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Experimental investigation on impact behavior of automotive bumper beam under crashes

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Abstract: - Automotive bumper beams are designed with different materials and their impact behaviors are studied. Four different shapes of bumper beams are designed using CATIA V5 R11 and their cross sections are varied with materials like Steel, Aluminium, Poly propylene (PEP) and Sheet Molding Compound (SMC). Finite element analysis is applied for studying the impact behavior of beams. Meshing the models are done by using HYPERMESH 9.0 software. LS-DYNA program manager and LS-DYNA pre/post processor helps for solving and generation results respectively. Maximum kinetic energy, maximum internal energy and maximum displacements of the beams are evaluated..

Keywords: - *Finite element analysis; bumper beams; kinetic energy; internal energy; displacement*

I. INTRODUCTION

A bumper is a shield made of steel, aluminum, rubber, or plastic that is mounted on the front and rear of a passenger car. When a low speed collision occurs, the bumper system absorbs the shock to prevent or reduce damage to the car. A modified SMC bumper beam can minimize the bumper beam deflection, impact force and stress distribution and also maximize the elastic strain energy [1]. The bio-based composite material has a potential to be used in automotive structural components by structural optimization [2]. This implies that a hybrid of kenaf/glass fibers could be utilized in automotive structural components such as bumper beam by improving the impact property [3].

II. EXPERIMENTAL PROCEDURE

A. Mechanics and modelling

Design process starts with the 2D sketch of standard possible sections which are converted into 3D design as it Sweep around a curve using CATIA V5 R11 software. Parameters Length, Height, Width are considered in the limits as per the standards. Based on the standards mentioned above, some of the Assumptions have been made to design the Bumper beam model. Length, Width and height are kept within the limits. i.e. Length is designed between 1000 mm to 1500 mm, width is designed between 50 mm to 75 mm and Height is restricted to 250 mm. These values are helpful in such a way designing Bumper is about to match the conditions of Impactor during crashes in simulation. Then the Curve Radius of the Bumper is fixed as 1514 mm based on the Bumper barrier used in RCAR bumper test.

Considering the above values, four Bumper beam models are designed by varying the sections within the values limits mentioned above using CATIA V5 R11 software. This software has more advantage in case of sheet metal design compared to others. 2D sketch is drawn in a plane and the 2D profile is swept in perpendicular Plane in the radius mentioned. Sweep is an option which helps to flow the Section over a curve makes a 3D Figure. The Four Profiles are shown below.

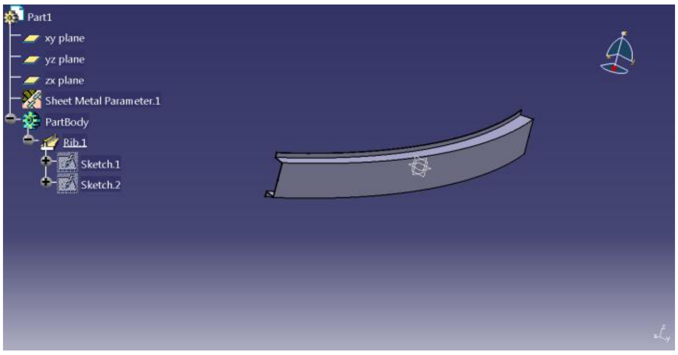

Fig.1.Simple hat profile

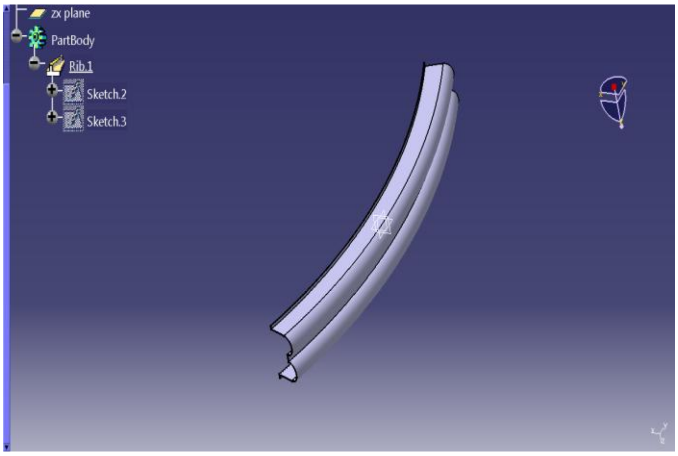

Fig.2. Double hat profile

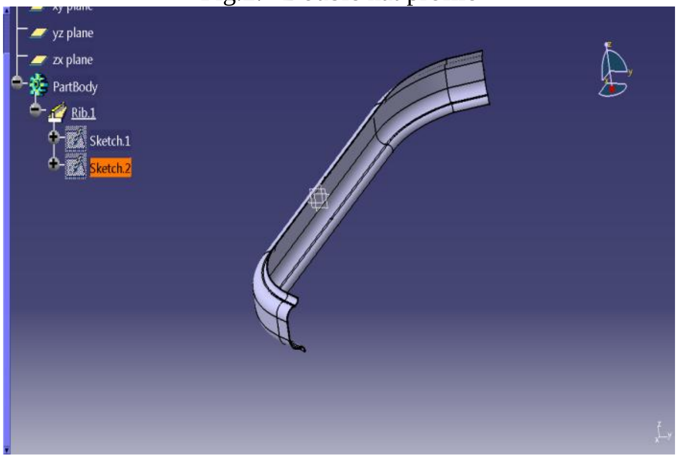

Fig. 3. Oblique profile

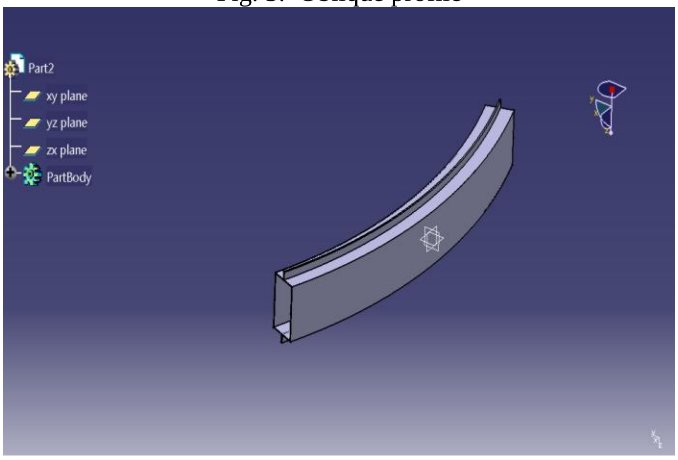

Fig. 4.Closed profile

Simple hat profile is basic profile having C shape section. Double hat profile is slight modifications of the above as it having W shape section. Oblique profile has Bends over both the sides. Closed profile is nothing but the two Simple hat profile is attached oppositely to form this profile. All these four profiles have the same sweep curve of radius 1514 mm.

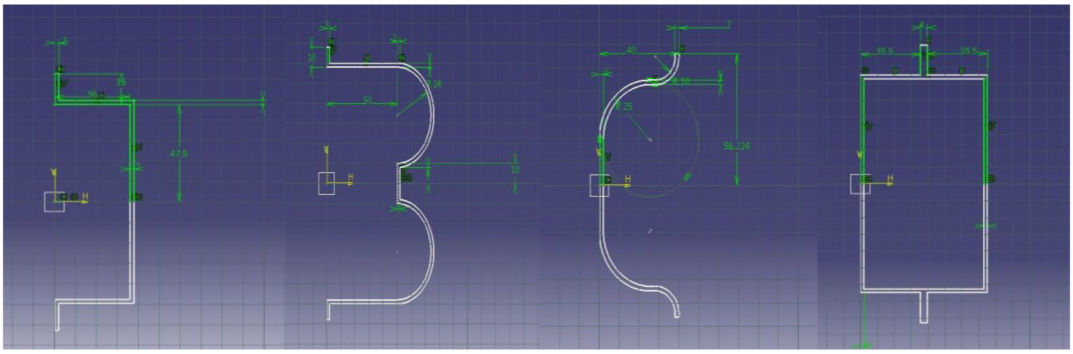

Fig. 5.Sections of simple hat, double hat, oblique and closed profile

Bumper beam profiles generated from CATIA V5 R11 as part file is converted into common standards IGES file for further processing in Hypermesh Software for meshing purpose and Analysis. Aim of this work is to find the appropriate condition (Shape, Thickness and materials) based on the Energy absorption and displacement. . Initially, the sheet thickness of the four profiles is designed with 2 mm. Later, Thickness is varied from 2 mm to 4 mm in the variation of 0.25 mm in the Hypermesh software by changing the property of the part.A good design of this Bumper beam must prepare for safety and meanwhile should have low weight. Thickness plays an important role in reducing weight of the part. Commercially, Bumper beam thickness of above 4 mm is not preferable.

B. *Finite element modeling*

Hypermesh 9.0 Software is used for meshing the model and preprocessing (Element types, Loads, Constraints) are done here with the help of Cards available for LS-DYNA processing. Material properties data is taken from previous literature referred. Bumper beam is made of steel, Aluminium and Composites. In this work, material properties for steel, Al, Polypropylene (PEP) and Sheet Moulding Compound (SMC) are taken into study for analysis and Results are compared.

TABLE I. Material properties for the materials taken under study

Material used	**Young's modulus(E) Gpa**	**Poisson's Ratio (ϑ)**	**Density (ρ)Kg/mm^3**
Commercial Steel bare –Cs	207	0.3	7.8e-6
Aluminium 3105-H18	68.9	0.33	2.72e-6
PEP(Poly Ethylene Propylene)	1.2	0.4	0.9e-6
Sheet molding Compound(SMC)	20	0.33	1.8e-6

Commercially the four materials mentioned in the above table are used widely in the production of Bumper beams in Automobile Industry. SMC is the latest material developed for the improvement in strength to weight ratio. Young's modulus, Poisson ratio and Density are the important criteria in the material selection.

The impactor, as a steel structure, was modelled with rigid solid impact elements according to precise dimensional drawings from the E.C.E. Standard. Material type 20 (mat-rigid) is assigned for the impactor which cannot be Deformable under the application of Load.

Bumper beam is assigned as deformable structure is so called "MAT type 24"A, Piecewise Linear Isotropic model.This is an elastic model made of plastic which applies the young's modulus if the stress is lower than the yield stress and measured stress-strain-curves if the stress is greater than yield stress.

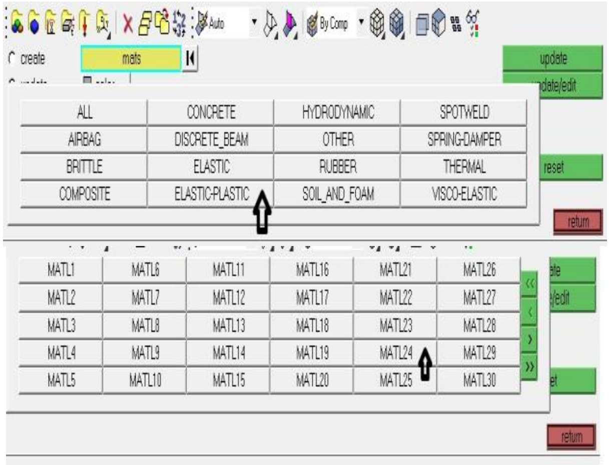

Fig. 6.Material properties cardsin hypermesh

The CAD data of the bumper structure was imported and the surfaces were created and meshed. Since the average thickness of bumper was much smaller than the other dimensions of the part, the best element for meshing was the *shell element*.

Standard procedures for meshing the model with Shell element in Hypermesh are as follows: Extract the midsurface of two given surfaces. A midsurface is the midplane layer of geometry that when meshed, can be used as a finite element shell representation of a given solid part.

Step 1:

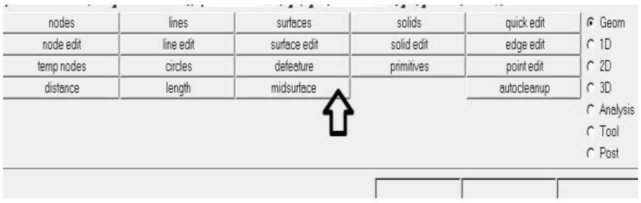

Fig. 7.Meshing the model

Step 2:

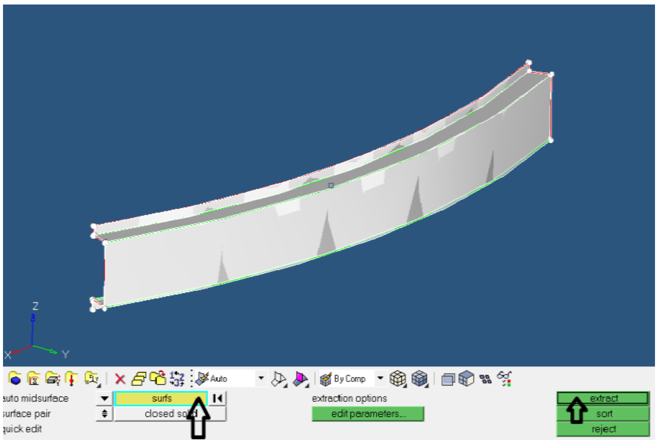

Fig. 8.Steps involved for meshing the model

Midsurface extraction can be used with sheet metal stampings, molded plastic parts with ribs, and other parts consisting of plates. As the Bumper Beam fall in this category it’s more convenient if we extract the midsurface and then later thickness is varied uniformly about a plane.

Mesh Element size is set to 5mm based on mesh Sensitivity study. Mesh sensitivity study is to ensure smallness of the element required in a model to ensure that the results of an analysis are not affected by changing the size of the mesh.

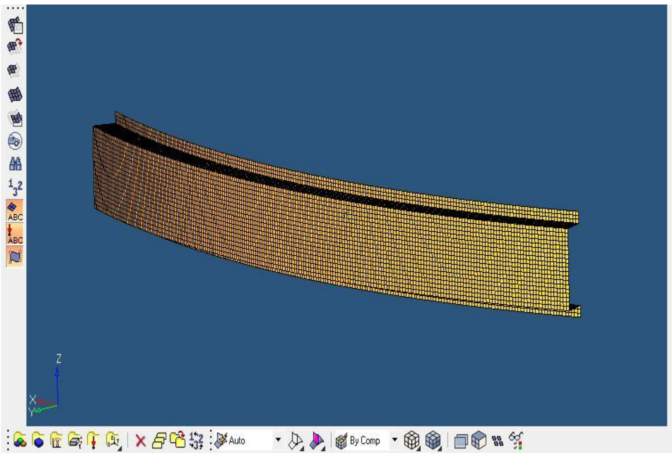

Fig. 9. Simple hat profile (meshed)

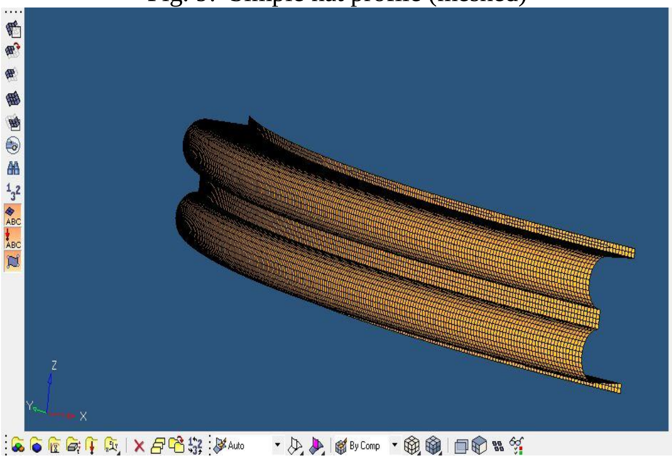

Fig. 10.Double hat profile (meshed)

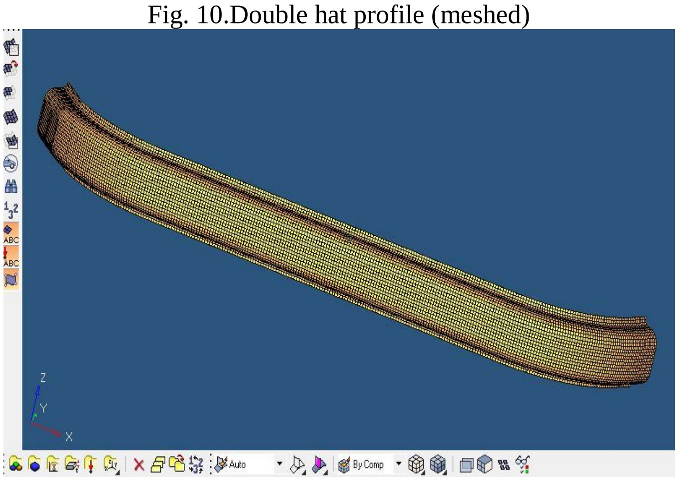

Fig.11. Oblique profile (meshed)

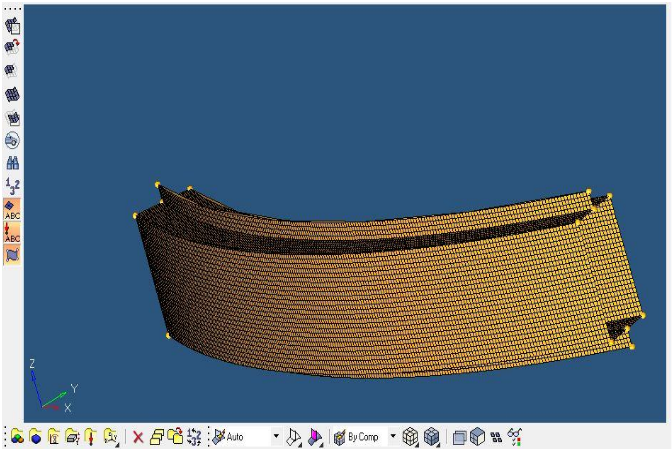

Fig. 12. Closed profile (meshed)

When solving Dynamic problems with Fem, it must be remembered that we use FEM only for spatial Discretization and the temporal time discretization is always using the FEM. We divide the total response time into much smaller intervals called time steps or increments. The equilibrium equation and the value of unknowns are determined at (t+At) based on value of time t. Explicit methods are those in which the information at time step n+1 can be obtained in terms of previous time steps and there is no dependence on the current time step.
As shown in figure 6.1, the impactor collide to the bumper beam in straight direction and perpendicularly.

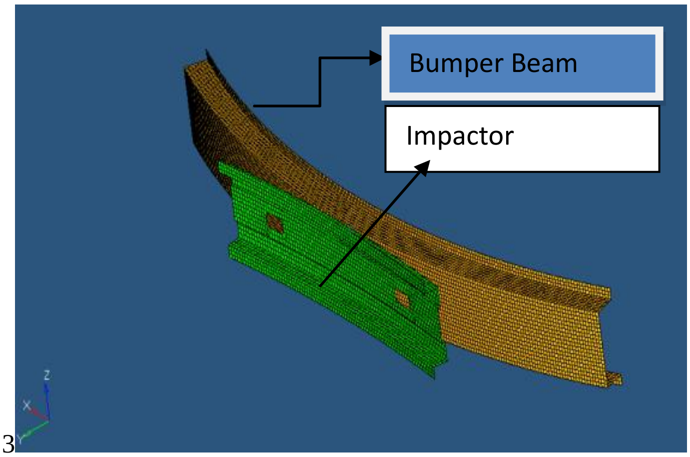

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Fig. 13. Bumper beam and impactor

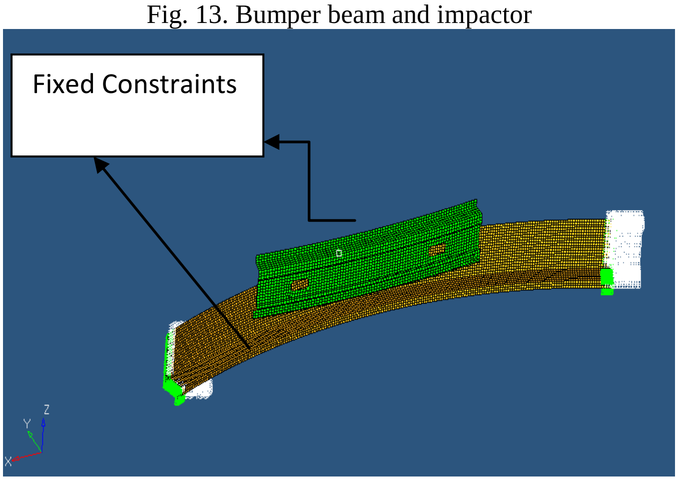


Fig. 14. Fixed constraints for simple hat profile bumper beam

Usually in all finite element study constraints plays a major role. Bumper beam is constrained at both the ends as shown in fig 6.2. The impactor collides to the bumper perpendicularly with 18.6 mm/msec velocity (based on LS-DYNA standard units).Parameters such as thickness, shape are varied for carrying out the analysis with constant velocity (18.8 mm/msec). Characteristics of Impactor remain constant for all the cases. Thickness varies from 2 mm to 4 mm in the increasing order of 0.25 mm.Hypermesh software itself has the capability to assign all loads and constraints that will be used in LS-DYNA for analysis. Keyword file (".k file") is exported from Hypermesh that is used in LS-DYNA solution manager to solve and LS-DYNA Pre/Post processor to produce results.

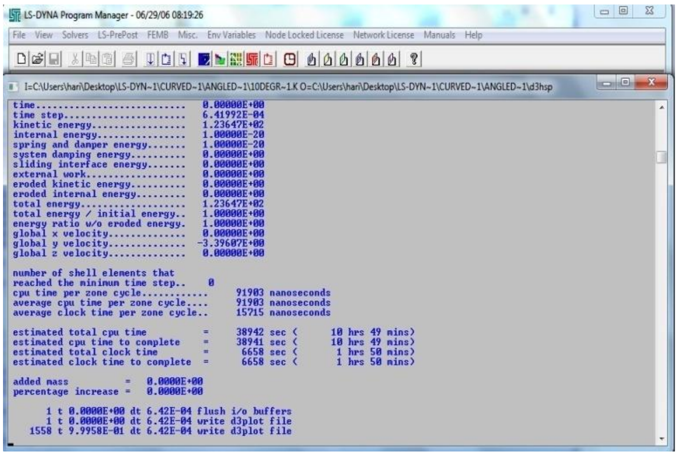


Fig. 15. LS-DYNA program manager

The time history analysis is limited to 20 milliseconds. Impactor is moving at constant velocity of 18.66 mm/msec which is app. to 65km/hr. Energy Absorption behaviour, Displacement plot are taken over the period of time. Results are obtained by varying Thickness, Shape and material.

III. RESULTS AND DISCUSSIONS

Bumper beam is designed and tested for the impact behaviour as per the conditions mentioned above. LS-DYNA program manager is used for solving and LS-DYNA pre/post processor helps to review the results. Parameters like Displacement, Internal energy absorption, Kinetic Energy etc are plotted and analysed to find the best appropriate conditions among the various cases mentioned in previous chapter.
Displacement is found to be maximum at the time of Bumper Rebound condition i.e. separation point of Bumper beam and Impactor. Values of Displacement are taken at the time of Bumper rebound condition.
Displacement sample results taken from LS-DYNA pre/post processor:

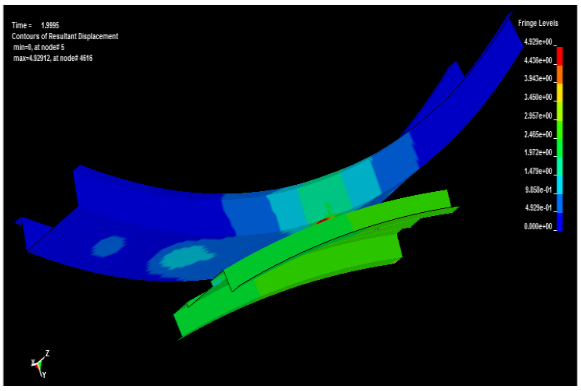


Fig. 16. Displacement plot for simple hat profile (steel material – 2mm)

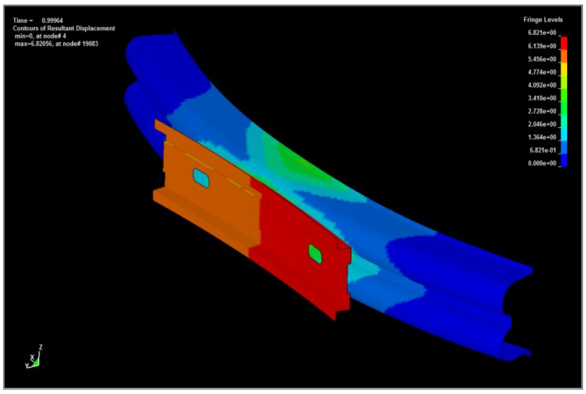


Fig. 17. Displacement plot for double hat profile (steel material – 2mm)

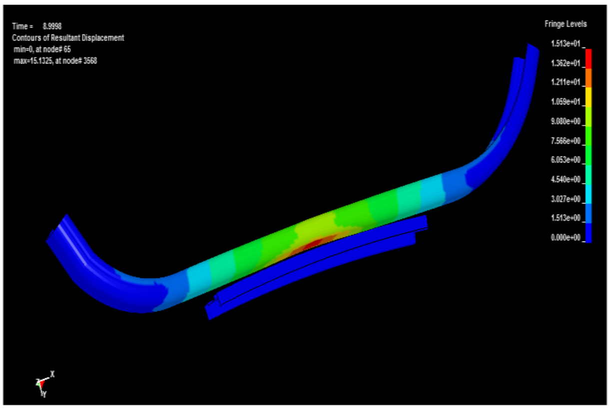


Fig. 18. Displacement plot for oblique profile (steel material – 2mm)

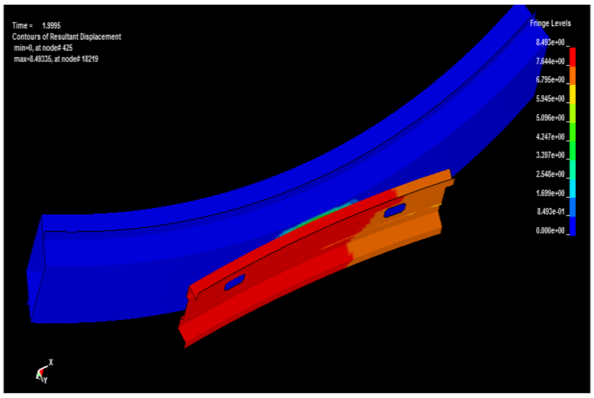


Fig. 19. Displacement plot for closed profile (steel material – 2mm)

Internal energy absorption plays a major role in selecting the best Bumper beam. So the condition with maximum internal energy absorption in thickness 2mm to 4mm with conditional displacement value is found to best condition. Internal energy is plotted against the time of 20 milliseconds. Max. Reach/ max value are taken from the graph for comparison. Kinetic Energy is decreasing as internal energy of the Beam increasing. Kinetic energy of the impactor decreases from a velocity 65km/hr (as per the conditions) to zero under collisions. Max. Kinetic energy value is taken for comparing using LS-DYNA pre/post processor.

A. *LS-Pre/Post processor results of simple hat profile*

TABLE II.Results of simple hat profile - Steel material

S.No.	Thickness mm	Max. Internal Energy (Joules)	Max. kinetic Energy (Joules)	Max. Displacement (mm)
1	2(3.24kg)	116.22	39.225	4.93
2	2.25	114.42	41.076	3.15
3	2.5	115.152	41.076	3.15
4	2.75	113.08	41.427	4.57
5	3	111.05	41.35	3.99
6	3.25	108.28	41.046	2.18
7	3.5	108.52	40.57	2.72
8	3.75	109.21	39.545	3.00
9	4	107.62	39.102	3.21

TABLE III. Results of simple hat profile - Aluminium material

S.No.	Thickness mm	Max. Internal Energy (Joules)	Max. kinetic Energy (Joules)	Max. Displacement (mm)
1	2	116.54	35.745	13.30
2	2.25	115.73	36.88	6.76
3	2.5 (1.395kg)	113.12	38.16	5.98
4	2.75	111.87	40.519	6.07
5	3	113.86	49.926	8.67
6	3.25	112.6	44.37	11.34
7	3.5	114.31	44.92	12.73
8	3.75	112.31	44.769	13.8
9	4	110.61	45.869	3.94

TABLE IV. Results of simple hat profile – Polypropylene (PEP) material

S.No.	Thickness mm	Max. Internal Energy (Joules)	Max. kinetic Energy (Joules)	Max. Displacement (mm)
1	2	318.57	76.71	192.29
2	2.25	276.29	70.76	182.72
3	2.5	255.32	59.52	167.13
4	2.75	244.19	54.67	157.33
5	3.25	273.48	59.13	144.10
6	3.5	196.57	38.46	132.28
7	3.75	181.61	38.98	121.26
8	4	172.06	40.42	107.92

TABLE V. Results of simple hat profile – Sheet moulding compound (SMC) material

S.No.	Thickness mm	Max. Internal Energy (Joules)	Max. kinetic Energy (Joules)	Max. Displacement (mm)
1	2	107.19	105.77	107.32
2	3	35.96	44049	51.16
3	4	20.69	15.12	8.73

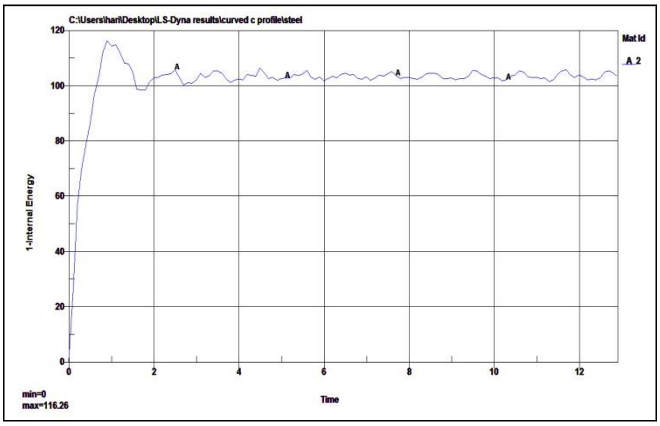


Fig. 20. Internal energy vs. Time plot for simple hat profile (steel -2mm)

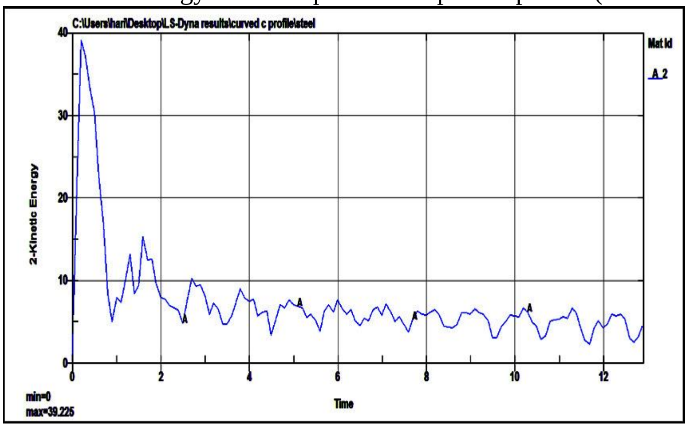


Fig. 21.Kinetic energy VS. Time plot for simple hat profile (steel -2mm)

- For Designing a Bumper beams two conditions are usually considered. First the Bumper beams are supposed to absorb Impact Energy of the Impactor. Second the deflection should not exceed to damage the adjacent component. The optimal design with these conditions satisfied should have low weight for easier manufacturing.
- Results obtained from LS-DYNA Pre/Post processor helpful to infer that Simple hat profile is best suited at thickness 2 mm, steel material and thickness 2.5 mm, Aluminium material based on maximum internal energy absorption and displacement criteria compared to other thickness values. As weight is also important criteria for manufacturing, Aluminium material is most preferred in this case (simple hat profile) based on Energy absorption and Displacement values (113.12 J and 5.98 mm) obtained.

B.*LS-PRE/POST PROCESSOR RESULTS OF DOUBLE HAT PROFILE*

TABLE VI.Results of double hat profile - Steel material

S.No.	**Thickness mm**	**Max. Internal Energy (Joules)**	**Max. kinetic Energy (Joules)**	**Max. Displacement (mm)**
1	2	114.92	25.60	6.00
2	2.25 (5.36kg)	115.27	27.71	4.52
3	2.5	113.74	28.13	3.42
4	2.75	114.33	26.76	2.72
5	3	114.52	26.56	2.35
6	3.25	115.52	28.42	2.28
7	3.5	115.21	29.10	2.13
8	3.75	115.19	28.68	1.92
9	4	114.9	27.74	1.952

TABLE VII. Results of double hat profile - Aluminium material

S.No.	**Thickness mm**	**Max. Internal Energy (Joules)**	**Max. kinetic Energy (Joules)**	**Max. Displacement (mm)**
1	2	111.88	30.09	12.50
2	2.25	112.85	31.69	10.91
3	2.5	110.07	32.12	8.79
4	2.75	110.73	34.12	7.08
5	3 (2.492kg)	108.72	37.23	5.62
6	3.25	107.08	37.75	5.37
7	3.5	107.31	39.38	5.97
8	3.75	106.91	40.88	6.48
9	4	105.38	46.42	6.93

TABLE VIII. Results of double hat profile – Polypropylene (PEP) material

S.No.	**Thickness mm**	**Max. Internal Energy (Joules)**	**Max. kinetic Energy (Joules)**	**Max. Displacement (mm)**
1	2	191.5	37.10	155.963
2	2.25	197.42	46.57	128.06
3	2.5	183.03	47.35	114.32
4	2.75	163.45	39.31	104.82
5	3	141.94	43.96	93.22
6	3.25	142.89	45.32	80.01
7	3.5	121.45	44.10	64.781
8	3.75	113.58	46.45	43.92
9	4	110.59	48.11	38.78

TABLE IX. Results of double hat profile – Sheet moulding compound (SMC) material

S.No.	**Thickness mm**	**Max. Internal Energy (Joules)**	**Max. kinetic Energy (Joules)**	**Max. Displacement (mm)**
1	2	110.32	38.00	14.85
2	3	105.36	43.98	10.54
3	4 (2.199kg)	102.37	60.88	5.61

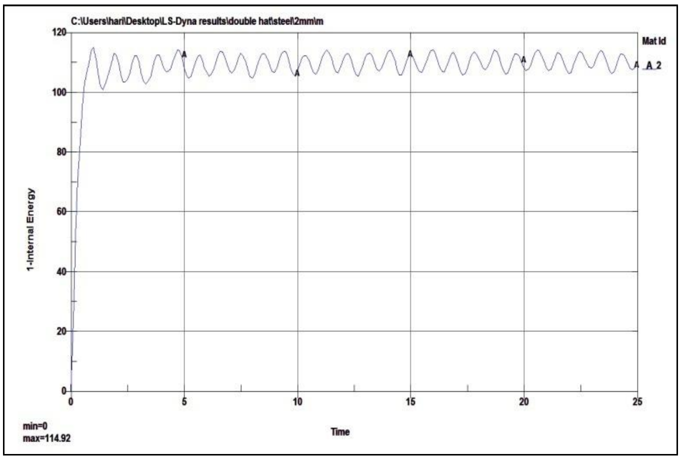
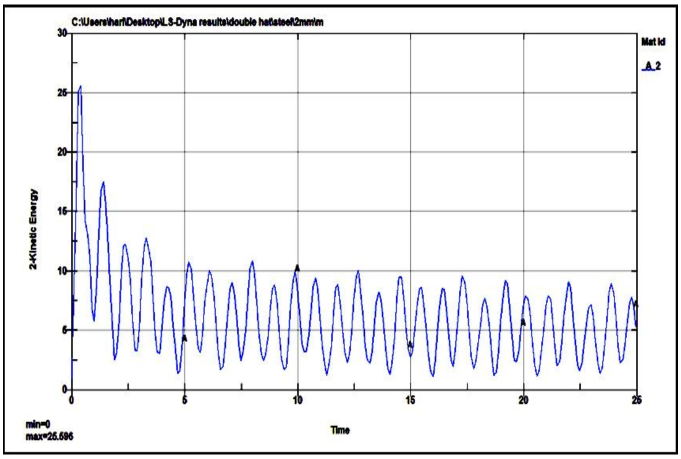


Fig. 22.Internal energy VS. Time plot for double hat profile (steel -2mm)



Fig, 23.Kinetic energy plot VS. Time for double hat profile (steel -2mm)

- In case of double hat profile, best suited thickness is 2.25 mm for steel and 3 mm thick for aluminium and 4 mm thick for SMC is found more preferred based on Energy absorption and Displacement values (102.31 J and 5.61 mm) obtained. As, SMC is most recently developed reinforced composite material which has high strength to weight ratio compared to aluminium. So, SMC is preferred for easier manufacturing and availability at lower cost in this case (Double Hat profile) based on Energy absorption and Displacement values obtained.

C.*LS-PRE/POST PROCESSOR RESULTS OF OBLIQUE PROFILE*

TABLE X. Results of oblique profile - Steel material

S.No.	**Thickness mm**	**Max. Internal Energy (Joules)**	**Max. kinetic Energy (Joules)**	**Max. Displacement (mm)**
1	2	120.52	40.153	16.17
2	2.25 (5.36kg)	120.04	39.089	12.582
3	2.5	119.95	37.62	9.94
4	2.75	119.74	37.15	7.89
5	3	119.27	36.54	6.32
6	3.25 (5.265kg)	119.25	35.24	4.72
7	3.5	119.23	34.42	3.42
8	3.75	119.21	33.99	2.37
9	4	117.28	34.56	1.75

TABLE XI. Results of oblique profile - Aluminium material

S.No.	**Thickness mm**	**Max. Internal Energy (Joules)**	**Max. kinetic Energy (Joules)**	**Max. Displacement (mm)**
1	2	117.1	41.52	35.15
2	2.25	117.15	42.67	29.95
3	2.5	116.35	42.38	25.19
4	2.75	116.16	44.60	19.18
5	3	114.84	48.04	15.03
6	3.25	113.44	49.67	12.376
7	3.5	113.08	50.57	9.05
8	3.75	113.03	54.12	7.25
9	4 (2.260kg)	110.75	55.81	5.23

TABLE XII. Results of oblique profile – Polypropylene (PEP) material

S.No.	Thickness mm	Max. Internal Energy (Joules)	Max. kinetic Energy (Joules)	Max. Displacement (mm)
1	2	156.95	37.90	305.27
2	2.25	185.16	39.81	285.95
3	2.5	196.87	153.46	291.90
4	2.75	220.4	65.79	247.17
5	3	254.86	104.26	247.72
6	3.25	253.77	72.62	241.78
7	3.5	294.94	66.09	252.87
8	3.75	270.06	53.02	258.09
9	4	363.67	79.47	252.48

TABLE XIII. Results of oblique profile – Sheet moulding compound (SMC) material

S.No.	Thickness mm	Max. Internal Energy (Joules)	Max. kinetic Energy (Joules)	Max. Displacement (mm)
1	2	103.85	43.36	52.36
2	3	103.06	55.60	22.306
3	4	99.09	68.23	19.21

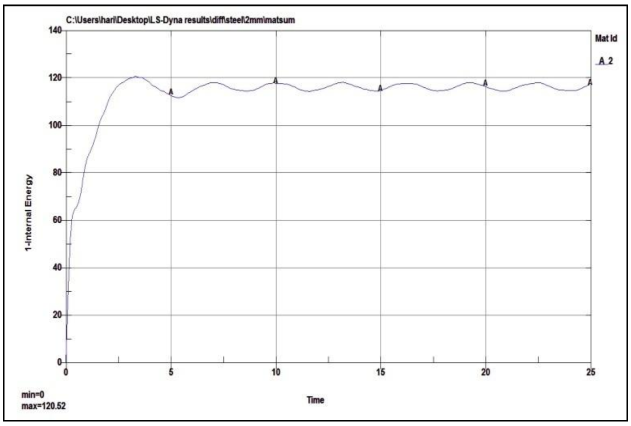


Fig. 24.Internal energy VS. Time plot for oblique profile (steel -2mm)

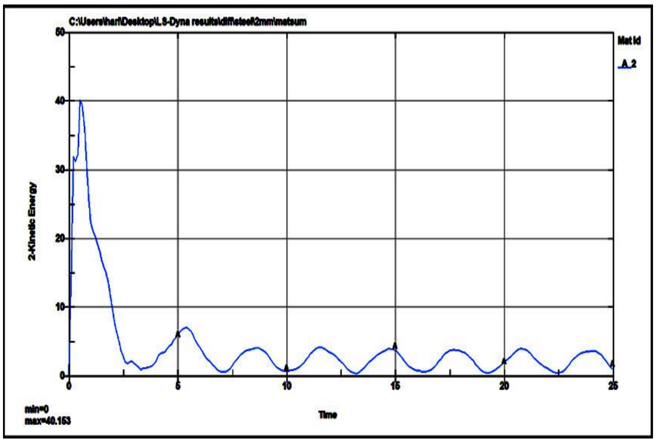


Fig. 25.Kinetic energy VS. Time plot for oblique profile (steel -2mm)

In case of oblique profile energy absorption value seems increasing than other profiles and best suited thickness at 3.25 mm for steel and 4 mm thick for aluminium is more preferred based on Energy absorption and

Displacement values (110.75 J and 5.23 mm) obtained. SMC is found to show higher values as it tends to cause damage to adjacent component.

D.*LS-PRE/POST PROCESSOR RESULTS OF CLOSED PROFILE*

TABLE XIV.Results of closed profile - Steel material

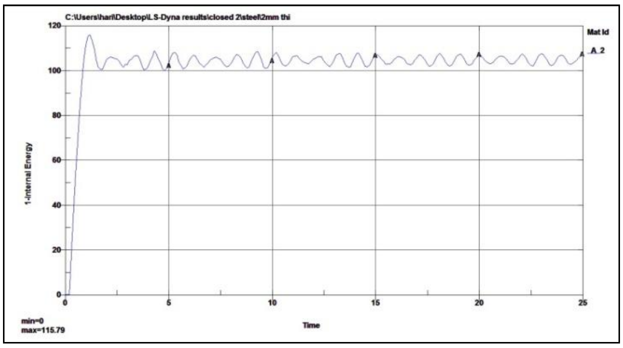
S.No.	Thickness mm	Max. Internal Energy (Joules)	Max. kinetic Energy (Joules)	Max. Displacement (mm)
1	2	115.79	37.45	8.49
2	2.5 (8.53kg)	115.14	39.56	5.55
3	3	112.28	39.87	4.89
4	3.5	110.57	39.98	3.54
5	4	105.94	40.99	2.56

TABLE XV. Results of closed profile - Aluminium material

S.No.	Thickness mm	Max. Internal Energy (Joules)	Max. kinetic Energy (Joules)	Max. Displacement (mm)
1	2 (2.38kg)	116.45	32.92	5.73
2	2.5	115.29	34.74	7.74
3	3	112.17	37.98	7.62
4	3.5	108.57	36.47	3.74
5	4	104.94	44.60	2.70

TABLE XVI. Results of closed profile – Sheet moulding compound (SMC) material

S.No.	Thickness mm	Max. Internal Energy (Joules)	Max. kinetic Energy (Joules)	Max. Displacement (mm)
1	2	112.43	40.272	12.61
2	2.5	112.76	43.49	3.84
3	3 (2.36kg)	108.1	46.6	5.61
4	3.5	102.04	51.03	7.23
5	4	95.12	66.06	4.87



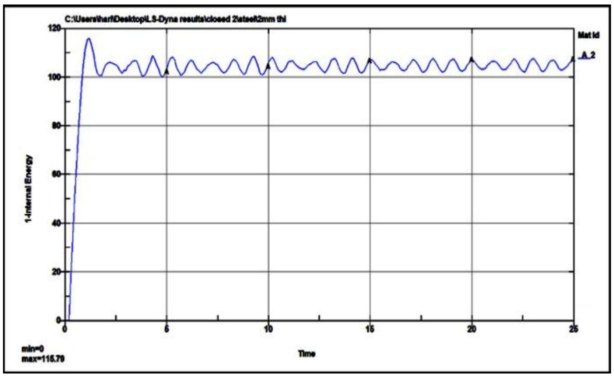


Fig.26.Internal energy VS. Time plot for closed profile (steel -2mm)

Fig.27.Kinetic energy VS. Time plot for closed profile (steel -2mm)

- In the case of closed profile, Thickness 2.5 mm, steel material shows good energy absorption and displacement values compared to other simulations. SMC material at thickness 3mm shows results of energy absorption and displacement 108.1J and 5.61 mm respectively are found suitable. Also at thickness 2 mm, Aluminium material is most preferred than others and also less weight based on Energy absorption and Displacement values (116.45 J and 5.43 mm) obtained.
- Overall, PEP material is unsuited material as the displacement value is high for all the four profiles as it tends to cause damage to adjacent component and also it tends to break at certain point of load conditions. Hence, it is found that PEP is unsuitable material for Bumper beam.
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IV. CONCLUSION

Reviewing the results among four bumper profiles with four different materials and varying thickness from 2mm to 4mm shows appropriate conditions to be chosen. Increasing bumper thickness causes a rise in bumper rigidity and impact force. Consequently, it results in reduction in bumper deflection and stress. Aluminium is found to be best choice between the metals for weight reduction. In case of Composites, SMC is found to be best choice as per weight reduction, cost effectiveness study and easier manufacturing.

Double hat profile and closed profile shows better results in energy absorption and displacement study in minimal thickness value. But considering the weight Double hat profile is found to be better choice among the profiles under study.

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