - \gamma - \eta)} < 1$ since $\alpha + 2\beta + 2\gamma + \delta + \eta \prec 1$

Therefore

$d\left(g_n(\xi), g_m(\xi)\right) \leq \frac{s^n}{1 - s} d\left(g_0(\xi), g_1(\xi)\right) \to 0$, as $m, n \to \infty$. Hence the sequence $\{g_n\}$is a Cauchy sequence, X being complete, there exist some $p \in X$ such that

$$E(\xi, u(\xi)) = E\left(\lim_{n \to \infty} g_n(\xi)\right) = \lim_{n \to \infty} . E\{\xi, g_n(\xi)\} = \lim_{n \to \infty} g_{n+1}(\xi) = u(\xi)$$

Therefore $u(\xi)$ is a fixed point of E.

Uniqueness: Let if possible there exist another fixed point $v(\xi)$ of E in X, such that $u(\xi) \neq v(\xi)$ then from (3.1.1) we have

$$d\left(u(\xi), v(\xi)\right) = d\left(E\{\xi, u(\xi)\}, E\{\xi, v(\xi)\}\right)$$

$$\leq \alpha \left[\frac{d\left(u(\xi), E\{\xi, u(\xi)\}\right) d\left(v(\xi), E\{\xi, v(\xi)\}\right) d\left(u(\xi), E\{\xi, v(\xi)\}\right) + d\left(u(\xi), v(\xi)\right) d\left(v(\xi), E\{\xi, u(\xi)\}\right) d\left(v(\xi), E\{\xi, v(\xi)\}\right)}{[d\left(u(\xi), v(\xi)\right)]^2 + d\left(u(\xi), E\{\xi, v(\xi)\}\right) d\left(v(\xi), E\{\xi, v(\xi)\}\right)} \right]$$
$$+ \beta\left[d\left(u(\xi), E\{\xi, u(\xi)\}\right) + d\left(v(\xi), E\{\xi, v(\xi)\}\right)\right]$$
$$+ \gamma\left[d\left(u(\xi), E\{\xi, v(\xi)\}\right) + d\left(v(\xi), E\{\xi, u(\xi)\}\right)\right]$$
$$+ \eta \frac{d\left(u(\xi), E\{\xi, u(\xi)\}\right) d\left(v(\xi), E\{\xi, v(\xi)\}\right)}{d\left(u(\xi), v(\xi)\right)}$$
$$+ \delta . d\left(u(\xi), v(\xi)\right)$$

$$\prec \gamma\left[d\left(u(\xi), v(\xi)\right) + d\left(v(\xi), u(\xi)\right)\right] + \delta . d\left(u(\xi), v(\xi)\right)$$
$$\leq (2\gamma + \delta) d\left(u(\xi), v(\xi)\right)$$

$$d\left(u(\xi), v(\xi)\right) \leq (2\gamma + \delta) d\left(u(\xi), v(\xi)\right)$$

Since $\alpha + 2\beta + 2\gamma + \delta + \eta \prec 1 \Rightarrow 2\gamma + \delta < 1$

⇨ $$d\left(u(\xi), v(\xi)\right) < d\left(u(\xi), v(\xi)\right)$$

This is contradiction, so $u(\xi) = v(\xi)$.

This completes the proof of the theorem (3.1.1). We now prove another theorem.

Theorem (3.2): Let Ebe a self-mapping of a complete metric space (X, d), if for some positive integer p, E^{p} is continuous, then E has unique fixed point in X.

Proof: Let $\{g_n\}$ be sequence which converges to some. $u \in X$. Therefore its subsequence $\{g_{n_k}\}$ also converges to u also

$$E^{p}(\xi, u(\xi)) = E^{p} \lim_{k\to\infty} g_{n_k}(\xi) = \lim_{k\to\infty} E^{p}\left(\xi, g_{n_k}(\xi)\right) = \lim_{k\to\infty} g_{n_{k+1}}(\xi) = u(\xi)$$

Therefore $u(\xi)$ is a fixed point of E^{p} , we now show that $E(\xi, u(\xi)) = u(\xi)$.

Let m be the smallest positive integer such that $E^{m}(\xi, u(\xi)) = u(\xi)$ and $E^{q} \neq u(\xi), 1 \le q \le m-1$, If m>1 then by (3.1.1) we get,

$$\begin{aligned}
d\left(u(\xi), E\{\xi, u(\xi)\}\right) &= d\left(E^{m}\{\xi, u(\xi)\}, E\{\xi, u(\xi)\}\right) \\
&= d\left(E(E^{m-1}\{\xi, u(\xi)\}), E\{\xi, u(\xi)\}\right) \\
&\le \alpha\left[\frac{\begin{array}{c} d\left(E^{m-1}\{\xi,u(\xi)\}, E^{m}\{\xi,u(\xi)\}\right).d\left(u(\xi), E\{\xi,u(\xi)\}\right).d\left(E^{m-1}\{\xi,u(\xi)\}, E\{\xi,u(\xi)\}\right) \\ + d\left(E^{m-1}\{\xi,u(\xi)\}, u(\xi)\right).d\left(u(\xi), E^{m}\{\xi,u(\xi)\}\right)d\left(u(\xi), E\{\xi,u(\xi)\}\right) \end{array}}{\left[d\left(E^{m-1}\{\xi,u(\xi)\}, u(\xi)\right)\right]^{2} + d\left(E^{m-1}\{\xi,u(\xi)\}, E\{\xi,u(\xi)\}\right)d\left(u(\xi), E\{\xi,u(\xi)\}\right)}\right] \\
&\quad + \beta\left[d\left(E^{m-1}\{\xi,u(\xi)\}, E^{m}\{\xi,u(\xi)\}\right) + d\left(u(\xi), E\{\xi,u(\xi)\}\right)\right] \\
&\quad + \gamma\left[d\left(E^{m-1}\{\xi,u(\xi)\}, E\{\xi,u(\xi)\}\right) + d\left(u(\xi), E^{m}\{\xi,u(\xi)\}\right)\right]
\end{aligned}$$

$$+ \eta\left[\frac{d\left(E^{m-1}\{\xi,u(\xi)\}, E^{m}\{\xi,u(\xi)\}\right)d\left(u(\xi), E\{\xi,u(\xi)\}\right)}{d\left(E^{m-1}\{\xi,u(\xi)\}, u(\xi)\right)}\right]$$

$$+ \delta\left[d\left(E^{m-1}\{\xi,u(\xi)\}, u(\xi)\right)\right]$$

$$\le \alpha\left[\frac{\begin{array}{c} d\left(E^{m-1}\{\xi,u(\xi)\}, u(\xi)\right).d\left(u(\xi), E\{\xi,u(\xi)\}\right).d\left(E^{m-1}\{\xi,u(\xi)\}, E\{\xi,u(\xi)\}\right) \\ + d\left(E^{m-1}\{\xi,u(\xi)\}, u(\xi)\right).d\left(u(\xi), u(\xi)\right).d\left(u(\xi), E\{\xi,u(\xi)\}\right) \end{array}}{\left[d\left(E^{m-1}\{\xi,u(\xi)\}, u(\xi)\right)\right]^{2} + d\left(E^{m-1}\{\xi,u(\xi)\}, E\{\xi,u(\xi)\}\right)d\left(u(\xi), E\{\xi,u(\xi)\}\right)}\right]$$

$$+ \beta\left[d\left(E^{m-1}\{\xi,u(\xi)\}, u(\xi)\right) + d\left(u(\xi), E\{\xi,u(\xi)\}\right)\right]$$

$$+ \gamma\left[d\left(E^{m-1}\{\xi,u(\xi)\}, E\{\xi,u(\xi)\}\right) + d\left(u(\xi), u(\xi)\right)\right]$$

$$+ \eta\left[\frac{d\left(E^{m-1}\{\xi,u(\xi)\}, u(\xi)\right)d\left(u(\xi), E\{\xi,u(\xi)\}\right)}{d\left(E^{m-1}\{\xi,u(\xi)\}, u(\xi)\right)}\right]$$

$$+ \delta\left[d\left(E^{m-1}\{\xi,u(\xi)\}, u(\xi)\right)\right]$$

$$\le \alpha\left[d\left(E^{m-1}\{\xi,u(\xi)\}, u(\xi)\right)\right]$$

$$+ \beta\left[d\left(E^{m-1}\{\xi,u(\xi)\}, u(\xi)\right) + d\left(u(\xi), E\{\xi,u(\xi)\}\right)\right]$$

$$+ \gamma\left[d\left(E^{m-1}\{\xi,u(\xi)\}, d(u(\xi)\right) + d\left(u(\xi), E\{\xi,u(\xi)\}\right)\right]$$

$$+ \eta\left[d\left(u(\xi), E\{\xi,u(\xi)\}\right)\right]$$

$$+ \delta\left[d\left(E^{m-1}\{\xi,u(\xi)\}, u(\xi)\right)\right]$$

$$= (\alpha + \beta + \gamma + \delta) d\left(E^{m-1}\{\xi, u(\xi)\},\ u(\xi)\right) + (\beta + \gamma + \eta) d\left(u(\xi),\ E\{\xi, u(\xi)\}\right)$$

$$(1 - \beta - \gamma - \eta) d\left(u(\xi),\ E\{\xi, u(\xi)\}\right) \le (\alpha + \beta + \gamma + \delta) d\left(E^{m-1}\{\xi, u(\xi)\},\ u(\xi)\right)$$

⇨ $$d\left(u(\xi),\ E\{\xi, u(\xi)\}\right) \le \frac{(\alpha + \beta + \gamma + \delta)}{(1 - \beta - \gamma - \eta)} d\left(E^{m-1}\{\xi, u(\xi)\},\ u(\xi)\right)$$

⇨ $d\left(u(\xi),\ E\{\xi, u(\xi)\}\right) \le s.d\left(E^{m-1}\{\xi, u(\xi)\},\ u(\xi)\right)$ where $s = \frac{(\alpha + \beta + \gamma + \delta)}{(1 - \beta - \gamma - \eta)} < 1$

Further,

$$\begin{aligned} d\left(E^{m-1}\{\xi, u(\xi)\},\ u(\xi)\right) &= d\left(E^{m}\{\xi, u(\xi)\},\ E^{m-1}\{\xi, u(\xi)\}\right) \\ &\le s.d\left(E^{m-1}\{\xi, u(\xi)\},\ E^{m-2}\{\xi, u(\xi)\}\right) \\ &\quad \text{.......... ....} \\ &\le s^{m-1} d\left(u(\xi),\ E\{\xi, u(\xi)\}\right) \end{aligned}$$

Therefore

$$\begin{aligned} d\left(u(\xi),\ E\{\xi, u(\xi)\}\right) &\le s^{m} d\left(u(\xi),\ E\{\xi, u(\xi)\}\right) \\ &\prec d\left(u(\xi),\ E\{\xi, u(\xi)\}\right) \end{aligned}$$

Which is contradiction, therefore $u(\xi)$ is fixed point of E.That is $u(\xi) = E\{\xi, u(\xi)\}$ .This completes the proof.

Theorem (3.3): Let E be a self-mapping of a complete metric space X such that for some positive integer m, E^{m} Satisfies

$$\begin{aligned} & d\left(E^{m}\{\xi, g(\xi)\},\ E^{m}\{\xi, h(\xi)\}\right) \\ & \le \alpha \left[\frac{\begin{array}{c} d\left(g(\xi),\ E^{m}\{\xi, g(\xi)\}\right).d\left(h(\xi),\ E^{m}\{\xi, h(\xi)\}\right).d\left(g(\xi),\ E^{m}\{\xi, h(\xi)\}\right) \\ + d\left(g(\xi),\ h(\xi)\right).d\left(h(\xi),\ E^{m}\{\xi, g(\xi)\}\right).d\left(h(\xi),\ E^{m}\{\xi, h(\xi)\}\right) \end{array}}{\left[d\left(g(\xi),\ h(\xi)\right)\right]^{2} + d\left(g(\xi),\ E^{m}\{\xi, h(\xi)\}\right).d\left(h(\xi),\ E^{m}\{\xi, h(\xi)\}\right)} \right]. \\ & + \beta\left[d\left(g(\xi),\ E^{m}\{\xi, g(\xi)\}\right) + d\left(h(\xi),\ E^{m}\{\xi, h(\xi)\}\right)\right] \\ & + \gamma\left[d\left(g(\xi),\ E^{m}\{\xi, h(\xi)\}\right) + d\left(h(\xi),\ E^{m}\{\xi, g(\xi)\}\right)\right] \\ & + \eta\left[\frac{d\left(g(\xi),\ E^{m}\{\xi, g(\xi)\}\right)d\left(h(\xi),\ E^{m}\{\xi, h(\xi)\}\right)}{d\left(g(\xi),\ h(\xi)\right)}\right] \qquad (3.3.1) \\ & + \delta\left[d\left(g(\xi),\ h(\xi)\right)\right] \end{aligned}$$

For all $g(\xi)$) and $h(\xi) \in X$ with $g(\xi) \ne h(\xi)$, where $\alpha, \beta, \gamma, \eta, \delta : R^{+} \to [0,1)$ are such that $\alpha + 2\beta + 2\gamma + \delta + \eta \prec 1$,if for some positive integer m,E^m is continuous, then E has a unique fixed point in X.

Proof: E^{m} has unique fixed point $u(\xi)$ in X follows from theorem (3.2).

$E\{\xi, u(\xi)\} = E(E^{m}(\xi, u(\xi))) = E^{m} E\{\xi, u(\xi)\}$, Which implies that $E\{\xi, u(\xi)\}$ is fixed point of E^{m} but has unique fixed point $u(\xi)$, so $E\{\xi, u(\xi)\} = u(\xi)$.

Since any fixed point of E is also a fixed point of E^{m} . It follows that $u(\xi)$ is unique fixed point of E. This completes the proof of (3.3).We now prove another theorem.

Theorem 3.4: Let E & F be a pair of self-mappings of a complete metric space X, satisfying the following conditions:

$$d\left(E\{\xi, g(\xi)\},\ F\{\xi, h(\xi)\}\right)$$

$$\leq \alpha\left[\frac{d(g(\xi),E\{\xi,g(\xi)\}).d(h(\xi),F\{\xi,h(\xi)\}).d(g(\xi),F\{\xi,h(\xi)\}) + d(g(\xi),h(\xi)).d(h(\xi),E\{\xi,g(\xi)\}).d(h(\xi),F\{\xi,h(\xi)\})}{[d(g(\xi),h(\xi))]^2 + d(g(\xi),F\{\xi,h(\xi)\}).d(h(\xi),F\{\xi,h(\xi)\})}\right]$$

$$+\beta\left[d(g(\xi),E\{\xi,g(\xi)\}) + d(h(\xi),F\{\xi,h(\xi)\})\right]$$

$$+\gamma\left[d(g(\xi),F\{\xi,h(\xi)\}) + d(h(\xi),E\{\xi,g(\xi)\})\right]$$

$$+\eta\frac{d(g(\xi),E\{\xi,g(\xi)\})d(h(\xi),F\{\xi,h(\xi)\})}{d(g(\xi),h(\xi))} \qquad (3.4.1)$$

$$+\delta.d(g(\xi),h(\xi))$$

For all $g(\xi)$), $h(\xi)\in X$ with $g(\xi)\neq h(\xi)$,where $\alpha,\beta,\gamma,\eta,\delta : R^{+}\rightarrow[0,1)$ are such that $\alpha+2\beta+2\gamma+\delta+\eta\prec 1$,if E & F are continuous on X , then E& F have a unique fixed point in X.

Proof: Let $\{g_n\}$ be a continuous sequence defined as

$$g_n(\xi)=\begin{cases} E\{\xi,g_{n-1}(\xi)\} & \text{n is even} \\ F\{\xi,g_{n-1}(\xi)\} & \text{n is odd}\end{cases}$$ and $g_n\{\xi\}\neq g_{n-1}\{\xi\}$ for all n.

Now $d(g_{2n}(\xi),g_{2n+1}(\xi)) = d(E\{\xi,g_{2n-1}(\xi)\},\ F\{\xi,g_{2n}(\xi)\})$

$$\leq \alpha\left[\frac{d(g_{2n-1}(\xi),E\{\xi,g_{2n-1}(\xi)\}).d(g_{2n}(\xi),F\{\xi,g_{2n}(\xi)\}).d(g_{2n-1}(\xi),F\{\xi,g_{2n}(\xi)\}) + d(g_{2n-1}(\xi),g_{2n}(\xi)).d(g_{2n}(\xi),E\{\xi,g_{2n-1}(\xi)\})d(g_{2n}(\xi),F\{\xi,g_{2n}(\xi)\})}{[d(g_{2n-1}(\xi),g_{2n}(\xi))]^2 + d(g_{2n-1}(\xi),F\{\xi,g_{2n}(\xi)\}).d(g_{2n}(\xi),F\{\xi,g_{2n}(\xi)\}).}\right]$$

$$+\beta\left[d(g_{2n-1}(\xi),E\{\xi,g_{2n-1}(\xi)\}).+d(g_{2n}(\xi),F\{\xi,g_{2n}(\xi)\})\right]$$

$$+\gamma\left[d(g_{2n-1}(\xi),E\{\xi,g_{2n}(\xi)\})+d(g_{2n}(\xi),E\{\xi,g_{2n-1}(\xi)\})\right]$$

$$+\eta\frac{d(g_{2n-1}(\xi),E\{\xi,g_{2n-1}(\xi)\}).d(g_{2n}(\xi),F\{\xi,g_{2n}(\xi)\})}{d(g_{2n-1}(\xi),g_{2n}(\xi))}$$

$$+\delta.d(g_{2n-1}(\xi),g_{2n}(\xi))$$

$$\leq \alpha\left[\frac{d(g_{2n-1}(\xi),g_{2n}(\xi)).d(g_{2n}(\xi),g_{2n+1}(\xi)).d(g_{2n-1}(\xi),g_{2n+1}(\xi)) + d(g_{2n-1}(\xi),g_{2n}(\xi)).d(g_{2n}(\xi),g_{2n}(\xi))d(g_{2n}(\xi),g_{2n+1}(\xi))}{[d(g_{2n-1}(\xi),g_{2n}(\xi))]^2 + d(g_{2n-1}(\xi),g_{2n+1}(\xi)).d(g_{2n}(\xi),g_{2n+1}(\xi))}\right]$$

$$+\beta\left[d(g_{2n-1}(\xi),g_{2n}(\xi))+d(g_{2n}(\xi),g_{2n+1}(\xi))\right]$$

$$+\gamma\left[d(g_{2n-1}(\xi),g_{2n+1}(\xi))+d(g_{2n}(\xi),g_{2n}(\xi))\right]$$

$$+\eta\frac{d(g_{2n-1}(\xi),g_{2n}(\xi)).d(g_{2n}(\xi),g_{2n+1}(\xi))}{d(g_{2n-1}(\xi),g_{2n}(\xi))}$$

$$+\delta.d(g_{2n-1}(\xi),g_{2n}(\xi))$$

$$\leq \alpha\left[(g_{2n-1}(\xi),g_{2n}(\xi))\right]$$

$$+\beta\left[d(g_{2n-1}(\xi),g_{2n}(\xi))+d(g_{2n}(\xi),g_{2n+1}(\xi))\right]$$

$$+\gamma\left[d(g_{2n-1}(\xi),g_{2n}(\xi))+d(g_{2n}(\xi),g_{2n+1}(\xi))\right]$$

$$+ \eta . d\left(g_{2n}(\xi), g_{2n+1}(\xi)\right) + \delta . d\left(g_{2n-1}(\xi), g_{2n}(\xi)\right)$$

$$= (\alpha + \beta + \gamma + \delta) d\left(g_{2n-1}(\xi), g_{2n}(\xi)\right) + (\beta + \gamma + \eta) d\left(g_{2n}(\xi), g_{2n+1}(\xi)\right)$$

$$(1 - \beta - \gamma - \eta) d\left(g_{2n}(\xi), g_{2n+1}(\xi)\right) \leq (\alpha + \beta + \gamma + \delta) d\left(g_{2n-1}(\xi), g_{2n}(\xi)\right)$$

$$d\left(g_{2n}(\xi), g_{2n+1}(\xi)\right) \leq \frac{(\alpha + \beta + \gamma + \delta)}{(1 - \beta - \gamma - \eta)} d\left(g_{2n-1}(\xi), g_{2n}(\xi)\right)$$

⇨ $d\left(g_{2n}(\xi), g_{2n+1}(\xi)\right) \leq s . d\left(g_{2n-1}(\xi), g_{2n}(\xi)\right)$ where $s = \frac{(\alpha + \beta + \gamma + \delta)}{(1 - \beta - \gamma - \eta)} < 1$

Similarly

$$d\left(g_{2n}(\xi), g_{2n+1}(\xi)\right) \leq s^2 . d\left(g_{2n-2}(\xi), g_{2n-1}(\xi)\right)$$

....

....

$$\leq s^{2n} . d\left(g_0(\xi), g_1(\xi)\right)$$

Hence $d\left(g_{2n+1}(\xi), g_{2n+2}(\xi)\right) \leq s^{2n+1} . d\left(g_0(\xi), g_1(\xi)\right)$

Hence the sequence $\{g_n\}$ is a Cauchy sequence in X and X being complete, therefore there exist $u(\xi)$ in X such that $\lim_{n \to \infty} g_n(\xi) = u(\xi)$ the subsequence $g_{nk} \to u(\xi)$

Now, if EF is continuous on X then

$$EF(\xi, u(\xi)) = EF\{\lim_{k \to \infty} g_{n_k}(\xi)\} = \lim_{k \to \infty} g_{n_{k+1}}(\xi) = u(\xi)$$

Thus $EF\{\xi, u(\xi)\} = u(\xi)$ i.e. $u(\xi)$ is fixed point of EF.

Now we show that $F\{\xi, u(\xi)\} = u(\xi)$. If $F\{\xi, u(\xi)\} \neq u(\xi)$, then

$$d\left(u(\xi), F\{\xi, u(\xi)\}\right) = d\left(EF\{\xi, u(\xi)\}, F\{\xi, u(\xi)\}\right)$$

$$\leq \alpha \left[\frac{d\left(F\{\xi,u(\xi)\}, EF\{\xi,u(\xi)\}\right) d\left(u(\xi), F\{\xi,u(\xi)\}\right) d\left(F\{\xi,u(\xi)\}, F\{\xi,u(\xi)\}\right) + d\left(F\{\xi,u(\xi)\}, u(\xi)\right) d\left(u(\xi), EF\{\xi,u(\xi)\}\right) d\left(u(\xi), F\{\xi,u(\xi)\right)}{\left[d\left(F\{\xi,u(\xi)\}, u(\xi)\right)\right]^2 + d\left(F\{\xi,u(\xi)\}, F\{\xi,u(\xi)\}\right) d\left(u(\xi), F\{\xi,u(\xi)\}\right)} \right]$$

$$+ \beta\left[d\left(F\{\xi,u(\xi)\}, EF\{\xi,u(\xi)\}\right) + d\left(u(\xi), F\{\xi,u(\xi)\}\right)\right]$$

$$+ \gamma\left[d\left(F\{\xi,u(\xi)\}, F\{\xi,u(\xi)\}\right) + d\left(u(\xi), EF\{\xi,u(\xi)\right)\right]$$

$$+ \eta \frac{d\left(F\{\xi,u(\xi)\}, EF\{\xi,u(\xi)\}\right) d\left(u(\xi), F\{\xi,u(\xi)\}\right)}{d\left(F\{\xi,u(\xi)\}, u(\xi)\right)}$$

$$+ \delta\left[d\left(F\{\xi,u(\xi)\}, u(\xi)\right)\right]$$

$$\leq \alpha \left[\frac{d\left(F\{\xi,u(\xi)\}, u(\xi)\right) d\left(u(\xi), F\{\xi,u(\xi)\}\right) d\left(F\{\xi,u(\xi)\}, F\{\xi,u(\xi)\}\right) + d\left(F\{\xi,u(\xi)\}, u(\xi)\right) d\left(u(\xi), u(\xi)\}\right) d\left(u(\xi), F\{\xi,u(\xi)\right)}{\left[d\left(F\{\xi,u(\xi)\}, u(\xi)\right)\right]^2 + d\left(F\{\xi,u(\xi)\}, F\{\xi,u(\xi)\}\right) d\left(u(\xi), F\{\xi,u(\xi)\}\right)} \right]$$

$$+ \beta\left[d\left(F\{\xi,u(\xi)\}, u(\xi)\right) + d\left(u(\xi), F\{\xi,u(\xi)\}\right)\right]$$

$$+ \gamma\left[d\left(F\{\xi,u(\xi)\}, F\{\xi,u(\xi)\}\right) + d\left(u(\xi), u(\xi)\right)\right]$$

$$+\eta\frac{d(F\{\xi,u(\xi)\},u(\xi))d(u(\xi),F\{\xi,u(\xi)\})}{d(F\{\xi,u(\xi)\},u(\xi))}$$

$$+\delta[d(F\{\xi,u(\xi)\},u(\xi))]$$

$$\leq\alpha\left[\frac{d(F\{\xi,u(\xi)\},u(\xi))d(u(\xi),F\{\xi,u(\xi)\})d(F\{\xi,u(\xi)\},F\{\xi,u(\xi)\})}{[d(F\{\xi,u(\xi)\},u(\xi))]^2}\right]$$

$$+2\beta[d(F\{\xi,u(\xi)\},u(\xi))]+\gamma[0]+\eta.d(F\{\xi,u(\xi)\},u(\xi))+\delta[d(F\{\xi,u(\xi)\},u(\xi))]$$

$$=(2\beta+\eta+\delta)d(F\{\xi,u(\xi)\},u(\xi))$$

$$d(F\{\xi,u(\xi)\},u(\xi))\leq(2\alpha+\eta+\delta)d(F\{\xi,u(\xi)\},u(\xi))$$
$$<d(F\{\xi,u(\xi)\},u(\xi))$$

Since $\alpha+2\beta+2\gamma+\delta+\eta\prec 1$ implies that $(2\beta+\eta+\delta)<1$

Hence $F\{\xi,u(\xi)\}=u(\xi)$.

Further $d(E\{\xi,u(\xi)\},u(\xi))<d(E\{\xi,u(\xi)\},u(\xi))$ implies that $E\{\xi,u(\xi)\}=u(\xi)$.

So u is common fixed point of E & F

Uniqueness:

Let $v(\xi)$ is another common fixed point of E & F we have

$$d(u(\xi),v(\xi))=d(E\{\xi,u(\xi)\},F\{\xi,v(\xi)\})$$

$$\leq\alpha\left[\frac{d(u(\xi),E\{\xi,u(\xi)\})d(v(\xi),F\{\xi,v(\xi)\})d(u(\xi),F\{\xi,v(\xi)\})+d(u(\xi),v(\xi))d(v(\xi),E\{\xi,u(\xi)\})d(v(\xi),F\{\xi,v(\xi)\})}{[d(u(\xi),v(\xi))]^2+d(u(\xi),F\{\xi,v(\xi)\})d(v(\xi),F\{\xi,v(\xi)\})}\right]$$

$$+\beta[d(u(\xi),E\{\xi,u(\xi)\})+d(v(\xi),F\{\xi,v(\xi)\})]$$

$$+\gamma[d(u(\xi),F\{\xi,v(\xi)\})+d(v(\xi),E\{\xi,u(\xi)\})]$$

$$+\eta\frac{d(u(\xi),E\{\xi,u(\xi)\})d(v(\xi),F\{\xi,v(\xi)\})}{d(u(\xi),v(\xi))}$$

$$+\delta[d(u(\xi),v(\xi))]$$

$$\leq\alpha\left[\frac{d(u(\xi),u(\xi))d(v(\xi),v(\xi))d(u(\xi),v(\xi))+d(u(\xi),v(\xi))d(v(\xi),u(\xi)\})d(v(\xi),v(\xi))}{[d(u(\xi),v(\xi))]^2+d(u(\xi),v(\xi))d(v(\xi),v(\xi))}\right]$$

$$+\beta[d(u(\xi),u(\xi))+d(v(\xi),v(\xi))]$$

$$+\gamma[d(u(\xi),v(\xi))+d(v(\xi),u(\xi))]$$

$$+\eta\frac{d(u(\xi),u(\xi))d(v(\xi),v(\xi))}{d(u(\xi),v(\xi))}$$

$$+\delta[d(u(\xi),v(\xi))]$$

$$\leq(2\gamma+\delta)d(u(\xi),v(\xi))$$

$$d(u(\xi),v(\xi))\leq(2\gamma+\delta)d(u(\xi),v(\xi))$$
$$\prec d(u(\xi),v(\xi))$$

Because $(2\gamma+\delta)<1$. This implies $u(\xi)=v(\xi)$.This completes the proof of (3.4)

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