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Studying The Effect Of Discharging For The Boundary Layer Flow Of A Nanofluid Past A Stretching Sheet

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Abstract:- The boundary layer issue of a Nanofluid Past a Stretching Sheet with regard to speed which is perpendicular to the Stretching Sheet along the negative Y axis was studied numerically.Regulated partial differential equations are became to ordinary differential equations by using of suitable parallel variables and have been resolved simultaneously by the relations of the changing properties of Nanofluids using finite difference method. Changing in surface temperature, volume fraction of nanoparticle have been investigated. The results showed a broadly declining in dimensionless values of volume fraction of nanoparticle and surface temperature by increasing of velocity perpendicular to Stretching Sheet**.**

Keywords: boundary layer, Nanofluids, fluid velocity of Stretching Sheet.

I. INTRODUCTION

Fluid mechanics, as well as many other sciences, thanks its progress to revolutionary ideas like the boundary layer theory and the thin film theory. Among these theories, undoubtedly boundarylayer theory enjoys a special place in this science. First, boundary layer was introduced by the Prandtl. Prandtl divided the flow field into two areas and simplified the regulated equations of flowing fluid [1]. These two areas are composed of internal area inside of boundary layer which viscosity is regulated in it and external area which the flow is potential.By using this theory, for solving many problems there is no need to solve the permanence equations completely. The Heat transfer processes have a special importance in the industries. Therefore, researchers are trying to find procedures which improves heat transfer. One of these methods is using of Nanofluids (fluid contains nanoparticles) [2-5].For the first time choi [6, 7] used Nanofluids and showed that adding a small amount of nanoparticles in fluids (Less than 1% by volume) improves the heat transfer coefficient and thus will increase the heat transfer. In addition to the changes that can now be applied on physical models, changes in the boundary conditions may also cause improvement in the heat transfer rate. One of these changes is regarding the velocity which is perpendicular to Stretching Sheet. A study by Steinrück [8] was proposed about boundary layer flow through a horizontal cold plane, it is shown that in the case of which flow is considered in a horizontal warm plane, a parallel solution of boundary layer equations for when there is the buoyancy in the flow direction and flow in the boundary layer is accelerated; however for a flow above a cold plane (buoyancy adverse effects) progressive flow have an adverse pressure gradient, therefore, it is expected that the flow separation will occurs. Arifin et al [9] presented a similarity solution of forced heat transfer of Nanofluid in the boundary layer through a horizontal flat plane. Results showed that increasing of nanoparticles, *Al2O3* and *Cu*, caused more heat transfers. The aim of this paper is studying the effect of speed which is perpendicular to the Stretching Sheet on the value of heat transfer and mass transfer in the Pop and Khanphysical model.

Nomenclature

a	Constant
b	Constant
c	Nanoparticle volume fraction
c_w	Nanoparticle volume fraction at the stretching sheet
c_∞	Ambient nanoparticle volume fraction
$f(\eta)$	Dimensionless stream function
K	Thermal conductivity
Le	Lewis number
N_b	Brownian motion parameter
N_t	Thermophoresis parameter
Nu	Nusselt number
Pr	Prandtl number
u,v	Velocity components along x – axes and y - axes

Greek Symbols

α	Thermal diffusivity
$\varphi(\eta)$	Rescaled nanoparticle volume fraction
η	Similarity variable
$\theta(\eta)$	Dimensionless temperature
ρ_F	Fluid density
ρ_p	Nanoparticle mass density
ρc_F	Heat capacity of the fluid
ρc_p	Effective heat capacity of the nanoparticle material
Ψ	Stream function

II. MATHEMATICAL MODEL OFRESEARCH METHOD

To investigate the effect of speed perpendicular to the Stretching Sheet on the value of heat transfer and mass transfer in the physical model, the governingequations will be as follows: [10-13]

$$f''' + ff'' - f'^2 = 0 \tag{1}$$

$$\frac{1}{pr}\theta'' + f\theta' + N_b\varphi'\theta' + N_t\theta'^2 = 0 \tag{2}$$

$$\varphi'' + Le\, f\varphi' + \frac{N_t}{N_b}\theta'' = 0 \tag{3}$$

According to the following boundary conditions:

$$f(o)=0,\ f'(0)=1,\ \theta(0)=1,\ f(0)=1 \tag{4}$$

$$f'(\infty)=0,\ \theta(\infty)=0,\ \varphi(\infty)=0 \tag{5}$$

2-1.The basic equations and physical model

A two dimensional boundary layer of a Nanofluid past a Stretching Sheet with linear velocity $u_w(x) = ax$ where a is a constant and x axis is in the plane of the Stretching Sheet and as shown in Figure 1, also $v = -b(a\nu)^{1/2}$ speed(b is a constant and ν kinematic viscosity) perpendicular to the Stretching Sheet is negative direction of the Y axis is considered, anequal uniform stress and contrary to the forceis usedto maintain the datum of Stretching Sheet. T temperature and volume fraction of C nanoparticles on Stretching Sheet respectively have been assumed by the fixed amount of$T_w$andC_w. Limited quantities ofC_∞, T_∞ areconsidered respectively for T temperature and fractions of C nanoparticles when Y tend to infinity.

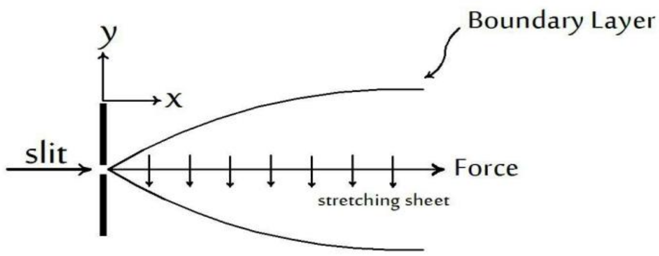

Figure 1: speed perpendicular to the Stretching Sheet and boundary layer

Continuity basic equations of mass flow, momentum, heat energy and nanoparticles for Nanofluids can consider as follows: [11]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_f}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{7}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho_f}\frac{\partial p}{\partial y}+v\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{8}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial^2 y}\right)+\tau\left\{D_B\left(\frac{\partial c}{\partial x}\frac{\partial T}{\partial x}+\frac{\partial c}{\partial y}\frac{\partial T}{\partial y}\right)+\left(\frac{D_T}{T_\infty}\right)\left[\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right]\right\} \tag{9}$$

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D_B\left(\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial y^2}\right)+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{10}$$

Due to boundary conditions:

$$v=-b\,(a v)^{1/2},\ u=u_w(x)=a\,x,\ T=T_w,\ C=C_w \quad \text{at } y=0$$

$$u=v=0,\ T=T_\infty,\ C=C_\infty,\quad \text{as } y\to\infty \tag{11}$$

Here u and v respectivelyare the velocity components along X and Y axis. P is fluid pressure, ρ_f is density of base fluid, α is thermal conductivity, *v*is kinematic viscosity, a and b are positive constants, D_B is Brownian diffusion coefficient, D_T is Thermophoretic diffiusion coefficient and $\tau=\frac{(\rho c)_p}{(\rho c)_f}$ is the ratio between effective thermal capacity of nanoparticles materials and heat capacity of the fluid with ρ density.To solve by similarity method we use Khan and Pop expressed form, as follows [12-15]:

$$\psi=(a\nu)^{\frac{1}{2}}\,x\,f(\eta),\qquad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}$$

$$\varphi(\eta)=\frac{C-C_\infty}{C_w-C_\infty},\qquad \eta=(a/\nu)^{1/2}y \tag{12}$$

Flow function ψ is defined as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$.
Due to the constant pressure outside the boundary layer by substituting equations (12) in (6) - (10), *PDE* equations is became into *ODE* equations and ordinary differential equations obtained as follows:

$$f'''+ff''-f'^2=0 \tag{13}$$

$$\frac{1}{pr}\theta''+f\theta'+N_b\varphi'\theta'+N_t\theta'^2=0 \tag{14}$$

$$\varphi''+Le\,f\,\varphi'+\frac{N_t}{N_b}\theta''=0 \tag{15}$$

According to boundary conditions:

$$f(0)=b,\ f'(0)=1,\ \theta(0)=1,\ \varphi(0)=1$$

$$f'(\infty)=0,\ \theta(\infty)=0,\ \varphi(\infty)=0 \tag{16}$$

So that the (′)indicate differentiation in terms of η and four parameters, Pr, Le, N_b andN_t, are defined as follows:

$$N_t=\frac{(\rho C)_p D_T(T_w-T_\infty)}{\nu(\rho C)_f\,T_\infty}\qquad N_b=\frac{(\rho C)_p D_B(\varphi_w-\varphi_\infty)}{\nu(\rho C)_f}$$

$$Le=\frac{\nu}{D_B}\qquad Pr=\frac{\nu}{\alpha} \tag{17}$$

Which Pr, Le, N_b and N_t respectively are Prandtl number, Lewis number, Brownian move parameters and Thermophoresis parameter.

III. RESULTS AND DISCUSSION

The results are obtained using *Rung Kutta* with *Newton Raphson* shows in Fig.2.

First, we solved equations (1) - (3) numerically by using the finite element method due to boundary conditions in (4) and (5) in three modes $(Pr = 10,\ Le = 10, N_t = 0, N_b = 0.1)$, $(Pr = 10,\ Le = 10, N_t = 0.1, N_b = 0.1)$ *and* $(Pr = 10,\ Le = 100, N_t = 0.5, N_b = 0.5)$ with *MATLAB* software and then by taking into account (*b=1*, *b=2*) for each expressed mode of $(Pr = 10,\ Le = 10, N_t = 0, N_b = 0.1)$, $(Pr = 10, Le = 10, N_t = 0.1, N_b = 0.1)$ *and* $(Pr = 10,\ Le = 100, N_t = 0.5, N_b = 0.5)$ for the purpose of comparison, we draw the related graphs.it is observed that $\theta(\eta)$ and $\varphi(\eta)$profiles are extremely convergent also temperature increases with N_t and N_b increasing.

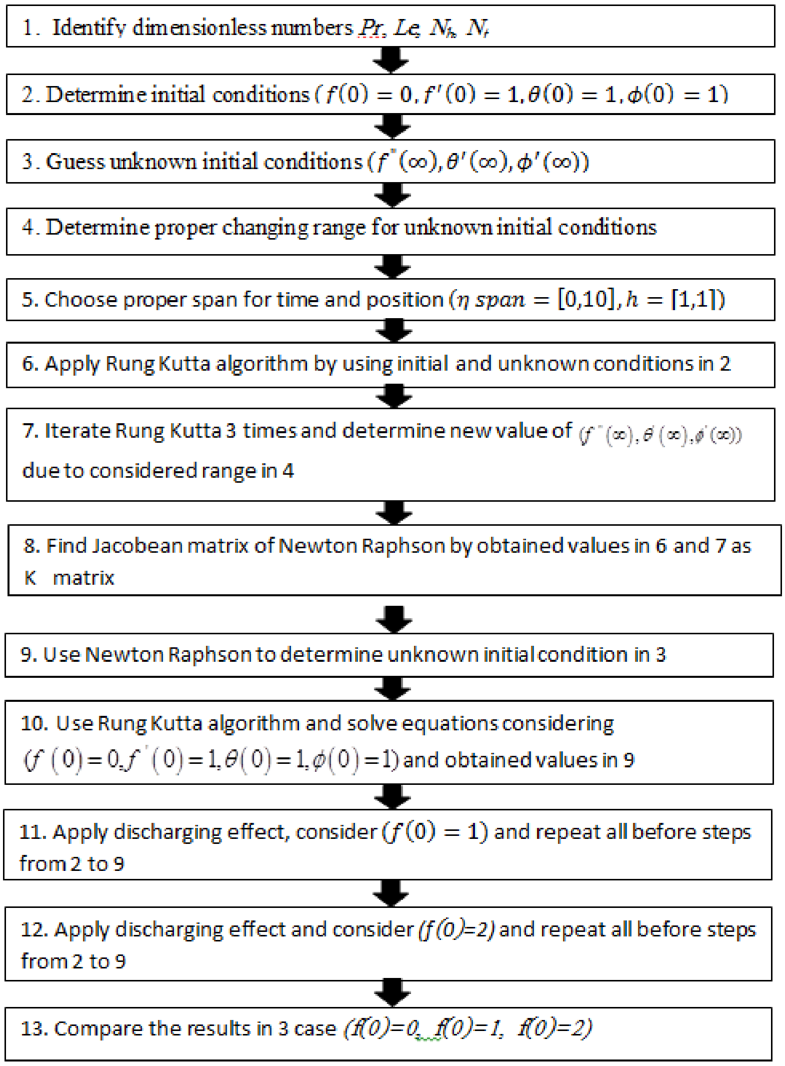

Fig. 2. Flow chartsolution of problems

Figures (3) and (4) plot are the variation graph of θ and φ by η in $(Pr = 10,\ Le = 10, N_t = 0, N_b = 0.1)$ .It can be seen that $\theta(\eta)$ and $\varphi(\eta)$ are descendingfunctionsin terms of η. Of course, this issue can be seen for any value of *P*rand *Le* and N_t and N_b. In addition, the corresponding values of $\theta(\eta)$ and $\varphi(\eta)$ in three modes *b=0*, *b=1*,*b=2*, respectively are declining. In other words increasingin speed perpendicular to the stretching sheet cause to reductioninθ and φ.

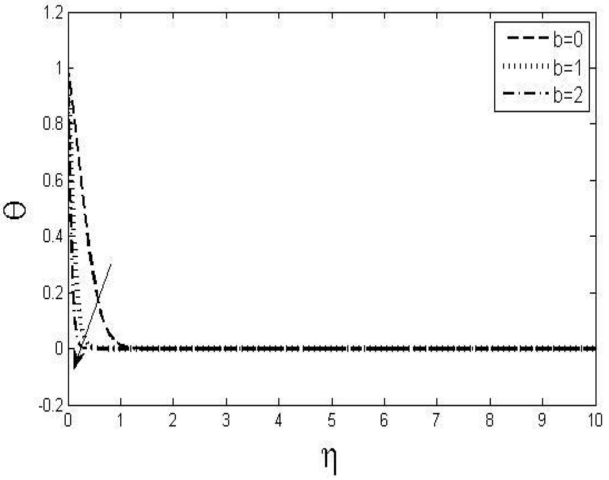

Fig. 3. □ changes in terms of □ in (Pr = 10, Le = 10,N_t=0,N_b=0.1) mode.

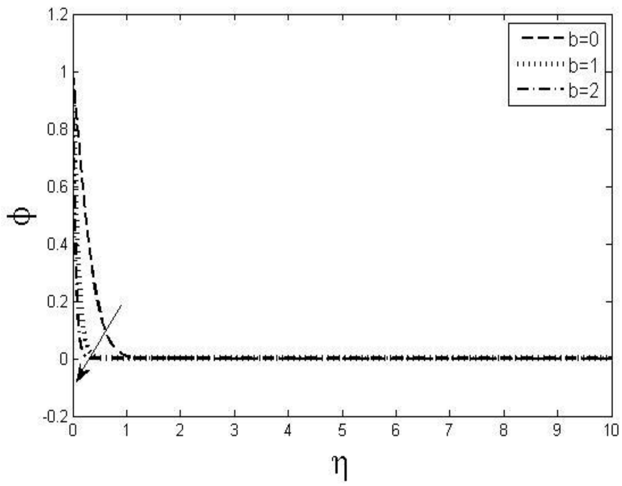

Fig. 4.φ Changes in terms of η in (*Pr = 10, Le = 10, N_t=0, N_b=0.1*) mode.

Figures (5) and (6) are the plot of changes of θ in terms of η and φ in terms of η in $(Pr = 10, Le = 10, N_t = 0.1, N_b = 0.1)$ mode. The results listed above are also visible in this mode.

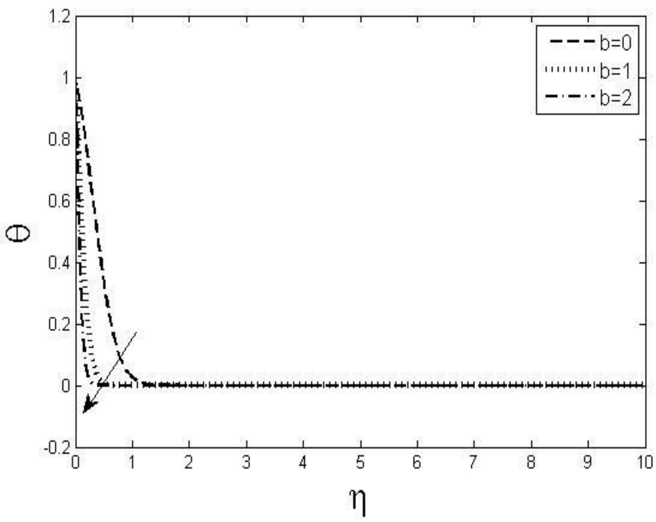

Fig. 5 .θ changes in terms of η in (*Pr = 10, Le = 10, $N_t = 0.1$, N_b=0.1*) mode.

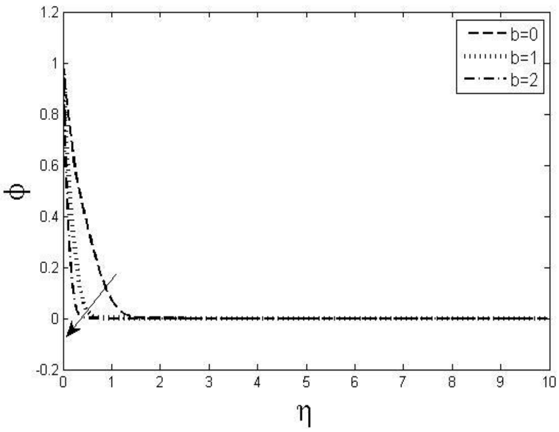

Fig. 6.φ Changes in terms of η in (Pr = 10, Le = 10, $N_t = 0.1$, N_b=0.1) mode.

figures (7) And (8) are θ changes plot in terms of η and φ changes plot in terms of η in $(Pr = 10, Le = 100, N_t = 0.5, N_b = 0.5)$ mode. In this case θ and φ are descending functions in terms of η and increasing in speed perpendicular to the stretching sheet will cause the reductionofθ and φ. Of course it shows increasing the amounts of Lewis number withincreasing of θ *(η)* compared to the two before modes and reduction of φ *(η)* significantly.

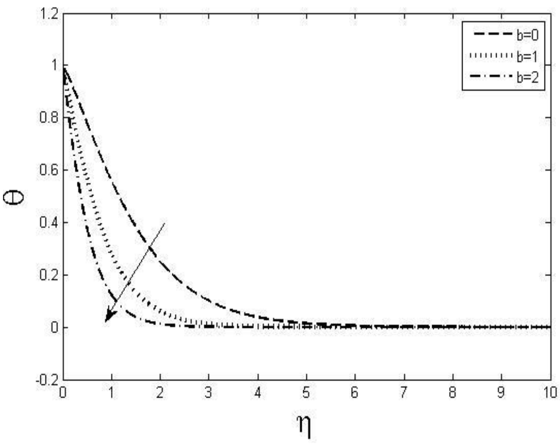

Fig. 7 .θ changes in terms of η in (*Pr = 10, Le = 100, N_t=0.5, N_b=0.5*) mode.

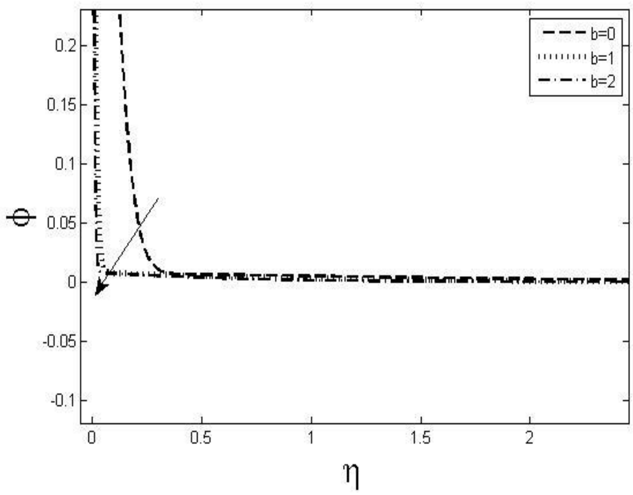

Fig. 8.φ Changes in terms of η in (*Pr = 10, Le = 100, N_t=0.5, N_b=0.5*)mode.

IV. CONCLUSIONS

- Boundary layer problem of a Nanofluid pass a stretching sheet is investigated bythe vspeed perpendicular to the stretching sheet, numerically.
- A similarity solution which dependence to the *Pr*, *Le*, N_b and $N_t$numbers, is presented.
- The results show that $\theta(\eta)$ and $\varphi(\eta)$changes are graphically presented. $\theta(\eta)$ and $\varphi(\eta)$Graphs are descending functions alsowith respect to increasing in velocity which is perpendicular to the stretching sheet, θ and φ values decreases.
- θ values in smallerLewisnumbers is smaller and in larger Lewis numbers is more alsoφvalues in smaller Lewis numbers is more and inlarger Lewis numbers is less.

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