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A Two-Echelon Supply Chain Model for Deteriorating Product with Time-Dependent Demand, Demand-Dependent Production Rate and Shortage

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Abstract: In this paper, we have developed a two- stage supply chain production inventory model for deteriorating product with time-dependent demand. Shortages are allowed in the retailer's inventory. This model is developed for finite time horizon. In reality, it is seen that if the manufacturer produces a huge amount of product, it may incur holding cost and moreover the items may get damaged or deteriorated incurring the cost for deterioration. On the other hand, an insufficient amount of product may result in shortage incurring penalty cost. Therefore, to balance these two unwanted situations, the manufacturer will fix the production rate as demand dependent. To make this study close to reality the production rate is assumed to be a function of demand rate. A numerical example and sensitivity analysis with respect to different associated parameter is also presented to illustrate the study.

Keywords: Supply chain, inventory, deterioration, time dependent demand, shortage.

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I. INTRODUCTION

A supply chain is a network of different stages, such as supplier, manufacturer, distributor and retailer that performs the functions of procurement of materials, transformation of these materials into intermediate and finished products and distribution of these finished products to customers. In traditional business environment, each business tries to minimize its cost only without considering the cost of the other supply chain members, although most of the decisions made by any of these stages have direct and indirect impacts on the profitably of the other members. So, the assumption that can greatly influence the optimal policies of any business organization is to take a supply chain perspective when analyzing pricing models. To date, very few studies on deteriorating inventory in two-stage supply chain systems have been carried out [1-5]. The most important factor of any supply chain system is demand and the profit of any supply chain depends mostly on it. The demand for any item varies depending on the product's characteristics and the consumption pattern followed by customers. A large group of researchers considers a constant rate of demand [6-10]. Some researchers studied supply chain model with time dependent demand [11], [12] and some authors developed supply chain model with stock dependent demand [4], [13], [14]. There are also some supply chain models where price sensitive demand rate has been assumed [15-18]. In our study, we have considered a supply chain model, where the demand rate is dependent on time. In any supply chain system, the manufacturer's profitably depends mostly on the production rate, because producing a large amount of product may increase the manufacturer's total cost as it may incur holding cost and the cost for deterioration. On the other hand, a lower rate of production may result in shortage and the ultimate effects of it, like loss due to lost sale and loss of goodwill. To deal with these unwanted situations, we have considered production rate as demand dependent.

The rest of this paper is organized as follows: in section 2 we have discussed the assumptions and notations to develop the model. In section 3 we have developed mathematical models for both the manufacturer and the retailer. A numerical example and sensitivity analysis with respect to different associated parameter is presented in sections 4 and 5 respectively. Finally, we have drawn a conclusion in section 6.

1.1Assumptions and Notations

1.2Assumptions:

The model is based on the following assumptions-

1. The deterioration rate is a constant fraction of on hand inventory.
2. The model is developed for finite time horizon.
3. The demand for the products is time dependent.

4. The production rate is also considered as a function of demand rate.
5. The shortages are allowed for retailer only.

1..2 Notations

The following notations are used to developed this present model-

1. The demand rate function D(t) is assumed to be a function of time in a polynomial form: R(t) = $\beta t^{\gamma-1}$, with $\gamma>1$ for increasing demand, $\gamma<1$ for decreasing demand and $\gamma=1$ for constant demand and β is a positive constant.
2. T = time horizon
3. K = production coefficient, $K\geq1$
4. θ = constant deterioration rate, $0<\theta<<1$
5. t_1 = production period for manufacturer
6. C_m = production cost per unit for the manufacturer
7. h_m holding cost per unit for the manufacturer
8. h_r = holding cost per unit for the retailer
9. A_m= set up cost per production run for the manufacturer
10. s = shortage cost per unit for the retailer
11. Q = initial inventory level for the retailer
12. p = purchasing cost per unit for the retailer
13. r = the time at which inventory level becomes zero for the retailer.
14. d_m = cost for deterioration for the manufacturer.
15. d_r = cost for deterioration for the retailer.

II. MATHEMATICAL MODEL

2.1 Manufacturer's model:

The manufacturer starts his production process at time t = 0 and continues up to time t =t_1, where the inventory level reaches its maximum level. Production then stops at t = t_1 and the inventory gradually depletes to zero at the end of cycle time t = T due to demand and deterioration. The change in inventory level can be described by the following differential equation.

$\frac{dI(t)}{dt}$ + θI(t) = (k – 1)β$t^{(\gamma-1)}$, $0 \leq t \leq t_1$……….. (1)

$\frac{dI(t)}{dt}$ + θI(t) = - $\beta t^{(\gamma-1)}$, $t_1 \leq t \leq T$ ………… (2)

With boundary conditions I(0) = 0, I(T) = 0 ……….. (3)

Solving (1) and (2) we have-

$I(t) = \beta(k-1)[\frac{t^{\gamma}}{\gamma} + \frac{\theta}{(\gamma+1)}t^{(\gamma+1)}]e^{-\theta t}$………. (4)

And, $I(t) = \beta[\frac{1}{\gamma}(T^{\gamma} - t^{\gamma}) + \frac{\theta}{(\gamma+1)}(T^{(\gamma+1)} - t^{(\gamma+1)})]e^{-\theta t}$ ………. (5)

Now, set up cost of the manufacturer –

$= A_m$

Production cost –

$= C_m \int_0^{t_1} k\beta t^{(\gamma-1)}\, dt$

$= \frac{C_m k\beta}{\gamma} {t_1}^{\gamma}$…………… (6)

Holding cost –

$= h_m[\int_0^{t_1} I(t)dt + \int_{t_1}^{T} I(t)dt]$

$= h_m[\int_0^{t_1} \beta(k-1)\{\frac{t^{\gamma}}{\gamma} + \frac{\theta}{(\gamma+1)}t^{(\gamma+1)}\}e^{-\theta t}dt + \int_{t_1}^{T} \beta\{\frac{1}{\gamma}(T^{\gamma} - t^{\gamma}) + \frac{\theta}{(\gamma+1)}\left(T^{(\gamma+1)} - t^{(\gamma+1)}\right)\}e^{-\theta t}dt]$

$= \beta h_m[\frac{(k-1)}{\gamma(\gamma+1)}{t_1}^{(\gamma+1)} - \frac{k\theta}{\gamma(\gamma+1)(\gamma+2)}{t_1}^{(\gamma+2)} + \frac{1}{(\gamma+1)}T^{(\gamma+1)} + \frac{\theta}{2(\gamma+2)}T^{(\gamma+2)} - \frac{1}{\gamma}(T^{\gamma}t_1 - \frac{{t_1}^{(\gamma+1)}}{(\gamma+1)} - \theta(\frac{1}{(\gamma+1)}T^{(\gamma+1)}t_1 - \frac{1}{2\gamma}T^{\gamma}{t_1}^{2}]$………… (7)

(Neglecting higher powers of θ)

Cost of deterioration –

$= d_m[\int_0^{t_1} \theta I(t)dt + \int_{t_1}^{T} \theta I(t)dt]$

$= \theta\beta d_m[\frac{(k-1)}{\gamma(\gamma+1)} t_1^{(\gamma+1)} - \frac{k\theta}{\gamma(\gamma+1)(\gamma+2)} t_1^{(\gamma+2)} + \frac{1}{(\gamma+1)} T^{(\gamma+1)} + \frac{\theta}{2(\gamma+2)} T^{(\gamma+2)} - \frac{1}{\gamma}(T^\gamma t_1 - \frac{t_1^{(\gamma+1)}}{(\gamma+1)} - \theta(\frac{1}{(\gamma+1)} T^{(\gamma+1)} t_1 - \frac{1}{2\gamma} T^\gamma t_1^2]$ ……………. (8)

Therefore, total cost of the manufacturer-

$TC_M = A_m + \frac{C_m k\beta}{\gamma} t_1^\gamma + \beta(h_m + \theta d_m) [\frac{(k-1)}{\gamma(\gamma+1)} t_1^{(\gamma+1)} - \frac{k\theta}{\gamma(\gamma+1)(\gamma+2)} t_1^{(\gamma+2)} + \frac{1}{(\gamma+1)} T^{(\gamma+1)} + \frac{\theta}{2(\gamma+2)} T^{(\gamma+2)} - \frac{1}{\gamma}(T^\gamma t_1 - \frac{t_1^{(\gamma+1)}}{(\gamma+1)} - \theta(\frac{1}{(\gamma+1)} T^{(\gamma+1)} t_1 - \frac{1}{2\gamma} T^\gamma t_1^2]$……… (9)

2.2 Retailer's Model:

The inventory cycle starts at t=0. During the time period [0, r], the inventory level decreases due to demand and deterioration. At t=r the inventory level becomes zero and after that shortages occur, which continues up to time t = T. The differential equations showing the behavior of the system are given as follow.

$\frac{dI(t)}{dt} + \theta I(t) = -\beta t^{(\gamma-1)}, \quad 0 \le t \le r$ ……….. (10)

$\frac{dI(t)}{dt} = -\beta t^{(\gamma-1)}, \quad r \le t \le T$ …………. (11)

With the boundary conditions, I(0) = Q, I(r) = 0, I(T) = -I_s ……….. (12)

The solutions of the equations (0) and (11) we have,

$I(t) = \beta\{\frac{1}{\gamma}(r^\gamma - t^\gamma) + \frac{\theta}{(\gamma+1)}(r^{(\gamma+1)} - t^{(\gamma+1)})\} e^{-\theta t}$…………. (13)

And, $I(t) = \frac{\beta}{\gamma}(r^\gamma - t^\gamma)$………….. (14)

Now, from (13), using I(0) = Q we have,

$Q = \beta \{\frac{1}{\gamma} r^\gamma + \frac{\theta}{(\gamma+1)} r^{(\gamma+1)}\}$ ………….. (15)

Now, purchasing cost –

= Q.p

$= \beta p\{\frac{1}{\gamma} r^\gamma + \frac{\theta}{(\gamma+1)} r^{(\gamma+1)}\}$ …………… (16)

Holding cost –

$= h_r \int_0^r I(t)dt$

$= \beta h_r \int_0^r\{\frac{1}{\gamma}(r^\gamma - t^\gamma) + \frac{\theta}{(\gamma+1)}(r^{(\gamma+1)} - t^{(\gamma+1)})\} e^{-\theta t} dt$

$= \beta h_r [\frac{1}{(\gamma+1)} r^{(\gamma+1)} + \frac{\theta}{2(\gamma+2)} r^{(\gamma+2)}]$ …………. (17)

(Neglecting higher powers of θ)

Cost of deterioration –

$= d_r \int_0^r \theta I(t)dt$

$= \theta\beta d_r [\frac{1}{(\gamma+1)} r^{(\gamma+1)} + \frac{\theta}{2(\gamma+2)} r^{(\gamma+2)}]$ …………. (18)

(Neglecting higher powers of θ)

Shortage cost-

$= -s\int_r^T I(t)dt$

$= -\frac{s\beta}{\gamma}\{r^\gamma T - \frac{1}{(\gamma+1)} T^{(\gamma+1)} - \frac{\gamma}{(\gamma+1)} r^{(\gamma+1)}\}$

Therefore, total cost of the retailer-

$TC_R = \beta p\{\frac{1}{\gamma} r^\gamma + \frac{\theta}{(\gamma+1)} r^{(\gamma+1)}\} + \beta(h_r + \theta d_r)\{\frac{1}{(\gamma+1)} r^{(\gamma+1)} + \frac{\theta}{2(\gamma+2)} r^{(\gamma+2)}\} + \frac{s\beta}{\gamma}\{r^\gamma T - \frac{1}{(\gamma+1)} T^{(\gamma+1)} - \frac{\gamma}{(\gamma+1)} r^{(\gamma+1)}\}$ ………….. (19)

Therefore, total cost of the entire supply chain –

$TC = TC_M + TC_r$

$$= A_m + \frac{C_m k\beta}{\gamma} t_1{}^{\gamma} + \beta(h_m + \theta d_m)\{\frac{(k-1)}{\gamma(\gamma+1)} t_1{}^{(\gamma+1)} - \frac{k\theta}{\gamma(\gamma+1)(\gamma+2)} t_1{}^{(\gamma+2)} + \frac{1}{(\gamma+1)} T^{(\gamma+1)} + \frac{\theta}{2(\gamma+2)} T^{(\gamma+2)} - \frac{1}{\gamma}(T^{\gamma} t_1 - \frac{t_1{}^{(\gamma+1)}}{(\gamma+1)} - \theta(\frac{1}{(\gamma+1)} T^{(\gamma+1)} t_1 - \frac{1}{2\gamma} T^{\gamma} t_1{}^2\} + \beta p\{\frac{1}{\gamma} r^{\gamma} + \frac{\theta}{(\gamma+1)} r^{(\gamma+1)}\} + \beta(h_r + \theta d_r)\{\frac{1}{(\gamma+1)} r^{(\gamma+1)} + \frac{\theta}{2(\gamma+2)} r^{(\gamma+2)}\} + \frac{s\beta}{\gamma}\{r^{\gamma} T - \frac{1}{(\gamma+1)} T^{(\gamma+1)} - \frac{\gamma}{(\gamma+1)} r^{(\gamma+1)}\} \quad \text{..............} \quad (20)$$

2.3 Numerical Example

We consider the following data to illustrate the model numerically-
A_m=100, C_m=7, h_m=2, d_m =5, k = 4, β=5, γ =0.8, θ=0.01, h_r=2, d_r=5, p=15, s=13and T=5.
Then we obtain, total supply chain cost TC =1084.94, t_1 = 3.14 and r = 3.28.

III. SENSITIVITY ANALYSIS

Table-1: sensitivity on θ

θ	t_1	r	TC
0.01	3.14	3.28	1084.94
0.02	3.26	3.23	1116.08
0.03	3.40	3.18	1149.46
0.04	3.54	3.14	1185.72
0.05	3.71	3.09	1225.78

Table-2: sensitivity on T

T	t_1	r	TC
4	2.06	2.43	790.45
5	3.14	3.28	1084.94
6	4.22	4.13	1401.28
7	5.31	4.98	1743.09
8	6.41	5.83	2112.15

Table-3: sensitivity on β

β	t_1	r	TC
4	3.14	3.28	887.95
5	3.14	3.28	1084.94
6	3.14	3.28	1281.92
7	3.14	3.28	1478.91
8	3.14	3.28	1675.90

Table-4: sensitivity on γ

γ	t_1	r	TC
0.4	3.05	3.28	1298.75
0.5	3.05	3.28	1178.62
0.6	3.07	3.28	1116.68
0.7	3.11	3.28	1089.17
0.8	3.14	3.28	1084.94

IV. OBSERVATIONS

1. Table-1 depicts the changes int_1, r and TC with the changes of the parameter θ. It is observed from this table that as the value of θ increases, the values of t_1 and TC of the system also increase, whereas the value of r decreases.
2. Form Table-2 it is seen that with the increase of the parameter T, the values of t_1, r and TC also increase.
3. From Table-3 and Table- 4 we observe the behavior of t_1, r and TC with the variation in demand parameters β and γrespectively. It is observed that with the increment in β, the values of TC increases. In contrast, with the increase of the parameter γ, the value of TC decreases.

V. CONCLUSIONS

In this paper, we have developed a two-echelon supply chain model for deteriorating items for manufacturer and retailer. In reality, it is seen that the demand of a product varies greatly with time. Therefore, demand rate is considered as time dependent. To keep the balance between production and demand, the production rate is taken as demand dependent. Under these circumstances we have determine the total supply chain cost, the optimum production time for the manufacturer and the time at which inventory level becomes zero for the retailer.

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