

A EPQ inventory model with Stochastic Demand, finite production rate of Deteriorating items, Shortages And two-level of credit financing

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Abstract:-An inventory model for deteriorating items and shortages with finite production rate and stochastic demand rate is developed when the supplier offers delay period to the retailer for due payment against purchases and the retailer in turn extends the trade credit offer to its customers. This policy of passing on of the credit period is well known as **two-level of credit financing**. Items in the system follow stochastic demand behavior that are produced with finite production rate and subjected to constant rate of deterioration. The model is developed with an objective to minimize total expected cost of retailer as it is assumed to be a dominant player in the supply chain. A special case is made by taking time dependent deterioration. A numerical example is provided using LINGO software. Sensitivity analysis of various parameters is carried out to gain meaningful managerial insights.

Keywords: *Inventory. Deterioration. Finite production. Stochastic demand. Trade credit. Two-level credit financing*

I. INTRODUCTION

It is a prudent and established trade practice that supplier offers time credit to retailer for settling dues and during this period retailer is not charged any interest by supplier. This works like a promotional tool in the hands of supplier to lure retailer to buy more during interest free credit period and settle its account later. Retailer is charged interest, normally higher the rate at which he will be able to earn otherwise, by supplier beyond free credit period. This in turn compels retailer economically to settle account at the end of free credit period. Retailer also offers credit period, less than what is offered by supplier to retailer, in turn to its customers to enhance demand. This is well known as two-level of credit financing in the literature of supply chain inventory modeling.

Goyal [7] developed first mathematical model when trade credit is offered to settle the payment. Chung [4] studied replenishment policies for deteriorating items and later it was extended by Chung et al. [5] for permissible delay in payment. Shah et al. [14] also extended it for progressive payment scheme under discounting. Shah et al. [15] gave an up-to-date review of available literature on inventory and trade credit. Most of the citations in the review article are based on the assumption that the demand rate is constant and known; i.e. deterministic. Shah et al. [8] developed inventory model that deals with temporary price discount under conditions deterioration. Cárdenas-Barrón [1, 2] developed model for finding optimal ordering policies in response to a discount offer. Later Cárdenas-Barrón et al. [3] extended to take advantage of a one-time discount offer with allowed backorders. Widyadana et al. [16] developed economic order quantity model for deteriorating items with planned backorder level.

However, the behavior of customer for a particular product is hardly known and remains constant. Thus the assumption of probabilistic demand is more appropriate to capture uncertain demand. Shah and Shah [12] formulated a probabilistic inventory model when delay in payments is permissible. Later on, it was extended by Shah [9] to study the effect of deterioration on the optimal solution. Shah and Shah [13] developed a probabilistic inventory model for deteriorating items under trade credit policy by considering discrete time. Shah [10, 11] allowed shortages to analyze the results when demand is stochastic and trade credit is offered. De and Goswami [6] analyzed an economic order quantity model for deteriorating items under scenario of credit financing, to minimize the joint relevant costs for two players in the supply chain, viz. retailer and customer. Shah et. al. [17] Considering deteriorating inventory model with finite production rate and two-level of credit financing for stochastic demand

The aim of this paper is to develop a deteriorating inventory model under the two-level trade credit policy when demand is stochastic and production rate is finite subjected to constant rate of deterioration with shortages and a special case of the model; with time dependent demand. By two-level trade credit policy we mean that supplier offers retailer a credit period which in turn is passed on to the customer to settle the accounts against the purchases made. Model is analyzed from the retailer's perspective to minimize total expected cost in

the hands of retailer. Results are validated with the help of numerical example. Sensitivity analysis is carried out on various parameters and managerial insights are discussed.

2. Notations and assumptions:

Following notations and assumptions are used to develop proposed model.

2.1 Notations:

- X Random demand during any scheduling period
- P Finite production rate
- A Ordering cost per order
- C_h Holding cost per unit per unit time
- C_d Deterioration cost per unit per unit time
- C_s Shortages cost per unit per unit time
- C Retailer’s Purchase cost per unit
- S Retailer’s selling price per unit offered to customer; S ≥ C
- I_e Interest earned by the retailer per \$ per year
- I_c Interest charged per \$ in stock per year by the supplier; I_c ≥ I_e
- M Credit period offered by the supplier to the retailer
- N Credit period offered by the retailer to the customer
- Q₁(t) Inventory level at any instant of time t during production period
- Q₂(t) Inventory level at any instant of time t during non-production period
- Q₃(t) Inventory level at any instant of time t during shortages period
- T Cycle time in years
- ETC Total expected cost per unit time
- θ Rate of deterioration (0 ≤ θ < 1)

2.2 Assumptions:

- (i) Supply chain under consideration consists of three players; supplier, retailer and customer.
- (ii) Inventory system deals with a single item.
- (iii) Production rate is finite and constant.
- (iv) Lead-time is zero or negligible.
- (v) Shortages are allowed.
- (vi) Demand x during any scheduling period T follows a probability distribution Function f(x), a(T) ≤ x ≤ b(T) where R = μ(T)/T is the average demand rate and

$$\mu(T) = E(x) = \int_{a(T)}^{b(T)} x f(x) dx \quad \dots\dots\dots (1)$$

as the mean demand during T. a(T) and b(T) denote the minimum and maximum demand respectively during cycle time T. R is assumed to be a known constant and the demand of x- units occur uniformly over the scheduling period T.

- (vii) Deterioration is considered to be constant.

II. Model developments:

The production starts at t=0 and continues up to t=t₁ at which maximum inventory level b(T) is attained. Production stops at t=t₁ and, thereafter, inventory depletes to zero at time t=t₂, due to demand and deterioration of units, shown in fig 1.

During production period, inventory level changes due to production, demand and constant rate of deterioration of units. Thus, the rate of change of inventory level at

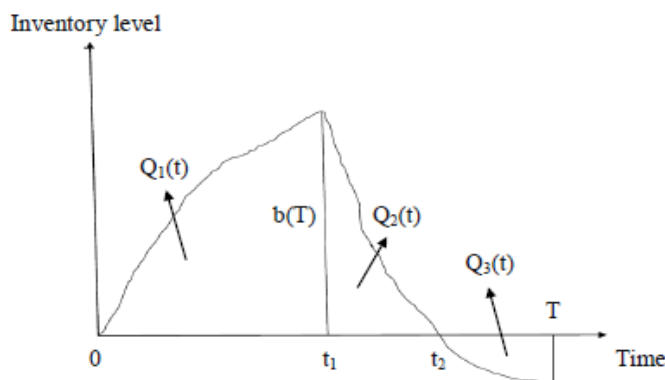


Fig.1 Inventory - time representation.

any instant of time t during production period [0, t1] can be described by the differential equation

$$\frac{dQ_1(t)}{dt} + \theta Q_1(t) = P - \frac{x}{T} \quad , 0 \leq t \leq t_1 \quad \dots\dots\dots (2)$$

With the initial condition Q1(0)=0.

During non-production period [t1, t2], inventory changes due to demand and deterioration of units. The rate of change of inventory level in this time period is governed by the differential equation

$$\frac{dQ_2(t)}{dt} + \theta Q_2(t) = -\frac{x}{T} \quad , t_1 \leq t \leq t_2 \quad \dots\dots\dots (3)$$

During shortages period [t2, T], inventory changes due to demand and deterioration of units. The rate of change of inventory level in this time period is governed by the differential equation

$$\frac{dQ_3(t)}{dt} = -\frac{x}{T} \quad , t_2 \leq t \leq T \quad \dots\dots\dots (4)$$

With the initial condition Q1(t2)= Q2(t1) and Q2(t2)=0.

The solutions of the differential Equations. (2), (3) and (4) are respectively as:

$$Q_1(t) = \frac{1}{\theta} (P - x/T) (1 - e^{-\theta t}) \quad , 0 \leq t \leq t_1 \quad \dots\dots\dots (5)$$

$$Q_2(t) = \frac{P}{\theta} (e^{-\theta(t-t_1)} - e^{-\theta t}) - \frac{x}{\theta T} (1 - e^{-\theta t}) \quad , t_1 \leq t \leq t_2 \quad \dots\dots\dots (6)$$

$$\text{And, } Q_3(t) = \frac{x}{T} (t_2 - t) \quad , t_2 \leq t \leq T \quad \dots\dots\dots (7)$$

From the boundary condition, Q2(t2)=0, we have

$$P e^{-\theta t_1} - \frac{x}{T} e^{-\theta t_2} - \left(P - \frac{x}{T} \right) = 0 \quad \dots\dots\dots (8)$$

Now, the expected holding cost per unit time are given by

$$\begin{aligned} \text{EHC} &= C_h \int_{a(T)}^{b(T)} \left\{ \int_0^{t_1} Q_1(t) dt + \int_{t_1}^{t_2} Q_2(t) dt \right\} f(x/T) dx \\ &= C_h \int_{a(T)}^{b(T)} \left\{ \int_0^{t_1} \frac{1}{\theta} (P - x/T) (1 - e^{-\theta t}) dt + \int_{t_1}^{t_2} \left[\frac{1}{\theta} P (e^{-\theta(t-t_1)} - e^{-\theta t}) - \frac{x}{\theta T} (1 - e^{-\theta t}) \right] dt \right\} f(x/T) dx \\ &= C_h \left[\frac{P}{\theta^2} \{ e^{-\theta t_2} - e^{-\theta(t_2-t_1)} + \theta t_1 \} - \frac{\mu(T)}{\theta^2 T} \{ e^{-\theta t_2} + \theta t_2 - 1 \} \right] \quad \dots\dots\dots (9) \end{aligned}$$

The cost due to deterioration per unit time are given by

$$\begin{aligned}
 \text{EDC} &= C_d \int_{a(T)}^{b(T)} \left\{ \int_0^{t_1} Q_1(t) dt + \int_{t_1}^{t_2} Q_2(t) dt \right\} f(x/T) dx \\
 &= C_d \int_{a(T)}^{b(T)} \left\{ \int_0^{t_1} \frac{1}{\theta} (P - x/T) (1 - e^{-\theta t}) dt + \int_{t_1}^{t_2} \left[\frac{1}{\theta} P (e^{-\theta(t-t_1)} - e^{-\theta t}) - \frac{x}{\theta T} (1 - e^{-\theta t}) \right] dt \right\} f(x/T) dx \\
 &= C_d \left[\frac{P}{\theta^2} \{ e^{-\theta t_2} - e^{-\theta(t_2-t_1)} + \theta t_1 \} - \frac{\mu(T)}{\theta^2 T} \{ e^{-\theta t_2} + \theta t_2 - 1 \} \right] \dots\dots (10)
 \end{aligned}$$

The cost due to shortages per unit time are given by

$$\begin{aligned}
 \text{ESC} &= C_s \int_{a(T)}^{b(T)} \left\{ \int_{t_2}^T Q_3(t) dt \right\} f(x/T) dx \\
 &= C_s \int_{a(T)}^{b(T)} \left\{ \int_{t_2}^T \frac{x}{T} (t_2 - t) dt \right\} f(x/T) dx \\
 &= C_s \frac{\mu}{T} \left\{ T t_2 - \frac{1}{2} (T^2 - t_2^2) \right\} \dots\dots\dots (11)
 \end{aligned}$$

And ordering cost is $OC = A/T$

Supplier gives a credit period M to the retailer and retailer offers a credit period N ($M > N$) to the customers. Hence, the retailer can generate revenue and earn interest at the rate I_e during the interval $[N, M]$. After that the interest is to be paid at a rate I_c per unit on the unsold stock in the inventory system. Depending on the lengths of M, N, t_1 and t_2 , four cases may arise. The interest earned and interests charged in each of the cases are computed as follows:

Case -I: $M \leq t_1 \leq t_2$ (Fig. 2)

Expected interest payable per unit time

$$\begin{aligned}
 &= \frac{CI_c}{T} \int_{a(T)}^{b(T)} \left\{ \int_M^{t_1} Q_1(t) dt + \int_{t_1}^{t_2} Q_2(t) dt \right\} f(x/T) dx \\
 &= \frac{CI_c}{T} \int_{a(T)}^{b(T)} \left\{ \int_M^{t_1} \frac{1}{\theta} (P - x/T) (1 - e^{-\theta t}) dt + \int_{t_1}^{t_2} \left[\frac{1}{\theta} P (e^{-\theta(t-t_1)} - e^{-\theta t}) - \frac{x}{\theta T} (1 - e^{-\theta t}) \right] dt \right\} f(x/T) dx \\
 &= \frac{CI_c}{\theta^2 T} \left\{ P (e^{-\theta t_2} - e^{-\theta M} - e^{-\theta(t_2-t_1)} + \theta t_1 - \theta M + 1) - \frac{\mu}{T} (e^{-\theta t_2} - e^{-\theta M} + \theta t_2 - \theta M) \right\} \dots\dots (12)
 \end{aligned}$$

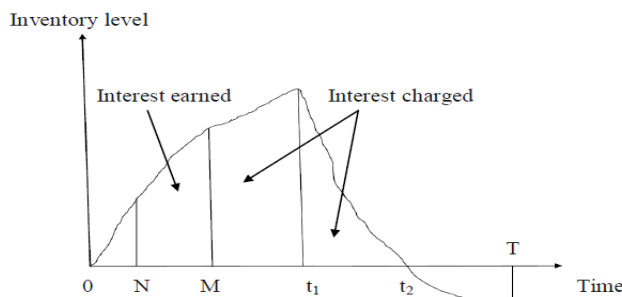


Fig. 2 Interest earned and charged when $M \leq t_1 \leq t_2$

And expected interest earned during $[N, M]$ per unit time

$$= \frac{SI_e}{T} \int_{a(T)}^{b(T)} \left\{ \int_N^M \frac{x}{T} dt \right\} f(x/T) dx = \frac{SI_e \mu (M^2 - N^2)}{2T^2} \dots\dots\dots (13)$$

Case -II: $t_1 \leq M \leq t_2$ (Fig. 3)

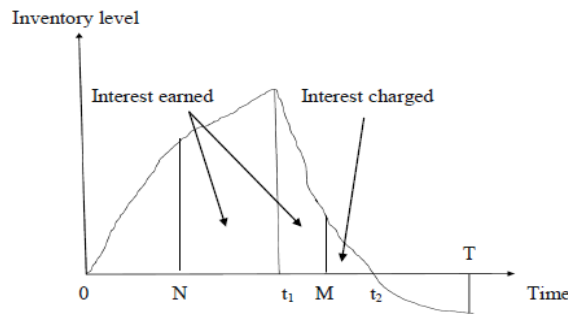
Expected interest charged per unit time

$$= \frac{CI_c}{T} \int_{a(T)}^{b(T)} \left\{ \int_M^{t_2} Q_2(t) dt \right\} f(x/T) dx$$

$$= \frac{CI_c}{T} \int_{a(T)}^{b(T)} \left\{ \int_M^{t_2} \left[\frac{P}{\theta} (e^{-\theta(t-t_1)} - e^{-\theta t}) - \frac{x}{\theta} (1 - e^{-\theta t}) \right] dt \right\} f(x/T) dx$$

$$= \frac{CI_c}{\theta^2 T} \left\{ P (e^{-\theta t_2} + e^{-\theta(M-t_1)} - e^{-\theta M} - e^{-\theta(t_2-t_1)}) - \frac{\mu}{T} (e^{-\theta t_2} - e^{-\theta M} + \theta T - \theta M) \right\} \dots (14)$$

Fig. 3 Interest earned and charged when $t_1 \leq M \leq t_2$



And expected interest earned during [N, M] per unit time is same as in case 1.

$$\frac{SI_e \mu (M^2 - N^2)}{2T^2} \dots\dots\dots (15)$$

Case -III: $N \leq t_2 \leq M$ (Fig. 4)

In this case, retailer's stock is zero before the payment is due. Retailer has sufficient amount to fully settle account and clear dues at time point M. The interest charged to retailer by supplier will be zero.

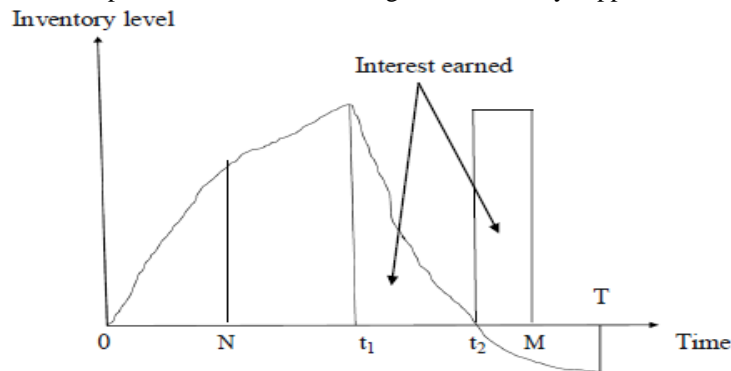


Fig. 4 Interest earned when $N \leq t_2 \leq M$

And expected interest earned per unit time is given by

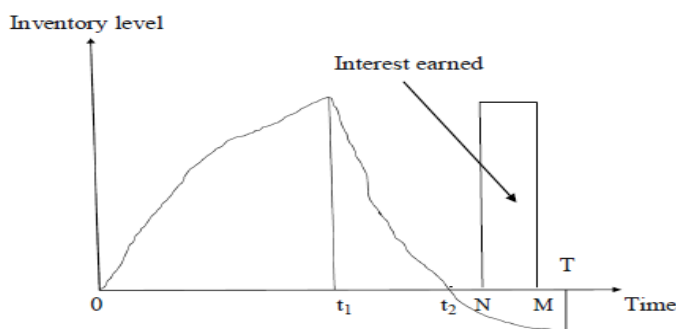
$$\begin{aligned}
 &= \frac{SI_e}{T} \int_{a(T)}^{b(T)} \left\{ \frac{x}{T} t dt + (M - t_2) \frac{x}{T} t_2 \right\} f(x/T) dx \\
 &= \frac{SI_e (2Mt_2 - N^2 - t_2^2) \mu}{2T^2} \dots\dots (16)
 \end{aligned}$$

Case -IV: $0 \leq t_2 \leq N$ (Fig. 5)

Similar to case 3, retailer will not have to pay any interest charges and expected interest earned per unit time

$$\begin{aligned}
 &= \frac{SI_e}{T} \int_{a(T)}^{b(T)} \left\{ (M - N) \frac{x}{T} t_2 \right\} f(x/T) dx \\
 &= \frac{SI_e (M - N) \mu}{T^2} \dots\dots\dots (17)
 \end{aligned}$$

Fig. 5 Interest earned when $0 \leq t_2 \leq N$



Hence, the total expected costs TC_i , $i=1, 2, 3, 4$ for four cases, per unit time are aggregate of ordering cost, holding cost, deterioration cost and shortages cost, interest charged less interest earned. Therefore, the total expected costs are:

$$\begin{aligned}
 TC_1 = & \frac{A}{T} + \frac{(C_h + C_d)}{T} \left[\frac{P}{\theta^2} \{ e^{-\theta t_2} - e^{-\theta(t_2-t_1)} + \theta t_1 \} - \frac{\mu(T)}{\theta^2 T} \{ e^{-\theta t_2} + \theta t_2 - 1 \} \right] + C_s \frac{\mu}{T} \left\{ T t_2 - \frac{1}{2} (T^2 - t_2^2) \right\} \\
 & + \frac{CI_c}{\theta^2 T} \left\{ P (e^{-\theta t_2} - e^{-\theta M} - e^{-\theta(t_2-t_1)} + \theta t_1 - \theta M + 1) - \frac{\mu}{T} (e^{-\theta t_2} - e^{-\theta M} + \theta t_2 - \theta M) \right\} \\
 & - \frac{SI_e \mu (M^2 - N^2)}{2T^2} \dots\dots\dots (18)
 \end{aligned}$$

$$\begin{aligned}
 TC_2 = & \frac{A}{T} + \frac{(C_h + C_d)}{T} \left[\frac{P}{\theta^2} \{ e^{-\theta t_2} - e^{-\theta(t_2-t_1)} + \theta t_1 \} - \frac{\mu(T)}{\theta^2 T} \{ e^{-\theta t_2} + \theta t_2 - 1 \} \right] + C_s \frac{\mu}{T} \left\{ T t_2 - \frac{1}{2} (T^2 - t_2^2) \right\} \\
 & + \frac{CI_c}{\theta^2 T} \left\{ P (e^{-\theta t_2} + e^{-\theta(M-t_1)} - e^{-\theta M} - e^{-\theta(t_2-t_1)}) - \frac{\mu}{T} (e^{-\theta t_2} - e^{-\theta M} + \theta T - \theta M) \right\} \\
 & - \frac{SI_e \mu (M^2 - N^2)}{2T^2} \dots\dots\dots (19)
 \end{aligned}$$

$$TC_3 = \frac{A}{T} + \frac{(C_h + C_d)}{T} \left[\frac{P}{\theta^2} \{e^{-\theta t_2} - e^{-\theta(t_2-t_1)} + \theta t_1\} - \frac{\mu(T)}{\theta^2 T} \{e^{-\theta t_2} + \theta t_2 - 1\} \right] + C_s \frac{\mu}{T} \left\{ T t_2 - \frac{1}{2} (T^2 - t_2^2) \right\} - \frac{S I_e (2 M t_2 - N^2 - t_2^2) \mu}{2 T^2} \dots\dots\dots (20)$$

$$TC_4 = \frac{A}{T} + \frac{(C_h + C_d)}{T} \left[\frac{P}{\theta^2} \{e^{-\theta t_2} - e^{-\theta(t_2-t_1)} + \theta t_1\} - \frac{\mu(T)}{\theta^2 T} \{e^{-\theta t_2} + \theta t_2 - 1\} \right] + C_s \frac{\mu}{T} \left\{ T t_2 - \frac{1}{2} (T^2 - t_2^2) \right\} - \frac{S I_e (M - N) \mu}{T^2} \dots\dots\dots (21)$$

Clearly, TC_i , $i=1, 2, 3, 4$ is continuous function of T and the second order derivatives of TC_i 's, $i=1, 2, 3, 4$ in (18) to (21) with respect to T are greater zero. Thus TC_i , $i=1, 2, 3, 4$ are convex functions and attain minimum values. Hence the optimal total cost, for a certain value of M and N we consider the minimum of the all total costs as:

$$TC = \min \begin{cases} TC_1 & M \leq t_1 \leq t_2 \\ TC_2 & t_1 \leq M \leq t_2 \\ TC_3 & N \leq t_2 \leq M \\ TC_4 & 0 \leq t_2 \leq N \end{cases} \dots\dots\dots (22)$$

4. Numerical examples:

An example is given to illustrate the results of the model developed in this study with the following data:

We Consider $C_h=6, C_d=4, C_s=3, \mu=40, \theta=0.4, P=400, I_c=0.8, I_e=0.5, A=200, c=10, b=10, s=15$ then,

For $M=0.05$ and $N=0.008$ **Optimum TC=127.40**
 For $M=0.2$ and $N=0.01$ **Optimum TC=127.40**

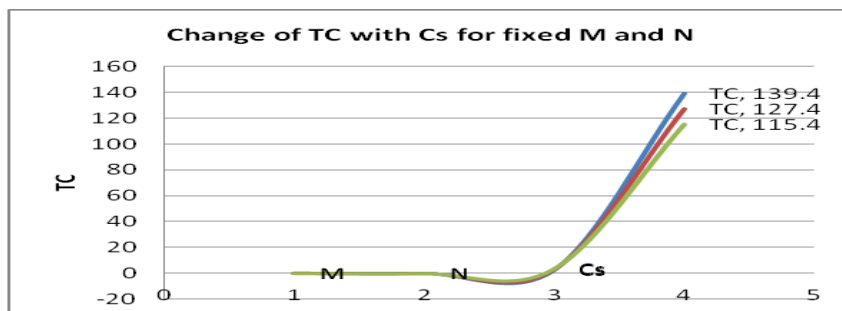
4.1 Sensitivity Analysis:

The sensitivity analysis of total cost with Shortages cost for fixed value of M and N :

Example 1:

M	N	Cs	ETC
0.05	0.008	2.4	139.40
0.05	0.008	3.0	127.40
0.05	0.008	3.6	115.40

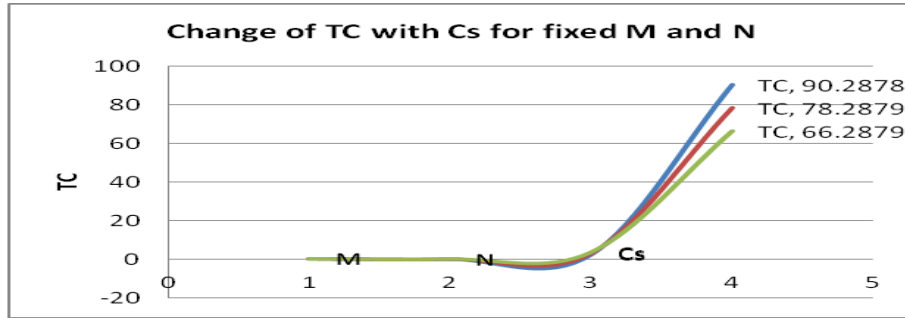
Graphically change ETC: (fig. 6) for $M=0.05$ & $N=0.008$



Example 2:

M	N	Cs	ETC
0.2	0.01	2.4	139.40
0.2	0.01	3.0	127.40
0.2	0.01	3.6	115.40

Graphically change ETC: (fig. 7) for M=0.2 & N=0.01



From these two examples we say that shortages has important role over total cost. i.e. total cost is sensitive with shortages. Total cost decreases with increase of shortages cost.

5. A Special Case of the Model (With time dependent deterioration):

In practical situation deterioration is not constant in major cases. We consider here in this special case model with time dependent deterioration as: $\theta(t) = kt$, $0 < k < 1$. Then the differential equations of the given model are as:

$$\frac{dQ_1(t)}{dt} + kt Q_1(t) = P - \frac{x}{T}, \quad 0 \leq t \leq t_1 \quad \dots\dots (23)$$

$$\frac{dQ_2(t)}{dt} + kt Q_2(t) = -\frac{x}{T}, \quad t_1 \leq t \leq t_2 \quad \dots\dots (24)$$

$$\frac{dQ_2(t)}{dt} = -\frac{x}{T}, \quad t_2 \leq t \leq T \quad \dots\dots (25)$$

With the initial conditions $Q_1(0)=0$, $Q_1(t_2)=Q_2(t_1)$ and $Q_2(t_2)=0$. The solutions of the differential equations are

$$Q_1(t) = \left(P - \frac{x}{T} \right) \left(t + \frac{kt^3}{6} \right) e^{-\frac{kt^2}{2}}, \quad 0 \leq t \leq t_1 \quad \dots\dots (26)$$

$$Q_2(t) = Q_2(t_1) e^{-k\left(\frac{t^2}{2} - \frac{t_1^2}{2}\right)} - \frac{x}{2} \left\{ (t-t_1) - \frac{k(t^3-t_1^3)}{6} \right\} e^{-\frac{kt^2}{2}}, \quad t_1 \leq t \leq t_2 \quad \dots (27)$$

$$\text{And, } Q_3(t) = x(t_2 - t), \quad t_2 \leq t \leq T \quad \dots\dots (28)$$

As, in the previous case, we calculate the different costs like ordering cost, holding cost, deterioration cost and shortages cost for the given this special case, we have the expected total cost is the sum of these costs is as:

$$ETC = \frac{A}{T} + (C_h + C_d) \left[P \left(\frac{t_1^2}{2} + \frac{kt_1^3}{6} - \frac{kt_1^4}{8} - \frac{k^2 t_1^5}{20} \right) + Q_2(t_1) \left\{ (t_2 - t_1) - \frac{kt_1^2(t_2 - t_1)}{2} - \frac{k(t_2^3 - t_1^3)}{6} \right\} \right] - \frac{\mu}{T} \left[\left(\frac{t_1^2}{2} + \frac{kt_1^3}{6} - \frac{kt_1^4}{8} - \frac{k^2 t_1^5}{20} \right) + \left(\frac{t_2^2 - t_1^2}{2} - (t_1 t_2 - t_1^2) - \frac{k(t_2^4 - t_1^4)}{8} + \frac{k(t_2^3 t_1 - t_1^4)}{6} \right) \right] + \left[\frac{k(t_2^4 - t_1^4)}{24} - \frac{k(t_2 t_1^3 - t_1^4)}{6} - \frac{k^2(t_2^6 - t_1^6)}{72} + \frac{k^2(t_2^3 t_1^2 - t_1^5)}{36} \right] + C_s \frac{\mu}{T} \left\{ T t_2 - \frac{1}{2} (T^2 - t_2^2) \right\} \dots\dots\dots (29) \mathbf{5.1}$$

Numerical examples:

An example is given to illustrate the results of the model developed in this study with the following data:
 We Consider $C_h=6, C_d=4, C_s=3, \mu=20, k=0.2, P=150, C_0=200$, then, the **Optimum Total Cost = 382.5704**

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