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A Supply Chain Inventory Model for Deteriorating Items with Price Dependent Demand and Shortage under Fuzzy Environment

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Abstract**:** This paper presents a supply chain inventory model for deteriorating items under fuzzy environment allowing shortage in inventory. Demand is taken as a function of selling price. Realistically it is observed that the cycle time of any supply chain system is uncertain, so we described the cycle time as triangular fuzzy number (symmetric) to make the model close to the reality. Signed distance method is used to defuzzify the cost function. The result is illustrated with the help of numerical example and sensitivity analysis with respect to different associated parameters has been presented.

Keywords**:** Inventory, price dependent demand, shortage, signed distance method, supply chain, triangular fuzzy number.

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I. INTRODUCTION

In any supply chain system it is very important to control and manage a good inventory so as to keep the smooth and efficient flow of physical goods or materials between the different stages of that system, such as supplier, distributor and retailer. The objective of maintaining such inventory is to gain the maximum profit by providing the right quantity of product at the right time and by taking strategic decision to lower the total cost of the system. Researchers have developed inventory models assuming various types of demand, such as constant [1,2] , time dependent[3 – 6], stock dependent [7 – 9)] or price dependent [10 – 13]

Since most of the physical goods deteriorate over time, various authors have shown keen interest in investigating inventory models for deteriorating items. Some of them assumed deterioration rate as constant, while others describe it as time dependent, two parameter Weibull deterioration etc.,[14 – 18].

Again, most business organizations face uncertainty associated with different parameters such as demand, raw materials supply, various relevant costs, cycle time, rate of deterioration etc., we use fuzzy set theory to solve this types of problems. Bellman and Zadeh[19] first introduced fuzzy set theory for solving decision making problems. After that, several authors have developed inventory models under fuzzy environment [20 – 24]

In this paper we have developed fuzzy inventory model for deteriorating items considering demand as selling price demand and allowing shortage in the inventory. In any supply chain system it is seen that we cannot predict the cycle time in prior, so keeping in mind this real life situation, we considered the cycle time as triangular fuzzy number (symmetric). The rest of this paper is organized as follows. In section 2, the assumption andnotations are given. In section 3, we have developed the mathematical models. In section 4, we provided numerical examples to illustrate the results. In addition, the sensitivity analysis of the optimal solution with respect to different parameters of the system is carried out in section 5. Finally, we drew the conclusions in section 6.

II. ASSUMPTIONS AND NOTATIONS

2.1 Assumptions:

This model is based on the following assumptions:

i) Demand is selling price dependent and is of the form $D(p) = a - bp$, where $a, b > 0$ and p is the selling price.
ii) The rate of deterioration is constant.
iii) Replenishment is instantaneous and lead time is zero.
iv) The cycle time is uncertain and we assume it as triangular fuzzy number.
v) Shortages are allowed in the inventory.

2.2. Notations:

Following notations are used to develop this model:

i) Demand D(p) = a - bp, where a, b > 0 and p is the selling price.
ii) θ is the rate of deterioration.
iii) q is the initial stock level at the beginning of every inventory.
iv) D is the total deteriorated items
v) T is the length of a cycle which is uncertain.
vi) I(t) is the inventory level at any time t.
vii) C_0 is the set up cost per cycle
viii) C_1 is the holding cost per unit time.
ix) C_2 is the deterioration cost per unit.
x) C_3 is the shortage cost per unit per unit time.
xi) TC is the total inventory cost.

III. MATHEMATICAL MODEL

Let I(t) be the on hand inventory at time t (0≤ t ≤T). The inventory cycle starts at t=0 with inventory level q. Then the inventory level decreases due to both demand and deterioration and ultimately it reaches to 0 level at time t_1. After that shortages start occurring and continues upto time T. The differential equation describing the instantaneous state of I(t) at any time t is given by :

$$\frac{dI(t)}{dt} + \theta I(t) = -(a-bp) \quad 0 \le t \le t_1 \quad \ldots (1)$$

$$\frac{dI(t)}{dt} = -(a-bp) \quad t_1 \le t \le T \quad \ldots (2)$$

With boundary conditions –

$$I(0) = q,\ I(t_1) = 0,\ I(T) = -s \quad \ldots (3)$$

Solving the equations (1) and (2) using (3) we have –

$$I(t) = qe^{-\theta t} + \frac{(a-bp)}{\theta}(e^{-\theta t}-1) \quad 0 \le t \le t_1 \quad \ldots (4)$$

$$I(t) = (a-bp)(t_1 - t) \quad t_1 \le t \le T \quad \ldots (5)$$

Using $I(t_1) = 0$ from (4) we have –

$$q = \frac{(a-bp)}{\theta}(e^{\theta t_1}-1) \quad \ldots (6)$$

Therefore, from (4) we have-

$$I(t) = \frac{(a-bp)}{\theta}(e^{\theta t_1}-1)\, e^{-\theta t} + \frac{(a-bp)}{\theta}(e^{-\theta t}-1)$$

$$= \frac{(a-bp)}{\theta}\{e^{\theta(t_1-t)} - 1\}$$

$$= (a-bp)\left\{(t_1-t) + \frac{(t_1-t)^2}{2}\theta + \frac{(t_1-t)^3\theta^2}{6}\right\} \quad \ldots (7)$$

(Neglecting higher powers of θ)

Inventory setup cost = C_0

Holding cost-

$$= C_1 \int_0^{t_1} I(t)dt$$

$$= C_1 (a-bp)\left[\int_0^{t_1}\left\{(t_1-t) + \frac{(t_1-t)^2}{2}\theta + \frac{(t_1-t)^3\theta^2}{6}\right\}dt\right]$$

$$= C_1(a-bp)\left[\frac{{t_1}^2}{2} + \frac{{t_1}^3}{6}\theta + \frac{{t_1}^4}{24}\theta^2\right] \quad \ldots (8)$$

Deterioration cost-

$$= C_2 \int_0^{t_1} \theta I(t)dt$$

$$= C_2(a-bp)\int_0^{t_1}\{e^{\theta(t_1-t)} - 1\}dt$$

$$= C_2(a-bp)\left[\frac{1}{\theta}(e^{\theta t_1} - 1) - t_1\right]$$

$$= C_2(a-bp)\frac{\theta {t_1}^2}{2} \quad \ldots (9)$$

Shortage cost-

$$= -C_3 \int_{t_1}^{T} I(t)dt$$

$$= -C_3 \int_{t_1}^{T} (a-bp)(t_1 - t)\, dt$$

$$= \frac{C_3}{2}(a-bp)(T - t_1)^2 \quad \ldots (10)$$

Therefore, average cost-

$$= \frac{1}{T}\left[C_0 + C_1(a-bp)\left\{\frac{{t_1}^2}{2} + \frac{{t_1}^3}{6}\theta + \frac{{t_1}^4}{24}\theta^2\right\} + C_2(a-bp)\frac{\theta {t_1}^2}{2} + \frac{C_3}{2}(a-bp)(T - t_1)^2\right] \quad \ldots (11)$$

Let, t_1 = βT , 0 <β <1, then,

Average cost –

$= \frac{1}{T}[C_0 + C_1 \text{ (a – bp)}\{\frac{\beta^2 T^2}{2} + \frac{\beta^3 T^3}{6}\theta + \frac{\beta^4 T^4}{24}\theta^2\} + C_2\text{(a – bp)}\frac{\theta\beta T^2}{2} + \frac{C_3}{2}\text{(a – bp)}(T - t_1)^2]$

$= \frac{C_0}{T} + C_1\text{(a – bp)}\beta^2\{\frac{T}{2} + \frac{\beta T^2}{6}\theta + \frac{\beta^2 T^3\theta^2}{24}\} + C_2\frac{\text{(a - bp)}}{2}\theta\beta T + \frac{C_3}{2}(\text{(a – bp)}(1 - \beta)T]$…………… (12)

3.1. Fuzzy model

Now we describe the cycle time as triangular fuzzy number. Let $\tilde{T}$ = (T- Δ, T, T + Δ). Then the cost function with fuzzy cycle time can be written as –

$\widetilde{TC} = \frac{C_0}{\tilde{T}} + C_1\text{(a – bp)}\beta^2\{\frac{\tilde{T}}{2} + \frac{\beta\tilde{T}^2}{6}\theta + \frac{\beta^2\tilde{T}^3\theta^2}{24}\} + C_2\frac{\text{(a - bp)}}{2}\theta\beta\tilde{T} + \frac{C_3}{2}(\text{(a – bp)}(1 - \beta)\tilde{T}$ ……….. (13)

From the definition of signed distance method we know that-

$d(\tilde{A}, \tilde{0}) = \frac{1}{2}\int_0^1 [A_L(\alpha) + A_U(\alpha)]d\alpha$

where, $\tilde{A}$=(a, b, c), $A_L(\alpha)$ = a + (b – a) α, $A_U(\alpha)$ =c – (c – b) α

Now, $T_L(\alpha)$ = (T – Δ)+ Δα

$T_U(\alpha)$ = (T + Δ)- Δα

Therefore, $d(\tilde{T},\tilde{0}) = \frac{1}{2}\int_0^1[(T - \Delta) + \Delta\alpha + (T + \Delta) - \Delta\alpha]\, d\alpha$

$= \frac{1}{2}\int_0^1 2T\, d\alpha = T$ ………………………………………………………. (14)

and $d(\frac{1}{\tilde{T}}, \tilde{0}) = \frac{1}{2}\int_0^1[(\frac{1}{T})_L(\alpha) + (\frac{1}{T})_U(\alpha)]d\alpha$

$= \frac{1}{2}\int_0^1[\frac{1}{T+\Delta-\Delta\alpha} + \frac{1}{T-\Delta+\Delta\alpha}]\, d\alpha$

$= \frac{1}{2\Delta}\ln(\frac{T+\Delta}{T-\Delta})$ ………………………………………………………………. (15)

From (13), (14) and (15) we have-

$\widetilde{TC} = \frac{C_0}{2\Delta}\ln(\frac{T+\Delta}{T-\Delta}) + C_1\text{(a – bp)}\beta^2\{\frac{T}{2} + \frac{\beta T^2}{6}\theta + \frac{\beta^2 T^3\theta^2}{24}\} + C_2\frac{\text{(a - bp)}}{2}\theta\beta T + \frac{C_3}{2}(\text{(a – bp)}(1 - \beta)T$ ……….…….. (16)

Here, $\frac{d(TC)}{dT} = \frac{C_0}{2\Delta}(\frac{1}{T+\Delta} - \frac{1}{T-\Delta}) + C_1(\text{a} - \text{bp})\beta^2\{\frac{1}{2} + \frac{\beta\theta T}{3} + \frac{\beta^2\theta^2 T^2}{8}\} + C_2\frac{\text{(a - bp)}}{2}\theta\beta + C_3\text{(a – bp)}(\beta - 1)$

And, $\frac{d^2(TC)}{dT^2} = 2C_0\frac{T}{(T^2-\Delta^2)^2} + C_1(\text{a} - \text{bp})\beta^2(\frac{\beta\theta}{3} + \frac{\beta^2\theta^2 T}{4}) > 0$

Hence, $\widetilde{TC}$ is strictly convex.

IV. NUMERICAL EXAMPLE

To illustrate this model numerically we consider the following numerical values of the parameters:

A = 100, p = 50, b = 0.01, θ = 0.5, β = 0.9, C_0 = 200, C_1 = 5, C_2 = 9, C_3 = 7 and Δ = 0.4

Then we have TC = 444.95 and T = 0.894 for crisp model and, $\widetilde{TC}$ = 445.10 and $\tilde{T}$ = 0.895 for fuzzy model.

V. SENSITIVITY ANALYSIS

Table – 1: Sensitivity on θ

Parameter (θ)	Crisp model		Fuzzy model	
	TC	T	$\widetilde{TC}$	$\tilde{T}$
0.03	437.04	0.912	437.18	0.913
0.05	444.95	0.894	445.10	0.895
0.07	452.69	0.877	452.85	0.878
0.09	460.28	0.861	460.45	0.862

Table – 2: Sensitivity on p

Parameter (p)	Crisp model		Fuzzy model	
	TC	T	$\widetilde{TC}$	$\tilde{T}$
45	446.11	0.892	446.26	0.893
50	444.95	0.894	445.10	0.895
55	443.78	0.896	443.93	0.897
60	442.61	0.899	442.76	0.900

Table – 3: Sensitivity on C_1

Parameter (C_1)	Crisp model		Fuzzy model	
	TC	T	$\widetilde{TC}$	$\tilde{T}$
4	408.574	0.974	408.69	0.974
5	444.95	0.894	445.10	0.895
6	478.54	0.832	478.73	0.832
7	509.90	0.780	510.13	0.782

Table – 4: Sensitivity onC_2

Parameter (C_2)	Crisp model		Fuzzy model	
	TC	T	$\widetilde{TC}$	$\tilde{T}$
9	444.95	0.894	445.10	0.895
10	446.86	0.890	447.01	0.891
11	448.76	0.887	448.91	0.888
12	450.65	0.883	45080	0.884

Table – 5: Sensitivity on C_3

Parameter (C_3)	Crisp model		Fuzzy model	
	TC	T	$\widetilde{TC}$	$\tilde{T}$
7	444.95	0.894	445.10	0.895
9	453.37	0. 878	453.52	0.879
11	461.63	0.862	461.80	0.863
13	469.76	0.848	469.93	0.849

5.1. Observations:
Based on the sensitivity analysis we observed that –

i) The fuzzy total cost and fuzzy cycle time is slightly higher than the crisp total cost and crisp cycle time.

ii) With the increase of the deterioration rate (θ) the total cost for both the models increase, whereas the cycle times decrease. But these two models show the different scenario in case of selling price, where the total costs decrease with the increase of selling price and the cycle times increase with the increase of this parameter (p).

iii) Both of fuzzy total cost and crisp total cost increases with the increase of the different costs, such as holding cost, deterioration cost and shortage cost. In contrast, the cycle times for both the models shows a decreasing trend with the increase of the cost parametersC_1, C_2 andC_3.

VI. CONCLUSIONS

In this paper we have developed a pricing model for deteriorating items considering price dependent demand and allowing shortage in the inventory under fuzzy environment. Here we have considered the cycle time as uncertain and described it as triangular fuzzy number (symmetric). We have observed that the cycle time and the total cost obtained by crisp model is less than those obtained by fuzzy model.From the sensitivity analysis it is observed that the total cost of both the models increase as the different costs associated with the model increase.

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