IOSR Journal of Engineering (IOSRJEN) **www.iosrjen.org**
ISSN (e): 2250-3021, ISSN (p): 2278-8719
Vol. 06, Issue10 (Oct. 2016), ||V2|| PP 49-52

Discreteness charge in Josephson juncture

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Abstract: - We discuss the connection between the inductance of LC circuit under the charge discreteness approach and the constitutive relation given by the inductance of a Josephson juncture where the discreteness of Cooper pair tunneling causes the periodic flux dependence of the current, with a period given by a universal quantum constant, the superconducting flux quantum h/2e.

Keywords: - *Charge, discreteness, juncture, inductance, Josephson*

I. INTRODUCTION

A fundamental concept which is responsible for sensible effects in electronic devices is the discreteness of the electric charge [1]. Quantizations of the conductance in units of e^2/h [2,3], or of the magnetic flux in units of h/e [4], can also be mentioned. Note that the charge of the electron is −e, as usual. Much progress has also been done in the field of miniaturization of circuits. In this context it has been found that the quantum mechanical description of LC [4-6]-, L [7,8]- and RLC [9,10]-circuits can be done by resorting once more again to the charge discretization. Studies in such fields are promising, as they produce ideas for further technological developments.We shall then use this opportunity to discuss in some more detail the quantum-mechanical description of the mesoscopic LC-circuit with a time independent voltage source V_B for the the Josephson junture with $e \to 2e$. So far the discrete Schrodinger-equation characterizing the LC-circuit has been established by starting from the concrete charge eigenvalue equation [7,8,9,10], $\hat{Q}\left|n\right\rangle = nq_e\left|n\right\rangle$, where $\hat{Q}$ denotes the Hermitian charge operator and where n is an integer playing the role of the discrete coordinate. In this paper, Equation (1) shows, of course, that the electric charge is quantized in units of the elementary electric charge $q_e = 2e$ .Consider a homogeneous classical circuit, where the every cell is constituted of a LC circuit with inductance L and capacitance C. Assume that the interaction between neighbor cells is trough the capacitors (direct line), As it has been proposed by Li and Chen [4, 5], and later by others [6-10], we treat our mesoscopic system as a quantum electrical circuit, with quantized charge. The classical Hamiltonian H_{cl} of the (circuit) model system, written in terms of the canonically conjugate variables Q and Φ is

$$H = \frac{\Phi^2}{2L} + \frac{Q^2}{2C} \qquad (1)$$

To quantize the system, the variables Q and Φ are replaced by the operators

$\hat{Q}$ and $\hat{\Phi}$. Further, to recover the quantization of charge within this electrical circuit approach, we introduce the replacement [6]

$$\hat{\Phi} = \frac{2\hbar}{q_e}\sin\left(\frac{q_e}{2\hbar}\hat{\phi}\right), \qquad (2)$$

where q_e is the quantum of charge. We remark that, if charge discreteness is neglected, the operator $\hat{\Phi}$ may be directly identified with the magnetic flux operator, and therefore directly related to the current; however, when one introduces charge discreteness via the replacement above, the simple relation to the current is lost, therefore, after replacement (2) the flux operator becomes the pseudo-flux. This pseudo-flux operator satisfies the usual commutation relation $\left[\hat{Q},\hat{\Phi}\right] = i\hbar$. Notice that, given the complexity of dealing with an operator such as the one above (2), it is simpler to work in the so-called pseudo-flux representation, in which the operator $\hat{\Phi}$ is replaced by its eigenvalue ϕ, while the charge operator is given by $\hat{Q} = i\hbar \frac{\partial}{\partial \phi}$. In this way, the resulting Hamiltonian is given by

$$\hat{H} = \frac{2\hbar^2}{q_e^2 L} \sin^2\left(\frac{q_e \hat{\phi}}{2\hbar}\right) + \frac{\hat{Q}^2}{2C} \quad (3)$$

The hamiltonian operator above constitutes our starting point, and our working hypothesis.

The current operator $\hat{I}$ is formally obtained from $\hat{I} = \frac{i}{\hbar}\left[\hat{H}, \hat{Q}\right]$ then, in the $\hat{\Phi}$ -representation, it becomes

$$\hat{I} = \frac{\hbar}{q_e L} \sin\left(\frac{q_e}{\hbar} \hat{\phi}\right) \quad (4)$$

The parameters of this theory, particularly L and C, are related to the geometry of the system, but are hard to compute for a given experimental system, therefore, they should be deduced from experimental observations. here, we want to discuss the Josephson non-linear inductance and according our knowledge we make for the first time, the connection between equation (4).with equation (6), (section 2.)

II. THE JOSEPHSON NON LINEAR INDUCTANCE

At low temperatures, and at the low voltages and low frequencies corresponding to quantum information manipulation, the Josephson tunnel junction behaves as a pure non-linear inductance (Josephson element) in parallel with the capacitance corresponding to the parallel plate capacitor formed by the two overlapping films of the junction (Fig. 1).

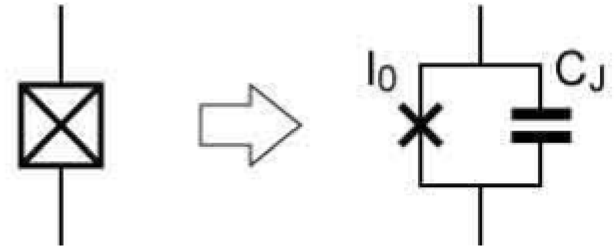

Figure 1. Equivalent circuit of Josephson juncture

The inductor can be characterized by its constitutive equation of the ordinary magnetic flux, which is defined by $\Phi(t) = \int_{-\infty}^{t} V(t_1)dt_1$ where $V(t)$ is the space integral of the electric field along a current line inside the element so the current I(t) flowing is

$$I(t) = \frac{1}{L_{J0}} \Phi(t) \quad (5)$$

The Josephson element behaves inductively, as its branch flux-current relationship [11] is:

$$I(t) = I_0 \sin(2\pi\Phi(t)/\Phi_0) \quad (6)$$

Where I_0 is the critical current. This inductive behavior, L_{J0}, is the manifestation, at the level of collective electrical variables, of the inertia of Cooper pairs tunneling across the insulator (kinetic inductance). The discreteness of Cooper pair tunneling causes the periodic flux dependence of the current, with a period given by a universal quantum constant, the superconducting flux quantum h/2e. Here we make the connection with equation (4) $I_0 = \hbar / q_e L$, $L = L_{J0}$ and $(q_e/\hbar)\hat{\phi} \rightarrow \varphi/\varphi_0$. The junction parameter I_0 is the critical current of the tunnel element. It scales proportionally to the area of the tunnel layer and diminishes exponentially with the tunnel layer thickness. Note that the constitutive relation Eq. (6) expresses in only one equation the two Josephson relations [11, 12].

The purely sinusoidal form of the constitutive relation Eq. (6) can be traced to the perturbative nature of Cooper pair tunneling in a tunnel junction. The quantity $2\pi\Phi(t)/\Phi_0 = \delta$ is called the gauge-invariant phase difference across the junction. It is important to realize that at the level of the constitutive relation of the Josephson element, this variable is nothing else than an electromagnetic flux in dimensionless units. In general, we have $\theta = \delta \bmod 2\pi$ where θ is the phase difference between the two superconducting condensates on both sides of the junction.

The Josephson element is also often described by two other parameters, each of which carry exactly the same information as the critical current. The first one is the Josephson effective inductance $L_{J0} = \varphi_0 / I_0$, where $\varphi_0 = \Phi_0 / 2\pi$ is the reduced flux quantum. The name of this other form becomes obvious if we expand the sine function in Eq. (6) in powers of Φ_0 around $\Phi_0 = 0$. Keeping the leading term, we have $I = \Phi / L_{J0}$. Note that the junction behaves for small signals almost as a point-like kinetic inductance: a 100nm×100nm area junction will have a typical inductance of 100nH. More generally, it is convenient to define the phase-dependent Josephson inductance,[12].

$$L_J(\delta) = \left(\frac{\partial I}{\partial \Phi}\right)^{-1} = \frac{L_{J0}}{\cos\delta} \tag{7}$$

Note that the Josephson inductance not only depends on δ, it can actually become infinite or negative! Thus, under the proper conditions, the Josephson element can become a switch and even an active circuit element. The other useful parameter is the Josephson energy $E_J = \varphi_0 I_0$. If we compute the energy stored in the junction $E(t) = \int_{-\infty}^{t} I(t_1)V(t_1)dt_1$, we find $E(t) = -E_J \cos(2\pi\Phi(t)/\Phi_0)$ which has the shape of a cosine washboard. The most simple theoretical example consists of a voltage biased placed across a spatial gap between bulk superconductors and corresponds to the Josephson pendulum Hamiltonian

$$H(V_{ext}) = (q^2 N^2 / 2C) - V_{ext} qN - E_j \cos\theta \tag{8}$$

where $q = 2e$ is the electron pair charge, C is the weak link capacitance, V_{ext} is an external voltage bias across the micro bridge, and $E_j = \varphi_0 I_o$ is the energy with which electron pair can tunnel across the micro bridge. Employing the number of pairs N stored in the micro bridge

$$N = -i(\partial / \partial\theta) \tag{9}$$

where $2\pi\Phi / \Phi_0 = \theta$, we can obtain the ground state energy

$$H(V_{ext})\psi_0(\theta) = E(V_{ext})\psi_0(\theta),\ \psi_0(\theta + 2\pi) = \psi_0(\theta) \tag{10}$$

which determines the charge Q, or the mean number of electron pairs $\langle N \rangle$ stored on the weak link capacitor

$$Q = q\langle N \rangle = -(\partial E(V_{ext}) / \partial V_{ext}) \tag{11}$$

The semi classical solution of (8) for $q = 2e$ is [13, 14]

$$\theta = \arcsin(\mathrm{sn}\, k\omega_0 t; 1/k) \tag{12}$$

with $k = \sqrt{V_{ext} qN / E_j}$

In Figure 2. We illustrate quantum steps of θ according equation (12).

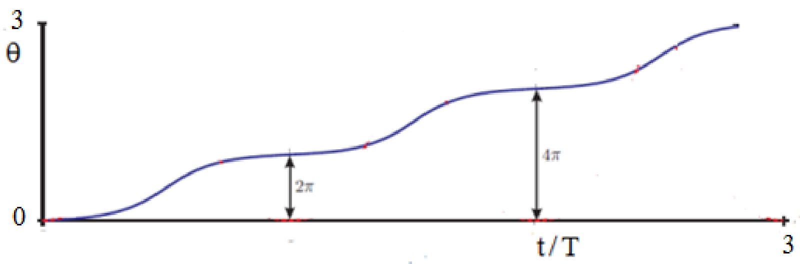

Figure 2. Theoretical quantum steps in the pair charge as a function of t/T. θ phase of a Josephson juncture with initial conditions $\theta|_{t=0} \approx \pi$, $d\theta/dt|_{t=0} \approx 0.15\omega_0$..

In Figure 2 we verify the condition $\psi_0(\theta + 2\pi) = \psi_0(\theta)$.The period T is given by $T = 4K(k_E^2)/\omega_0 = 4sn^{-1}(1,k)/\omega_0$. The function $sn^{-1}(1,k)$ is the inverse elliptic function.

III. CONCLUSION

We discuss the connection between the inductance of LC circuit under the charge discreteness approach and the constitutive relation given by the inductance of a Josephson juncture where the discreteness of Cooper pair tunneling causes the periodic flux dependence of the current, with a period given by a universal quantum constant, the superconducting flux quantum h/2e. We analyze the Josephson non-linear inductance with $q_e = 2e$ and to our knowledge we make for the first time, the connection between equation (4) with equation (6), (section 2.). This approach may be useful investigate a network of coupled superconducting transmission line resonators, qubit circuits and more realistic model of junction like bulk and weak-link superconductivity.

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