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A Generic Capacitated Multi-Period, Multi-Product, Integrated Forward-Reverse Logistics Network Design Optimization Model

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Abstract: *in this paper, a generic capacitated multi-product, multi-period, multi-echelon integrated forward-reverse logistics network design is developed. The proposed network structure consists of three echelons in the forward direction, (suppliers, factories, and distributors) and two echelons in the reverse direction (disassembly and redistribution centers) to provide the first customer zones with virgin products and the second customer zones with refurbished ones. The problem is formulated in a mixed integer linear programming (milp) decision-making form. The objective is to maximize the total profit. The performance of the developed model has been verified through two examples.*

Keywords : *Supply Chain; Location Allocation; Reverse Logistics; Forward-Reverse Logistics; Milp; Mixed Integer Linear Programming; Closed Loop.*

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I. INTRODUCTION

Closed loop or integrated forward reverse network establishes a relationship between the market that releases used or refurbished products and the market for new or virgin products.

Salema, M. I. G. et al. (2007) developed a mixed integer formulation for reverse distribution allows for any number of products, But the inventory was not taken into consideration.

Pishvaee, M. S. et al. (2009) developed a single period single product stochastic programming model for an integrated forward/reverse logistics network design under uncertainty.

El-Sayed, M. et al. (2010) developed a single product stochastic mixed integer linear programming for designing a forward–reverse logistics under demand risk.

Ramezani, M. et al. (2013) presented a single period stochastic multi-objective model for forward/reverse logistic network design under an uncertain environment.

Mutha, A., & Pokharel, S. (2013) proposed a mathematical RLN design model considering a third party collectors.

Hatefi, S. M., & Jolai, F. (2014) formulated a single period, single product robust and reliable model for an integrated forward–reverse logistics network design based on a recent robust optimization approach protecting the network against uncertainty.

Serdar E. T. & Al-Ashhab M. S. (2016) modeled a multi-product, multi-period supply chain network mathematically in a mixed integer linear programming (MILP) form deciding both location and allocation decisions which maximize the total profit.

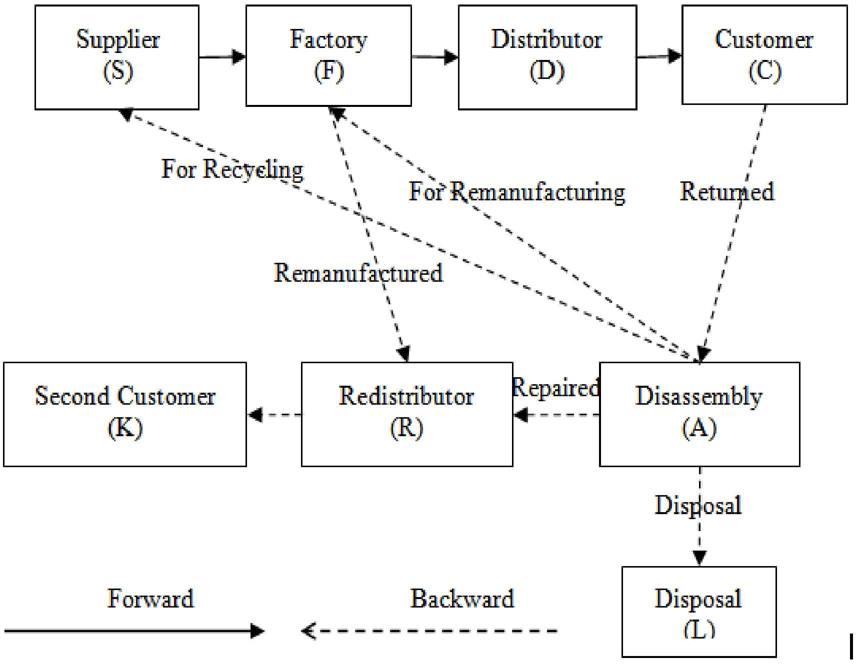
In this work, a generic multi-product, multi-period multi-echelon integrated forward-reverse logistics network design model is developed. The model is formulated in a mixed integer linear programming (MILP) decision-making form. The objective of the model is to maximize the total profit. Decisions are taken to determine the following:

- Suppliers, factories, distribution centers, disassembly, and redistribution centers locations,
- Production volume at each period in each location (what and how much to produce),
- Transported quantity of goods between locations, and
- The quantity of goods to hold as inventory at each period in both the facility and distributor stores.

II. MODEL DESCRIPTION

The model is a formulation for the integrated forward-reverse logistics network design problem. The network is a multi-product, multi-period, multi-echelon, where it consists of three suppliers, three factories, three distributors, and four first customers in the forward direction and it consists of three disassembles, three disposal centers, three redistribution locations and second customers in the reverse direction, as shown in Figure 1.

Figure 1: The proposed forward-reverse logistics network



In the forward direction, the suppliers supply the raw material to the factories which manufacture them and send them to the distributors to send them to the first customer considering their demand. In the reverse direction, the first customers return the used products to the disassembly locations for disassembling, and sorting for supplying the recyclable to the suppliers, the remanufacturable to the factories, the disposable to the disposal locations, and to repair the repairable products and supplying them directly to the redistribution locations. The recycled material is supplied to factories. The remanufactured and repaired products are supplied to the second customers through redistribution locations.
Costs incurred at different locations are as follows are shown in Table 1.

Table 1: Costs incurred at different locations of the network.

Forward Logistics Echelons Cost Elements			
Location	**Suppliers**	**Factories**	**Distributors**
Costs	1. Fixed 2. Materials 3. Recycling 4. Transportation	1. Fixed 2. Manufacturing 3. Remanufacturing 4. Non-utilized manufacturing capacity 5. Non-utilized remanufacturing capacity 6. Storage 7. Transportation	1. Fixed 2. Shortage 3. Storage 4. Transportation
Reverse Logistics Echelons Cost Elements			
Location	**Disassembly**	**Redistribution**	**Disposal**
Costs	1. Fixed 2. Returned price 3. Disassembly 4. Inspection 5. Sorting 6. Repairing 7. Transportation	1. Fixed 2. Transportation	1. Fixed 2. Disposing

Model Formulation
The model involves the following sets, parameters, and decision variables:
Sets:
S, F, D, and D: potential number of suppliers, factories, distributors, and first customers,
A, A, L, and K: potential number of disassembly, redistributors, locations, disposal, and second customers.
P: number of products,

T: number of periods.

Parameters:

Dcpt: demand of first customer *c* from product *p* in period *t*,
Dkpt: demand of the second customer *k* from product *p* in period *t*,
Ppct: unit price of product *p* at customer *c* in period *t*,
Ppkt: unit price of product *p* at second customer *k* in period *t*,
Fi: fixed cost of opening location *i*,
DSij: distance between any two locations *i* and *j*,
CAPSst: capacity of supplier s in period t (kg),
CAPMft: capacity of raw material store of facility *f* in period *t* (kg),
CAPHft: capacity in manufacturing hours of facility *f* in period *t*,
CAPFSft: capacity of final product store of facility *f* in period *t* (kg),
CAPDdt: capacity of distributor *d* in period *t* (kg),
CAPAat: capacity of disassembly *a* in period *t*,
CAPRCst: recycling capacity of supplier *s* in period *t* (kg),
CAPRMft: remanufacturing capacity in hours of factory *f* in period *t*,
CAPRrt: capacity of redistributor *r* in period *t*,
CAPLlt: capacity of disposal *p* in period *t*,

MatCst: material cost per unit supplied by supplier *s* in period *t*,
RECst: recycling cost per unit recycled by supplier *s* in period *t*,
MCft: manufacturing cost per hour for factory *f* in period *t*,
RMCft: remanufacturing cost per hour for factory *f* in period *t*,
DACat: disassembly cost per unit weight disassembled by disassembly location *a* in period *t*,
REPCat: repairing cost per unit repaired by disassembly location *a* in period *t*,
DISPClt: disposal cost per unit disposed of by disposal location *l* in period *t*,
NUCCf: non-utilized manufacturing capacity cost per hour of facility *f*,
NURCCf: non-utilized remanufacturing capacity cost per hour of factory *f*,
SCPUp: shortage cost per unit per period for product *p*,
MHp: manufacturing hours for product *p*,
RMHp: remanufacturing hours for product *p*,
FHf: holding cost per unit weight per period at the store of factory *f*,
DHd: holding cost per unit weight per period at distributor store *d*,
Bs, Bf, Bd, Ba & Br: batch size from supplier *s*, factory *f*, distributor *d*, disassembly *a*, and, redistributor *r* respectively,
Tc: transportation cost per unit per kilometer,
RR: return ratio at the first customers,
RC: recycling ratio,
RM: remanufacturing ratio,
RP: repairing ratio,
RD: disposal ratio,

Decision variables:

Li: binary variable equals 1 if location *i* is open and 0 otherwise,
Qijpt: flow of batches from location *i* to location *j* of product *p* in period *t*,
Ifpt: flow of batches from factory *f* to its store of product *p* in period *t*,
Ifdpt: flow of batches from store of factory *f* to distributor *d* of product *p* in period *t*,
Rfpt: the residual inventory of product *p* in the period *t* at store of factory *f*,
Rdpt: the residual inventory of product *p* in the period *t* at distributor *d*.

3.1. Objective Function

The objective of the model is to maximize the total profit of the forward-reverse network.
Total Profit = Total Revenue – Total Cost.

3.1.1. Total Revenue

Total Revenue = First Sales + Second Sales + Recycling Cost Saving.

$$\text{First Sales} = \sum_{d\in D}\sum_{c\in C}\sum_{p\in P}\sum_{t\in T} Q_{dcpt}\, B_{dp}\, P_{pct} \qquad (1)$$

$$\text{SecondSales} = \sum_{r\in R}\sum_{k\in C}\sum_{p\in P}\sum_{t\in T} Q_{rkpt}\ B_{rp}\ P_{pkt} \qquad (2)$$

$$\text{Recycling cost saving} = \sum_{a\in A}\sum_{s\in S}\sum_{p\in P}\sum_{t\in T} Q_{ast}\ B_{a}\ W_{p}\ (MatC_{st} \qquad (3)$$

3.1.2. Total Cost

Total cost = fixed costs + material costs + manufacturing costs + non-utilized capacity costs + shortage costs + Purchasing costs + Disassembly costs + Remanufacturing cost + Repairing cost + Disposal cost + Transportation costs + inventory holding costs.

The costs are as follows:

1) Fixed Costs

$$\sum_{s\in S} F_s L_s + \sum_{f\in F} F_f L_f + \sum_{d\in D} F_d L_d + \sum_{a\in A} F_a L_a + \sum_{r\in R} F_r L_r + \sum_{l\in L} F_l L_l \qquad (4)$$

2) Material Cost

$$\sum_{s\in S}\sum_{f\in F}\sum_{t\in T} Q_{sft}\ B_s\ MatC_{st} \qquad (5)$$

3) Manufacturing Costs

$$\sum_{f\in F}\sum_{d\in D}\sum_{p\in P}\sum_{t\in T} Q_{fdpt}\ B_{fp}\ MH_p\ MC_{ft} + \sum_{f\in F}\sum_{d\in D}\sum_{p\in P}\sum_{t\in T} I_{fpt}\ B_{fp}\ MH_p\ MC_{ft} \qquad (6)$$

4) Non-Utilized Manufacturing Capacity Cost (for factories)

$$\sum_{f\in F}(\sum_{t\in T}((CAPH_{ft})\ L_f - \sum_{d\in D}\sum_{p\in P}(Q_{fdpt}\ B_{fp} MH_p) - \sum_{d\in D}\sum_{p\in P}(I_{ffpt}\ B_{fp} MH_p))NUCC_f) \qquad (7)$$

5) Shortage Cost (for distributor)

$$\sum_{p\in P}(\sum_{c\in C}(\sum_{t\in T}(\sum_{1}^{t} DEMAND_{cpt} - \sum_{1}^{t}\sum_{d\in D} Q_{dcpt}\ B_{dp})))\ SCPU_p \qquad (8)$$

6) Purchasing Costs

$$\sum_{c\in C}\sum_{a\in A}\sum_{p\in P}\sum_{t\in T} Q_{capt}\ P_{pct}\ B_c\ QL_c \qquad (9)$$

7) Disassembly Costs

$$\sum_{c\in C}\sum_{a\in A}\sum_{p\in P}\sum_{t\in T} Q_{capt}\ B_c\ DAC_{at} \qquad (10)$$

8) Non-Utilized Remanufacturing Capacity Cost (for factories)

$$\sum_{f\in F}(\sum_{t\in T}((CAPRM_{ft})\ L_f - \sum_{r\in R}\sum_{p\in P}(Q_{frpt}\ B_{fp}\ RMH_p))NURCC_f) \qquad (11)$$

9) Remanufacturing Costs

$$\sum_{f\in F}\sum_{r\in R}\sum_{p\in P}\sum_{t\in T} Q_{frpt}\ B_{fp}\ RMH_p\ RMC_{ft} \qquad (12)$$

10) Repairing Costs

$$\sum_{a\in A}\sum_{r\in R}\sum_{p\in P}\sum_{t\in T} Q_{arpt}\ B_a\ W_p\ REPC_{at} \qquad (13)$$

11) Disposal Costs

$$\sum_{a\in A}\sum_{l\in L}\sum_{p\in P}\sum_{t\in T} Q_{alpt}\ B_a\ W_p\ DISPC_{lt} \tag{14}$$

12) Transportation Costs

$$\begin{aligned}&\sum_{t\in T}\sum_{s\in S}\sum_{f\in F} Q_{sft}\ B_s\ T_s\ DS_{sf} + \sum_{t\in T}\sum_{f\in F}\sum_{d\in D}\sum_{p\in P} Q_{fdpt}\ B_f W_p\ Tc\, DS_{fd} + \sum_{t\in T}\sum_{f\in F}\sum_{d\in D}\sum_{p\in P} I_{fdpt}\ B_{fp} W_p T_f\ D_{fd}(\\ &\sum_{d\in D}\sum_{c\in C}\sum_{p\in P}\sum_{t\in T} Q_{dcpt}\ B_{dp}\ W_p T_d\ D_{dc} + \sum_{t\in T}\sum_{d\in D}\sum_{a\in A}\sum_{s\in S} Q_{aspt}\ B_a W_p\ Tc\, DS_{as} + \sum_{t\in T}\sum_{d\in D}\sum_{a\in A}\sum_{f\in F} Q_{afpt}\ B_a W_p\\ &\sum_{t\in T}\sum_{d\in D}\sum_{f\in F}\sum_{r\in R} Q_{frpt}\ B_f W_p\ Tc\, DS_{fr} + \sum_{t\in T}\sum_{d\in D}\sum_{a\in A}\sum_{r\in R} Q_{arpt}\ B_a W_p\ Tc\, DS_{ar} + \sum_{t\in T}\sum_{d\in D}\sum_{a\in A}\sum_{l\in L} Q_{alpt}\ B_a W_p\\ &\sum_{t\in T}\sum_{d\in D}\sum_{r\in R}\sum_{k\in K} Q_{rkpt}\ B_r W_p\ Tc\, DS_{rk}\end{aligned} \tag{15}$$

13) Inventory Holding Costs

$$\sum_{p\in P} \left(\sum_{f\in F}\sum_{t\in T} R_{fpt}\ W_p\ HF_f + \sum_{d\in D}\sum_{t\in T} R_{dpt}\ W_p\ HD_d\right) \tag{16}$$

3.2. Constraints

This section is a representation of the constraints of the model:

3.2.1. Balance Constraints:

Balance constraints at for factories, stores, distributors, disassembly, and redistributors locations are given in the following equations (17-29).

3.2.1.1 Factory balance

$$\sum_{s\in S} Q_{sft} B_s = \sum_{d\in D}\ \sum_{p\in P} Q_{fdpt} B_{fp} W_p + I_{fpt} B_{fp} W_p, \forall t\in T, \forall f\in F \tag{17}$$

3.2.1.2 Factory store balance

$$I_{fpt} B_{fp} + R_{fp(t-1)} B_{fp} = R_{fpt} B_{fp} + \sum_{d\in D} I_{fdpt} B_{fp}, \forall t\in T, \forall f\in F, \forall p\in P \tag{18}$$

3.2.1.3 Distributor store balance

$$\sum_{f\in F} (Q_{fdpt} + I_{fdpt}) B_{fp} + R_{dp(t-1)} B_{dp} = R_{dpt} B_{dp} + \sum_{c\in C} Q_{dcpt} B_{dp}, \forall t\in 2\rightarrow T, \forall d\in D, \forall p\in P \tag{19}$$

3.2.1.4 Customer in balance

$$\sum_{d\in D} Q_{dcpt} B_{dp} \le DEMAND_{cpt} + \sum_{1\rightarrow t} DEMAND_{cp(t-1)} - \sum_{d\in D} Q_{dcp(t-1)} B_{dp}, \forall t\in T, \forall c\in C, \forall p\in P \tag{20}$$

3.2.1.5 Customer out balance

$$\sum_{a\in A} Q_{capt}\ B_c \le \left(\sum_{d\in D} Q_{dcpt} B_d\right) RR, \forall t\in T, \forall c\in C, \forall p\in C, \forall p\in P \tag{21}$$

3.2.1.6 Disassembly balance

$$\begin{aligned}\sum_{c\in C} Q_{capt}\ B_c = &\sum_{s\in S}(Q_{aspt}\ B_a) + \sum_{f\in F}(Q_{afpt}\ B_a) + \sum_{r\in R}(Q_{arpt}\ B_a) +\\ &\sum_{l\in L}(Q_{alpt}\ B_a), \forall t\in T, \forall a\in A, \forall p\in P\end{aligned} \tag{22}$$

3.2.1.7 Recycling balance

$$\sum_{c \in C}(Q_{capt}\ B_c\ RC) = \sum_{s \in S}(Q_{aspt}\ B_a), \forall t \in T, \forall a \in A, \forall p \in P \quad (23)$$

3.2.1.8 Remanufacturing balance

$$\sum_{c \in C}(Q_{capt}\ B_c\ RM) = \sum_{f \in F}(Q_{afpt}\ B_a), \forall t \in T, \forall a \in A, \forall p \in P \quad (24)$$

3.2.1.9 Return balance

$$\sum_{c \in C}(Q_{capt}\ B_c\ RP) = \sum_{r \in R}(Q_{arpt}\ B_a), \forall t \in T, \forall a \in A, \forall p \in P \quad (25)$$

3.2.1.10 Disposing balance

$$\sum_{c \in C}(Q_{capt}\ B_c\ RD) = \sum_{l \in L}(Q_{alpt}\ B_a), \forall t \in T, \forall a \in A, \forall p \in P \quad (26)$$

3.2.1.11 Remanufacturing balance

$$\sum_{a \in A}(Q_{afpt}\ B_a) = \sum_{r \in R}(Q_{frpt}\ B_f), \forall t \in T, \forall f \in F, \forall p \in P \quad (27)$$

3.2.1.12 Redistribution balance

$$\sum_{a \in A}(Q_{arpt}\ B_a) + \sum_{f \in F}(Q_{frpt}\ B_f) = \sum_{k \in K}(Q_{rkpt}\ B_r), \forall t \in T, \forall r \in R, \forall p \in P \quad (28)$$

3.2.1.13 Second customer balance

$$\sum_{r \in R}(Q_{rkpt}\ B_r) \le D_{kpt}, \forall t \in T, \forall k \in K, \forall p \in P \quad (29)$$

3.2.2. Capacity Constraints:

Capacity constraints for suppliers, factories, stores, distributors, disassembly, disposal, and redistributors locations are given in the following equations (30-38)

3.2.2.1 Supplier capacity

$$\sum_{f \in F} Q_{sft}\ B_s \le CAPS_{st}\ L_s, \forall t \in T, \forall s \in S \quad (30)$$

3.2.2.2 Factory material capacity

$$\sum_{s \in S} Q_{sft}\ B_s \le CAPM_{ft}\ L_f, \forall t \in T, \forall f \in F \quad (31)$$

3.2.2.3 Manufacturing hours capacity

$$\left(\sum_{d \in D} Q_{fdpt}\ B_{fp} + \sum_{d \in D} I_{fpt}\ B_{fp}\right) MH_p \le CAPH_{ft}\ L_f, \forall t \in T, \forall f \in F, \forall p \in P \quad (32)$$

3.2.2.4 Facility store capacity

$$\sum_{p \in P} R_{fpt} B_{fp} W_p \le CAPFS_{ft}\ L_f, \forall t \in T, \forall f \in F \quad (33)$$

3.2.2.5 Distributor store capacity

$$\sum_{f \in F}\sum_{p \in P}(Q_{fdpt} + I_{fdpt})\ B_{fp} W_p + \sum_{p \in P} R_{dpt-1}\ B_{dp} W_p \le CAPD_{dt}\ L_d, \forall t \in T, \forall d \in D \quad (34)$$

3.2.2.6 Disassembly capacity

$$\sum_{s\in S}\sum_{p\in P} Q_{aspt}\, B_a\, W_p + \sum_{f\in F}\sum_{p\in P} Q_{afpt}\, B_a\, W_p + \sum_{r\in R}\sum_{p\in P} Q_{arpt}\, B_a W_p + \sum_{l\in L}\sum_{p\in P} Q_{alpt}\, B_a W_p \le CAPA_{lt}, \forall t\in T, \forall a\in A$$

3.2.2.7 Redistributors capacity

$$\sum_{k\in K}\sum_{p\in P} Q_{rkpt}\, B_r\, W_p \le CAPR_{rt}, \forall t\in T, \forall r\in R$$

3.2.2.8 Recycling capacity

$$\sum_{a\in A}\sum_{p\in P} Q_{aspt}\, B_a\, W_p \le CAPRC_{st}, \forall t\in T, \forall s\in S$$

3.2.2.9 Disposal capacity

$$\sum_{a\in A}\sum_{p\in P} Q_{alpt}\, B_a\, W_p \le PC_{pt}, \forall t\in T, \forall l\in L$$

The model is built by using Mosel language [9], which can be work as both a modeling language and a programming language.

III. MODEL VERIFICATION RESULTS ANALYSIS

The effectiveness of the model has been verified through solving two examples with different demand patterns. Other parameters are assumed to be constant and having the values given in Table 2.

Table 2: Nominal values of the model parameters

Parameter	Value	Parameter	Value
Virgin products prices	100, 150 and 200	Supplier locations fixed costs.	10,000
Weights of the three products	1, 2 and 3 Kg.	Factory location fixed costs.	50,000
Manufacturing time of each product	1, 2 and 3 hr.	Distributor locations fixed costs.	5,000
Remanufacturing time of each product	2, 3 and 4 hr.	Disassembly location fixed costs.	2,000
Second customer demand for each product in each period	500	Redistribution location fixed costs.	2,000
Second products price ratio	80 %	Disposal location Fixed costs.	1,000
Returned products quality (may be random)	20 %	Supplier recycling capacity (kg)	2,000
Material cost per kilogram	10	Supplier capacity (kg)	4,000
Manufacturing costs per unit	10	Factory store capacity	2,000
Shortage cost for each product per period	5, 10 and 15	Factory raw material storing capacity (kg)	4,000
Non-Utilized manufacturing capacity cost	10	Factory manufacturing capacity (hours)	6,000
Non-Utilized remanufacturing capacity cost	10	Factory remanufacturing capacity (hours)	2,000
Factory holding cost	3	Distributor store capacity	4,000
Distributor holding cost	2	Disassembly location capacity	2,000
Disassembly cost per unit	3	Redistribution capacity	2,000
Recycling cost per unit	5	Disposal location capacity	1,000
Remanufacturing cost per unit	10	Max return ratio	50 %
Repairing cost	5	Repairing ratio	50%
Disposal cost	1	Recycling ratio	10%
Max number of operating suppliers, factories, distributors, disassembles and redistributors	3	Remanufacturing ratio	30%
Max number of first customers	4	Disposal ratio	10%
Max number of second customers	2	Batch sizes	1

3.1 Example 1

3.1.1 Example 1: Inputs

The model has been verified through the following case study where the input parameters are considered as showing in Table 2. The demand patterns are assumed for all customer as shown in Table 3.

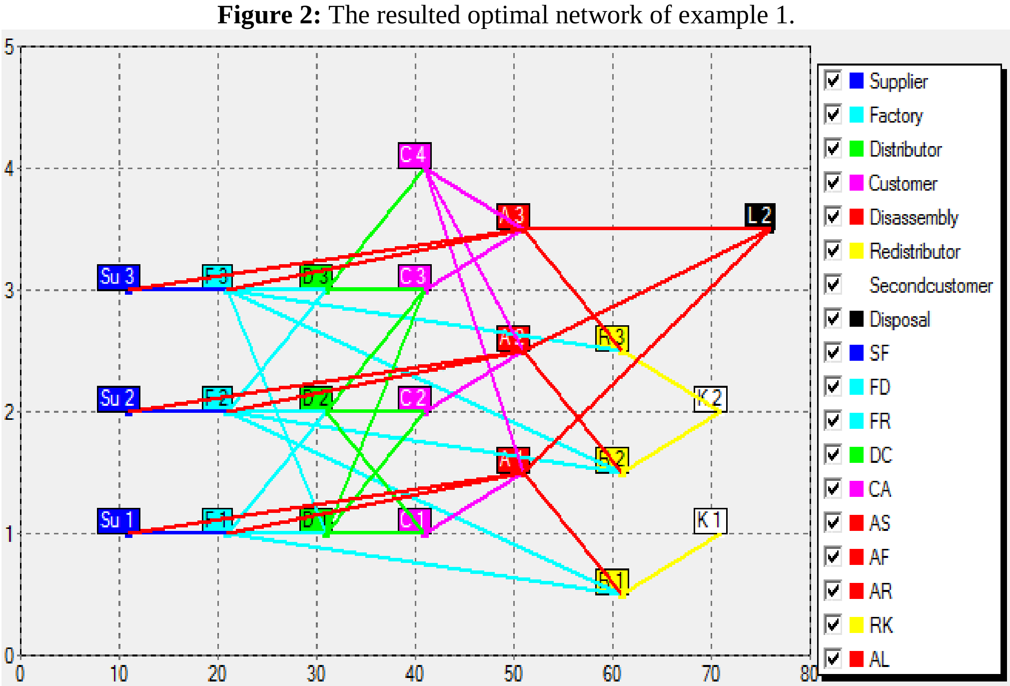
Table 3: Demand of each customer in all period for all products.

Period	Required Demand											
	Customer 1			Customer 2			Customer 3			Customer 4		
	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3
1	470	500	530	470	500	530	470	500	530	470	500	530
2	460	490	520	460	490	520	460	490	520	460	490	520
3	450	480	510	450	480	510	450	480	510	450	480	510

3.1.2 Example 1: Outputs And Discussion

The resulted optimal network is as shown in Figure 2.

Figure 2: The resulted optimal network of example 1.



Total profit, total cost, total revenue, and their elements are given in Table 4. Only the Inventory Holding Cost equals zero which means that there is no inventory at all in the network.

Table 4: Total profit, total cost, total revenue, and their elements.

Revenue	Value	Cost	Value	Cost	Value
First Sales	2,682,000	Fixed Cost	206,500	Purchasing Costs	268,200
Second Sales	858,240	Material Cost	360,000	Disassembly Cost	54,000
Recycling Profit	4,410	Manufacturing Cost	364,800	Remanufacturing Cost	80,460
		Non-Utilized Cost	274,740	Repairing Cost	45,000
		Shortage Cost	2,400	Disposal Cost	1,800
		Inventory Holding Cost	0	Transportation Costs	29,060
Total Revenue	3,544,650		Total Cost	1,686,960	
			Total Profit	1,857,690	

Where the quantities of batches transferred from suppliers to the factories and from factories to distributors are shown in Table 5. Flow balancing is noticed in Table 5 where the total weights of transferred materials are the same of 36000 kg.

Table 5: Number of batches transferred from suppliers and factories.

From	Suppliers			Factories								
	S1	S2	S3	F1			F2			F3		
Period	RM	RM	RM	P1	P2	P3	P1	P2	P3	P1	P2	P3
1	4000	4000	4000	940	735	530	1	744	837	919	501	693
2	4000	4000	4000	460	990	520	460	120	1100	940	750	520
3	4000	4000	4000	900	785	510	347	775	701	553	480	829
Weight	12000	12000	12000	2300	5020	4680	808	3278	7914	2412	3462	6126
Total W.	36000			36000								

The number of batches transferred from distributors to customers for all product in all period of also 36000 kg is shown in Table 6.

Table 6: Number of batches transferred from distributors to customers.

Given Quantities												
Period	**Customer 1**			**Customer 2**			**Customer 3**			**Customer 4**		
	P1	**P2**	**P3**	**P1**	**P2**	**P3**	**P1**	**P2**	**P3**	**P1**	**P2**	**P3**
1	470	500	530	470	490	470	470	490	530	450	500	530
2	460	490	520	460	500	580	460	380	520	480	490	520
3	450	480	510	450	480	510	450	600	510	450	480	510
Weight (Kg.)	1380	2940	4680	1380	2940	4680	1380	2940	4680	1380	2940	4680
Total Weights	36000											

The shortage can be calculated easily by subtracting given quantities shown in Table 6 from the required quantities (demand) shown in Table 3 and it is shown in Table 7. Table 7 shows that all shortages are compensated in the next periods and these shortages resulted in a shortage cost of 2400 as shown in Table 4.

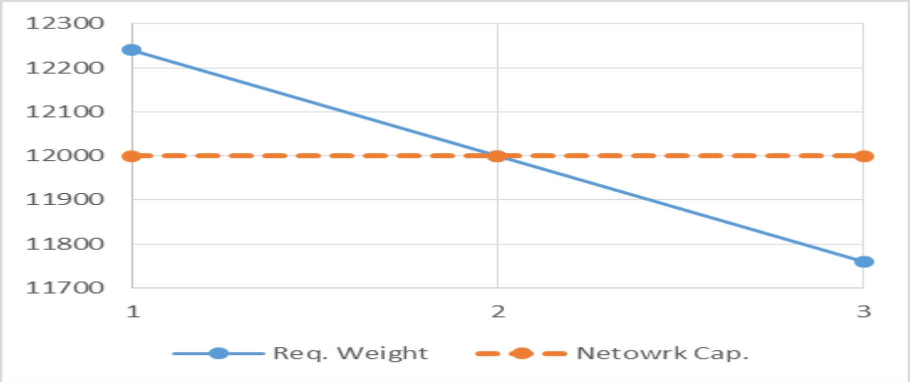
Table 7: Shortages.

Shortage per period												
Period	Customer 1			Customer 2			Customer 3			Customer 4		
	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3
1	0	0	0	0	10	60	0	10	0	20	0	0
2	0	0	0	0	-10	-60	0	110	0	-20	0	0
3	0	0	0	0	0	0	0	-120	0	0	0	0
Weight (Kg.)	0	0	0	0	0	0	0	0	0	0	0	0
Total Weights	0											

Figure 3 depicts the given quantities versus demand for all customer and all products. It can be noticed that they are all equal which means that there are no final shortages. Figure 4 shows that the total required weights are more than the network capacity in the first period, equals it at the second period, and less than it at the third period which explains shortage compensation.

Figure 3: Given quantities versus demand for all customer and all products.

Figure 4: Total required weights vs. network capacity



The flow in the reverse direction begins by receiving the returned products from the first customers by disassembly locations. Table 8 gives the maximum flow weights and the actual flow weights. It is noticed that the disassembly locations receives the maximum flow weights of 18000 kg. So, other actual weights of the repaired, recycled, remanufactured, disposed and redistributed equal the maximum flow weights. The number of products of 18000 kg weight purchased by disassembly locations from the first customers is shown in Table 9.

Table 8: Maximum and actual flow weights.

	Ratio	Max. flow weights		Actual flow weights	
Returned	0.5	18000		18000	
Redistributed	0.8	14400		14400	
Repaired	0.5	9000	18000	9000	18000
Recycled	0.1	1800		1800	
Remanufactured	0.3	5400		5400	
Disposed	0.1	1800		1800	

Table 9: Number of products purchased by disassembly locations from the first customers.

Retuned Quantities									
To	**A1**			**A2**			**A3**		
Period	**C1A1**			**C1A2**			**C1A3**		
	P1	**P2**	**P3**	**P1**	**P2**	**P3**	**P1**	**P2**	**P3**
1	235	250	265	0	0	0	0	0	0
2	230	245	260	0	0	0	0	0	0
3	225	240	255	0	0	0	0	0	0
Period	**C2A1**			**C2A2**			**C2A3**		
	P1	**P2**	**P3**	**P1**	**P2**	**P3**	**P1**	**P2**	**P3**
1	0	0	0	235	245	235	0	0	0
2	0	0	0	230	250	290	0	0	0
3	0	0	0	225	240	255	0	0	0
Period	**C3A1**			**C3A2**			**C3A3**		
	P1	**P2**	**P3**	**P1**	**P2**	**P3**	**P1**	**P2**	**P3**
1	0	0	0	0	0	0	235	245	265
2	0	0	0	0	0	0	230	190	260
3	0	0	0	0	0	0	225	300	255
Period	**C4A1**			**C4A2**			**C4A3**		
	P1	**P2**	**P3**	**P1**	**P2**	**P3**	**P1**	**P2**	**P3**
1	55	80	85	105	165	45	65	5	135
2	0	25	150	0	200	0	240	20	110
3	45	160	55	175	80	65	5	0	135
Weight (Kg.)	**790**	**2000**	**3210**	**970**	**2360**	**2670**	**1000**	**1520**	**3480**
Total Weights	**18000**								

The number and weights of remanufactured, repaired, and delivered to the second customers matching Table 8 are presented in Tables 10, 11 and 12 respectively.

Table 10: Flow of remanufactured products from factories to redistributors.

	Remanufactured								
Period	**F1R**			**F2R**			**F3R**		
	P1	**P2**	**P3**	**P1**	**P2**	**P3**	**P1**	**P2**	**P3**
1	99	127	149	180	170	160	0	0	0
2	129	109	151	150	170	168	0	0	2
3	85	120	161	185	186	145	0	0	0
Weight (Kg.)	313	712	1383	515	1052	1419	0	0	6
Total Weights	5400								

Table 11: Flow of repaired products from disassembly locations to redistributors.

	Repaired								
Period	**A1R**			**A2R**			**A3R**		
	P1	**P2**	**P3**	**P1**	**P2**	**P3**	**P1**	**P2**	**P3**
1	145	165	175	170	205	140	150	125	200
2	115	135	205	115	225	145	235	105	185
3	135	200	155	200	160	160	115	150	195
Weight (Kg.)	395	1000	1605	485	1180	1335	500	760	1740
Total Weights	9000								

The number of batches transferred to the second customer through the reverse chain is shown in Table 12.

Table 12: Flow of refurbished products from redistributors to second customers.

Second Products						
To	**K1**			**K2**		
Period	**R1K1**			**R1K2**		
	P1	**P2**	**P3**	**P1**	**P2**	**P3**
1	244	292	324	0	0	0
2	244	244	356	0	0	0
3	220	320	316	0	0	0
Period	**R2K1**			**R2K2**		
	P1	**P2**	**P3**	**P1**	**P2**	**P3**
1	0	0	0	350	375	300
2	0	0	0	265	395	313
3	0	0	0	385	346	305
Period	**R3K1**			**R3K2**		
	P1	**P2**	**P3**	**P1**	**P2**	**P3**
1	0	0	0	150	125	200
2	0	0	0	235	105	187
3	0	0	0	115	150	195
Weight (Kg.)	708	1712	2988	1500	2992	4500
Total Weights	14400					

4.2 Example 2

4.2.1 Example 2: Inputs

Demand patterns are assumed for all customer as shown in Table 13.

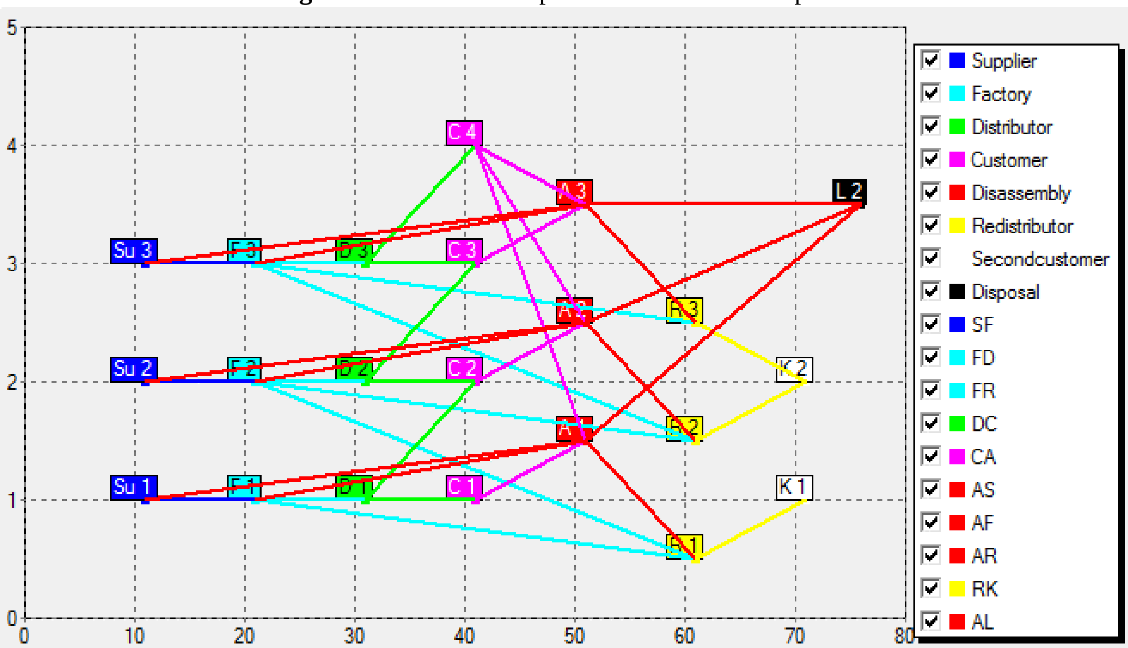
Table 13: Demand of each customer in all period for all products.

Required Demand												
Period	**Customer 1**			**Customer 2**			**Customer 3**			**Customer 4**		
	P1	**P2**	**P3**	**P1**	**P2**	**P3**	**P1**	**P2**	**P3**	**P1**	**P2**	**P3**
1	450	480	510	450	480	510	450	480	510	450	480	510
2	460	490	520	460	490	520	460	490	520	460	490	520
3	470	500	530	470	500	530	470	500	530	470	500	530

3.2.2 Example 2: Outputs And Discussion

The resulted optimal network is as shown in Figure 5.

Figure 5: The resulted optimal network of example 2.



Total profit, total cost, total revenue, and their elements are given in Table 14. Only the inventory holding cost equals zero which means that there is no inventory at all in the network.

Table 14: Total profit, total cost, total revenue, and their elements.

Revenue	Value	Cost	Value	Cost	Value
First Sales	2,666,000	Fixed Cost	206,500	Purchasing Costs	266,600
Second Sales	853,120	Material Cost	357,600	Disassembly Cost	53,640
Recycling Profit	4,390	Manufacturing Cost	377,200	Remanufacturing Cost	79,980
		Non-Utilized Cost	262,820	Repairing Cost	44,700
		Shortage Cost	1,200	Disposal Cost	1,788
		Inventory Holding Cost	0	Transportation Costs	28,589
Total Revenue	3,523,510		Total Cost	1,680,617	
		Total Profit	1,842,893		

Where the quantities of batches transferred from suppliers to the factories and from factories to distributors are shown in Table 15. Flow balancing is noticed in Table 15 where the total weights of transferred materials are the same of 35760 kg.

Table 15: Number of batches transferred from suppliers and factories.

From	Suppliers			Factories								
	S1	S2	S3	F1			F2			F3		
Period	RM	RM	RM	P1	P2	P3	P1	P2	P3	P1	P2	P3
1	3760	4000	4000	450	890	510	899	70	987	451	960	543
2	4000	4000	4000	920	490	700	0	710	860	920	760	520
3	4000	4000	4000	471	500	843	938	1000	354	471	500	843
Weight	11760	12000	12000	1841	3760	6159	1837	3560	6603	1842	4440	5718
Total W.	35760			35760								

The number of batches transferred from distributors to customers for all product in all period of also 35760 kg is shown in Table 16

Table 16: Number of batches transferred from distributors to customers.

Period	Given Quantities											
	Customer 1			Customer 2			Customer 3			Customer 4		
	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3
1	450	480	510	450	480	510	450	480	510	450	480	510
2	460	490	520	460	490	520	460	490	520	460	490	520
3	470	500	530	470	500	530	470	500	530	470	500	450
Weight (Kg.)	1380	2940	4680	1380	2940	4680	1380	2940	4680	1380	2940	4440
Total Weights	35760											

The shortage can be calculated easily by subtracting given quantities shown in Table 16 from the required quantities (demand) shown in Table 13 and it is shown in Table 17. Table 17 shows that all shortages occurred only on the third product for the fourth (the furthest) customer because the network capacity is lower than the required as shown in Figure 7 and this shortage resulted in a shortage cost of 1200 as shown in Table 14.

Table 17: Shortages.

Period	Shortage											
	Customer 1			Customer 2			Customer 3			Customer 4		
	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	80
Weight (Kg.)	0	0	0	0	0	0	0	0	0	0	0	240
Total Weights	240											

Figure 6 depicts the given quantities versus demand for all customer and all products. It can be noticed that they are all not equal which means that there is a final shortage for customer 4 from product 3. Figure 7 shows that the total required weights is more than the network capacity in the third period, equals it at the second period, and less than it at the first period which explains final shortage.

Figure 6: Given quantities versus demand for all customer and all products.

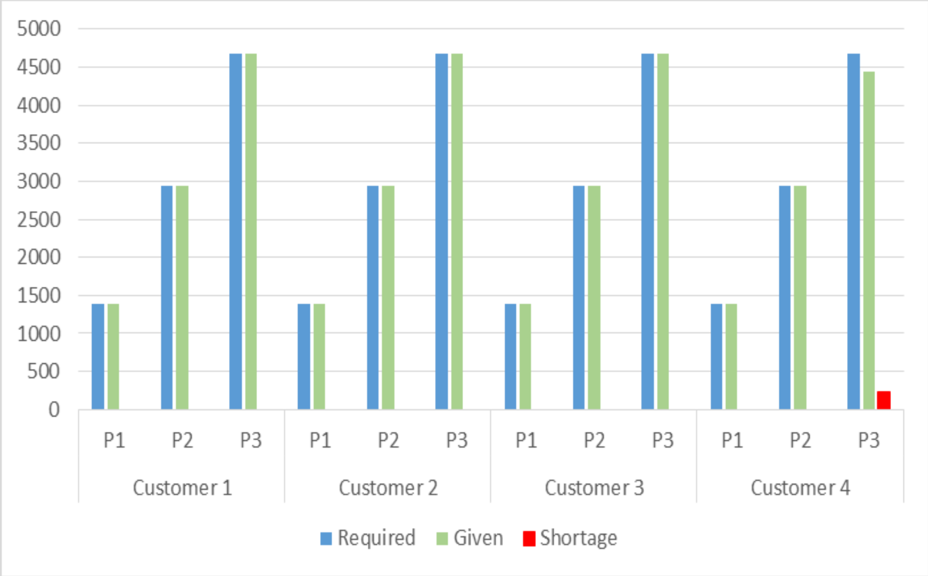
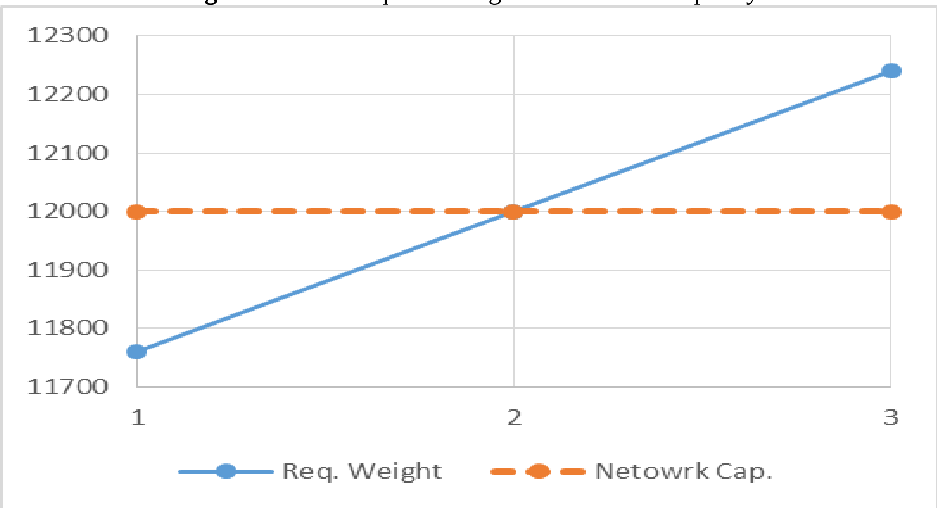


Figure 7: Total required weights vs. network capacity



The flow in the reverse direction begins by receiving the returned products from the first customers by disassembly locations. Table 18 gives the maximum flow weights and the actual flow weights. It is noticed that the disassembly locations receives the maximum flow weights of 17880 kg. So, other actual weights of the repaired, recycled, remanufactured, disposed and redistributed equal the maximum flow weights. The number of products of 17880 kg weight purchased by disassembly locations from the first customers is shown in Table 19.

Table 18: Maximum and actual flow weights.

<table>
<tr><th></th><th>Ratio</th><th colspan="2">Max. flow weights</th><th colspan="2">Actual flow weights</th></tr>
<tr><td>Returned</td><td>0.5</td><td colspan="2">17880</td><td colspan="2">17880</td></tr>
<tr><td>Redistributed</td><td>0.8</td><td colspan="2">14304</td><td colspan="2">14304</td></tr>
<tr><td>Repaired</td><td>0.5</td><td>8940</td><td rowspan="4">17880</td><td>8940</td><td rowspan="4">17880</td></tr>
<tr><td>Recycled</td><td>0.1</td><td>1788</td><td>1788</td></tr>
<tr><td>Remanufactured</td><td>0.3</td><td>5364</td><td>5364</td></tr>
<tr><td>Disposed</td><td>0.1</td><td>1788</td><td>1788</td></tr>
</table>

Table 19: Number of products purchased by disassembly locations from the first customers.

Retuned Quantities									
To	**A1**			**A2**			**A3**		
Period	**C1A1**			**C1A2**			**C1A3**		
	P1	**P2**	**P3**	**P1**	**P2**	**P3**	**P1**	**P2**	**P3**
1	225	240	255	0	0	0	0	0	0
2	230	245	260	0	0	0	0	0	0
3	235	250	265	0	0	0	0	0	0
Period	**C2A1**			**C2A2**			**C2A3**		
	P1	**P2**	**P3**	**P1**	**P2**	**P3**	**P1**	**P2**	**P3**
1	0	0	0	225	240	255	0	0	0
2	0	0	0	230	245	260	0	0	0
3	0	0	0	235	250	265	0	0	0
Period	**C3A1**			**C3A2**			**C3A3**		
	P1	**P2**	**P3**	**P1**	**P2**	**P3**	**P1**	**P2**	**P3**
1	0	0	0	0	0	0	225	240	255
2	0	0	0	0	0	0	230	245	260
3	0	0	0	0	0	0	235	250	265
Period	**C4A1**			**C4A2**			**C4A3**		
	P1	**P2**	**P3**	**P1**	**P2**	**P3**	**P1**	**P2**	**P3**
1	5	90	115	5	60	135	215	90	5
2	0	85	110	230	135	0	0	25	150
3	15	100	85	215	0	85	5	150	55
Weight (Kg.)	**710**	**2020**	**3270**	**1140**	**1860**	**3000**	**910**	**2000**	**2970**
Total Weights	**17880**								

The number and weights of remanufactured, repaired, and delivered to the second customers matching Table 18 are presented in Tables 20, 21 and 22 respectively.

Table 20: Flow of remanufactured products from factories to redistributors.

	Remanufactured								
Period	**F1R**			**F2R**			**F3R**		
	P1	**P2**	**P3**	**P1**	**P2**	**P3**	**P1**	**P2**	**P3**
1	105	103	131	165	155	175	0	30	0
2	121	119	147	155	175	165	0	0	0
3	127	125	141	155	175	165	0	0	0
Weight (Kg.)	353	694	1257	475	1010	1515	0	60	0
Total Weights	5364								

Table 21: Flow of repaired products from disassembly locations to redistributors.

	Repaired								
Period	**A1R**			**A2R**			**A3R**		
	P1	**P2**	**P3**	**P1**	**P2**	**P3**	**P1**	**P2**	**P3**
1	115	165	185	115	150	195	220	165	130
2	115	165	185	230	190	130	115	135	205
3	125	175	175	225	125	175	120	200	160
Weight (Kg.)	355	1010	1635	570	930	1500	455	500	1485
Total Weights	8940								

The number of batches transferred to the second customer through the reverse chain is shown in Table 22.

Table 22: Flow of refurbished products from redistributors to second customers.

Second Products						
To	**K1**			**K2**		
Period	**R1K1**			**R1K2**		
	P1	**P2**	**P3**	**P1**	**P2**	**P3**
1	220	268	316	0	0	0
2	236	284	332	0	0	0
3	252	300	316	0	0	0
Period	**R2K1**			**R2K2**		
	P1	**P2**	**P3**	**P1**	**P2**	**P3**
1	0	0	0	280	305	370
2	0	0	0	385	365	295
3	0	0	0	380	300	340
Period	**R3K1**			**R3K2**		

	P1	P2	P3	P1	P2	P3
1	0	0	0	220	195	130
2	0	0	0	115	135	205
3	0	0	0	120	200	160
Weight (Kg.)	708	1704	2892	1500	3000	4500
Total Weights	14304					

IV. Conclusion

From the previous study, the following conclusions can be derived:

1. The proposed model is successful in designing forward-reverse logistics networks while considering multi-product in multi-period with three echelons (suppliers, factories and distributors) in the forward direction and two echelons (disassemblies and re-distributors) in the reverse direction.
2. Quality level of the returned products, return rate, and others may be tackled as random value, but it is assumed as known to facilitate discussion.
3. This model can be developed easily to match a wide range of practical cases.

It is recommended to:

4. Take the time value of money into consideration.
5. Tackle the robustness of environmental parameters.
6. Take the percent defective of each facility into consideration.

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