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Distance Divisor Signed Graphs

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Abstract: In this paper we introduced a new notion distance divisor signed graph of a signed graph and its properties are obtained. Also, we obtained the structural characterization of distance divisor signed graphs. Further, we presented some switching equivalent characterizations.

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I. INTRODUCTION

For standard terminology and notion in group theory and graph theory, we refer the reader to the text-books of Herstein [3] and Harary [1] respectively. The non-standard will be given in this paper as and when required.

Let $G = (V, E)$ be a graph with $|V| = p$ and $|E| = q$. The shortest path P in G is said to be distance divisor path, if $l(P) \mid q$, where $l(P)$ denotes the length path P.

Let $G = (V, E)$ be a graph with $|V| = p$ and $|E| = q$. The distance divisor graph$DD(G)$of $G = (V, E)$ is a graph with V(DD(G)) = V (G) and any two vertices u and v in DD(G) are joined by an edge if there exists a distance divisor path between them in G. This concept was introduced by Saravanakumar and Nagarajan [6].

To model individual's preferences towards each other in a group, Harary [2] introduced the concept of signed graphs in 1953. A signed graph $S = (G, \sigma)$ is a graph $G = (V, E)$ whose edges are labeled with positive and negative signs (*i.e.,* $\sigma: E(G) \rightarrow \{+, -\}$). The vertexes of a graph represent people and an edge connecting two nodes signifies a relationship between individuals.Thesigned graph captures the attitudes between people, where a positive (negative edge) represents liking (disliking). An unsigned graph is a signed graph with the signs removed. Similar to an unsigned graph, there are many active areas of research for signed graphs.

The sign of a cycle (this is the edge set of a simple cycle) is defined to be the product of the signs of its edges; in other words, a cycle is positive if it contains an even number of negative edges and negative if it contains an odd number of negative edges. A signed graph S is said to be balanced if every cycle in it is positive. A signed graph S is called totally unbalanced if every cycle in S is negative. A chord is an edge joining two non adjacent vertices in a cycle.

A marking of S is a function $\mu : V(G) \rightarrow \{+, -\}$. Given a signed graph S one can easily define a marking $\mu\ of\ S$ as follows: For any vertex $v \in V(S)$,

$$\mu(v) = \prod_{uv \in E\ (S)} \sigma(uv),$$

the marking $\mu\ of\ S$ is called a canonical marking of S
The following are the fundamental results about balance, the second being a more advanced form of the first.
Note that in a bipartition of a set, $V = V_1 \cup V_2$ the disjoint subsets may be empty.

Theorem 1.1.A signed graph $S = (G, \sigma)$is balanced if and only if either of the following equivalent conditions is satisfied:

(i) Its vertex set has a bipartition $V = V_1 \cup V_2$ such that every positive edge joins vertices in V_1 or in V_2, and every negative edge joins a vertex of$V_1$with a vertex of V_2*(Harary [2])*.

(ii) There exists a marking μ of its vertices such that each edge uv in Ssatisfies $\sigma(uv) = \mu(u)\mu(v)$. (*Sampathkumar [4]*).

Switching S with respect to a marking μ is the operation of changing the sign of every edge of S to its opposite whenever its end vertices are of opposite signs. The resulting signed graph $S_\mu(S)$ is said to be a switched signed graph. A signed graph S is said to be switched to another signed graph S' written $S \sim S'$, whenever there exists a marking μ such that $S_\mu(S) \cong S'$, where $\cong$ denotes the usual equivalence relation of isomorphism in the class of signed graphs. Hence, if $S \sim S'$, we shall say that S and S' are switching equivalent. Two signed graphs S_1 and S_2 are signed isomorphic (written $S_1 \cong S_2$) if there is a one to one correspondence between there vertex sets which preserves adjacency as well as sign.

Two signed graphs $S_1 = (G_1, \sigma_1)$ and $S_2 = (G_2, \sigma_2)$ are said to be weakly isomorphic (see [5]) or cycle isomorphic (see [7]) if there exits an isomorphism $\emptyset : G_1 \to G_2$ such that the sign of every cycle Z in S_1 equals to the sign of $\emptyset(Z)$ in S_2. The following result is well known (see [7]):

Theorem 1.2: (T. Zaslavsky [7]) Given a graph G, any two signed graphs in $\psi(G)$, where $\psi(G)$ denotes the set of all the signed graphs possible for a graph G, are switching equivalent if and only if they are cycle isomorphic.

II. DISTANCE DIVISOR SIGNED GRAPH OF A SIGNED GRAPH

Motivated by the existing definition of complement of a signed graph, we now extend the notion of distant divisor graphs to signed graphs as follows: The distance divisor signed graph $DD(S)$ of a signed graph $S = (G, \sigma)$ is a signedgraph whose underlying graph is $DD(G)$ and sign of any edge uv is $DD(S)$ is $\mu(u)\mu(v)$, where μ is the canonical marking of S. Further, a signed graph $S = (G, \sigma)$ is called distance divisor signed graph, if $S \cong DD(S')$ for some signed graph S'. The following result restricts the class of distance divisorgraphs.

Theorem 2.1. For any signed graph $S = (G, \sigma)$, its distance divisor signed graph $DD(S)$ is balanced.

Proof. Since sign of any edge $e = uv$ in $DD(S)$ is $\mu(u)\mu(v)$, where μ is the canonical marking of S, by Theorem 1.1, $DD(S)$ is balanced.

For any positive integer k, the k^{th} iterated distance divisor signed graph, $DD^k(S)$ of S is defined as follows:

$$DD^0(S) = S, DD^k(S) = DD(DD^{k-1}(S)).$$

Corollary 2.2. For any signed graph $S = (G, \sigma)$ and for any positive integer k, $DD^k(S)$ is balanced.

The following result characterizes signed graphs which are distance divisor signed graphs.

Theorem 2.3. A signed graph $S = (G, \sigma)$ is a distance divisor signed graph if, and only if, S is balanced signed graph and its underlying graph G is a distance divisor graph.

Proof. Suppose that S is balanced and G is a distance divisor graph. Then there exists a graph G' such that $DD(G') \cong G$. Since S is balanced, by Theorem 1.1, there exists a marking μ of G such that each edge uv in S satisfies $\sigma(uv) = \mu(u)\mu(v)$. Now consider the signed graph $S' = (G', \sigma')$, where for any edge e in G', $\sigma'(e)$ is the marking of the corresponding vertex in G. Then clearly, $DD(S') \cong S$. Hence S is adistance divisor signed graph.

Conversely, suppose that $S = (G, \sigma)$ is a distance divisor signed graph. Then there exists a signed graph $S' = (G', \sigma')$ such that $DD(S') \cong S$. Hence, G is the distance divisor graph of G' and by Theorem 2.1, S is balanced.

Consider a graph $G = (V, E)$ with $|V| = p$ and $|E| = q$. Let $k_1, k_2, \ldots, k_\tau$ denote the positive divisors of q with $k_1 = 1, \ldots, k_\tau = q$ and $k_1 < k_2 \ldots < k_\tau$. In [6], the authors characterizes the graphs such that G and $DD(G)$ are isomorphic.

Theorem 2.4. Let $G = (V, E)$ be a graph with $|V| = p$ and $|E| = q$, where q is a composite number. Then G and $DD(G)$ are isomorphic if and only if the diameter of G is less than or equal to $k_2 - 1$.

In view of the above, we have the following result:

Theorem 2.5. For any signed graph $S = (G, \sigma)$ with $|V| = p$ and $|E| = q$, where q is a composite number. Then S and $DD(S)$ are cycle isomorphic if and only if S is balanced and the diameter of G is less than or equal to $k_2 - 1$.

Proof. Suppose $DD(S) \sim S$. This implies, $DD(G) \cong G$ and hence by Theorem 2.4, we see that the diameter of G is less than or equal to $k_2 - 1$. Now, if S is any signed graph with diameter of G is less than or equal to $k_2 - 1$.

Then *DD(S)* is balanced and hence if *S* is unbalanced and its distance divisor signed graph *DD(S)* being balanced cannot be switching equivalent to *S* in accordance with Theorem 1.2. Therefore, *S* must be balanced.

Conversely, suppose that *S*is a balanced signed graph with the underlying graph *G* satisfies the conditions of Theorem 2.4. Since *DD(S)* is balanced as per Theorem 2.1, the result follows from Theorem 1.2 again.

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