Plane Strain Deformation of an Elastic Half-Space Due to a **Rectangular Thermal Inclusion**

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Abstract: In the present work, closed form analytical solutions for the displacement and strain fields due to a rectangular thermal inclusion in the interior of an isotropic elastic half-space are obtained. The thermoelastic displacement potential functions are used to determine the two dimensional plane strain deformation of an elastic half-space within the framework of steady-state linear uncoupled thermoelasticity. The thermal displacement field is generated due to differences in the coefficients of linear thermal expansion between a subregion and the surrounding material. The analytical expressions for the displacement and strain fields are derived for displacement free boundary conditions.

Keywords: Uncoupled linear thermoelasticity, rectangular inclusion, potential functions, displacement and strain fields, isotropic half-space

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I. **INTRODUCTION**

The determination of elastic stresses caused by an inclusion in a medium is a classical problem in the field of mechanics of solids. Comprehensive reviews of the micromechanics of inclusions and related problems have been documented in several classical texts such as Mura, 1982; Weng et al., 1990; Nemat-Nasser and Hori, 1993; Markov and Preziosi, 2000; and others. Considerable attention has been paid to the deformation of a body due to inclusions of different material in it. In general, an inclusion can have an arbitrary shape and volume. Goodier (1937) solved the static uncoupled thermal stress problem of an infinite elastic solid at zero temperature except for a heated region using displacement potential functions. Using Galerkin vector stress function, Mindlin and Cheng (1950) analysed the thermoelastic stress for the centre of dilatation in a semi-infinite elastic solid with the traction free surface. The case of a spherical inclusion near the surface of a semi-infinite elastic medium was given as an application. Using biharmonic potential functions, Eshelby (1957) solved the problem of an ellipsoidal inclusion in an infinite homogeneous and isotropic elastic medium, subjected to a uniformly distributed stress-free transformation strain and determined that the strain and stress fields inside the inclusion were uniform. The elastic fields outside an ellipsoidal inclusion was also addressed by Eshelby (1959) using harmonic potential functions. Ignaczak and Nowacki (1958) determined the state of stresses in an elastic space and semi-space for two cases of discontinuous temperature field- (i) rectangular parallelepiped thermal inclusion; (ii) cylindrical thermal inclusion, using integral representations of thermoelastic displacement potential functions. Bhargava and Kapoor (1966) studied the problem of two dimensional rectangular inclusion using complex variable method. Chang and Conway (1968, 1969) presented a solution method to analyze the stresses in an infinite elastic plate containing a rectangular inclusion subjected to a uniform stress field using Muskhelishvili's complex variable method in combination with the Schwartz- Christoffel conformal transformation technique. Based upon the Papkovich- Neuber stress function approach, Aderogba and Berry (1971) obtained the elastic field generated in two bonded isotropic elastic half-spaces containing either a plane circular inclusion or a rectangular inclusion.

Chiu (1980) obtained the closed form analytic solutions for the internal stresses due to a rectangular inclusion in an infinite plane, a half plane and a layer with specified boundary conditions using Airy stress function and the method of Fourier transforms. Hu (1989) derived analytical solution for the stresses due to a parallelepipedic thermal inclusion in a semispace using Goodier's method of nucleus of thermal strain or center of dilatation. Some thermoelastic problems in the half-space have been studied by Min-zhong and Ke-fu (1991) using general solutions of elasticity based on the Boussinesq solutions. Vihak et al. (1998) proposed a new method for solving the plane thermoelasticity problem for a rectangular domain in the absence of body forces and with traction-free boundary conditions. Using Green's function approach, Cheng et al. (1999) derived the

closed form solutions of the stress field due to a rectangular inclusion with quadratic eigenstrains embedded in an infinite isotropic elastic domain. The cases of non-uniform heating and pure shear distortion in a rectangular region were given as examples. Davies (2003) derived the elastic field due to a non-uniform temperature or a coherently misfitting inclusion in a semi-infinite region from the corresponding field in an infinite region. Liu and Wang (2005) determined the elastic field caused by the eigenstrains with arbitrary distribution in a halfspace. Wang and Pan (2008) derived closed-form solutions for displacement, strains and stresses due to a rectangular inclusion with uniform antiplane eigenstrains in an orthotropic quarter plane and in a bimaterial consisting of two orthotropic quarter planes using Green's function method. Meleshko (2011) demonstrated the method for determining the two-dimensional thermal stresses inside the rectangle subjected to an arbitrary prescribed temperature distribution. Zou *et al.* (2013) presented a general method to study the two-dimensional Eshelby's problem of thermal inclusions inside a bounded homogeneous anisotropic medium for the Dirichlet and Neumann boundary conditions. Nenashev and Dvurechenskii (2017) derived closed form analytical solutions for the three-dimensional elastic strain distribution in an isotropic infinite elastic medium containing a polyhedral-shaped inclusion with a coordinate-dependent lattice mismatch.

In the present paper, an attempt has been made to obtain thermoelastic displacement and strain fields due to a rectangular inclusion in the interior of an isotropic elastic half-space. Firstly, the displacement components for an infinite medium have been derived using thermoelastic displacement potential functions. Then following the method opted by Collins (1960), the displacement field for a half-space is obtained from the corresponding fields for an infinite medium. The closed form analytical expressions for displacement field are obtained for the plane strain problem. The strain components are also derived using strain-displacement relations. The surface of half-space is considered to be a displacement free surface. Numerical results for strain field are also shown graphically.

II. FORMULATION AND SOLUTION OF PROBLEM

We consider the problem of a rectangular thermal inclusion in the upper half-space ($z \ge 0$) having different coefficient of thermal expansion to that of the half-space but have same elastic constants. Due to this difference in the coefficients of thermal expansion between a sub region and its surrounding material, say η_0 , thermoelastic stress field is generated. The rectangular column (parallel to the surface of half-space) is of infinite extent along y-direction, having cross- section L < x < R, B < z < D, where $B \ge 0$ as shown in Figure 1. If all the components of an elastic material are independent of coordinate y, then we have a plane strain problem in two-dimensional Cartesian coordinates (x, z). Then, the thermoelastic potential function ϕ

satisfies the following Poisson's equations, when temperature difference of the semi-infinite region is T_0 ,

$$\nabla^2 \phi = \frac{1+\nu}{1-\nu} \eta_0 T_0 \qquad \text{for} \quad L < x < R , \quad B < z < D \tag{1}$$

and
$$\nabla^2 \phi = 0 \qquad , \quad \text{otherwise} \tag{2}$$

where α is the coefficient of linear thermal expansion and V is Poisson's ratio.





Then the thermoelastic potential function ϕ for this problem can be written as (Davies, 2003):

$$\phi(x,z) = f(x-L,z-B) - f(x-L,z-D) - f(x-R,z-B) + f(x-R,z-D)$$
(3)

where
$$f(x,z) = K \left[xz \log(x^2 + z^2) - 3xz + x^2 \tan^{-1} \frac{z}{x} + z^2 \tan^{-1} \frac{x}{z} \right]$$
 (4)

and
$$K = \left(\frac{1+\nu}{1-\nu}\right)\eta_0 T_0$$
 (5)

Boundary conditions: We assume that the boundary z=0 of the half-space is displacement free boundary, i.e., $u_x = u_z = 0$ on z = 0(6)

Following Collins (1960), the displacement components $\mathbf{u} = (u_x, u_z)$ within the semi-infinite region $z \ge 0$ with the displacement free boundary in terms of displacement components (u_0, w_0) for an infinite region in plane strain problem in xz-plane are reduced in the form:

$$u_x = u_0 + u_1$$
 (7a)
 $u_z = w_0 + w_1$ (7b)

where (u_1, w_1) are the additional displacement components and T_1 is the excess temperature that are added respectively to (u_0, w_0) and T_0 to obtain the corresponding components for the same temperature distribution in the semi-infinite region $z \ge 0$ with z = 0 as displacement free boundary.

The additional displacement components in terms of those of an infinite region for the plane strain problem in the *xz*-plane can be reduced in the form (Collins, 1960):

$$u_{1}(z) = -u_{0}(-z) + \frac{2z}{(3-4\nu)} \frac{\partial}{\partial x} w_{0}(-z) + \frac{\beta z^{2}}{2(3-4\nu)(1-\nu)} \frac{\partial}{\partial x} T_{0}(-z)$$

$$w_{1}(z) = -w_{0}(-z) + \frac{2z}{(3-4\nu)} \frac{\partial}{\partial z} w_{0}(-z) - \frac{(1-2\nu)\beta z}{(3-4\nu)(1-\nu)} T_{0}(-z) + \frac{\beta z^{2}}{2(3-4\nu)(1-\nu)} \frac{\partial}{\partial z} T_{0}(-z)$$
(8a)
where $\beta = 2\alpha (1+\nu)$
(9)

where $\beta = 2\alpha(1+\nu)$

Now the displacement components (u_0, w_0) in an infinite region and those at the image point are obtained on using $\mathbf{u}^{(\infty)} = \nabla \phi$ (Timoshenko & Goodier, 1951):

$$u_{0}(z) = \frac{\partial \phi}{\partial x} = K \begin{bmatrix} (z-B) \log \frac{(x-L)^{2} + (z-B)^{2}}{(x-R)^{2} + (z-B)^{2}} + (z-D) \log \frac{(x-R)^{2} + (z-D)^{2}}{(x-L)^{2} + (z-D)^{2}} \\ + 2(x-L) \left\{ \tan^{-1} \frac{z-B}{x-L} - \tan^{-1} \frac{z-D}{x-L} \right\} - 2(x-R) \left\{ \tan^{-1} \frac{z-B}{x-R} - \tan^{-1} \frac{z-D}{x-R} \right\} \end{bmatrix}$$
(10a)
$$w_{0}(z) = \frac{\partial \phi}{\partial z} = K \begin{bmatrix} (x-L) \log \frac{(x-L)^{2} + (z-B)^{2}}{(x-L)^{2} + (z-D)^{2}} + (x-R) \log \frac{(x-R)^{2} + (z-D)^{2}}{(x-R)^{2} + (z-B)^{2}} \\ + 2(z-B) \left\{ \tan^{-1} \frac{x-L}{z-B} - \tan^{-1} \frac{x-R}{z-B} \right\} - 2(z-D) \left\{ \tan^{-1} \frac{x-L}{z-D} - \tan^{-1} \frac{x-R}{z-D} \right\} \end{bmatrix}$$
(10b)

$$u_{0}(-z) = -K \begin{bmatrix} (z+B)\log\frac{(x-L)^{2} + (z+B)^{2}}{(x-R)^{2} + (z+B)^{2}} + (z+D)\log\frac{(x-R)^{2} + (z+D)^{2}}{(x-L)^{2} + (z+D)^{2}} \\ + 2(x-L)\left\{\tan^{-1}\frac{z+B}{x-L} - \tan^{-1}\frac{z+D}{x-L}\right\} - 2(x-R)\left\{\tan^{-1}\frac{z+B}{x-R} - \tan^{-1}\frac{z+D}{x-R}\right\} \end{bmatrix}$$
(11a)

$$w_{0}(-z) = K \begin{bmatrix} (x-L) \log \frac{(x-L)^{2} + (z+B)^{2}}{(x-L)^{2} + (z+D)^{2}} + (x-R) \log \frac{(x-R)^{2} + (z+D)^{2}}{(x-R)^{2} + (z+B)^{2}} \\ + 2(z+B) \left\{ \tan^{-1} \frac{x-L}{z+B} - \tan^{-1} \frac{x-R}{z+B} \right\} - 2(z+D) \left\{ \tan^{-1} \frac{x-L}{z+D} - \tan^{-1} \frac{x-R}{z+D} \right\} \end{bmatrix}$$
(11b)

$$T_0(-z) = 0$$
 (12)

Further,

$$\frac{\partial}{\partial x}w_{0}(-z) = K \left[\log \frac{\left\{ (x-L)^{2} + (z+B)^{2} \right\} \left\{ (x-R)^{2} + (z+D)^{2} \right\} }{\left\{ (x-L)^{2} + (z+D)^{2} \right\} \left\{ (x-R)^{2} + (z+B)^{2} \right\} } \right]$$
(13a)
$$\frac{\partial}{\partial z}w_{0}(-z) = 2K \left[\tan^{-1}\frac{x-L}{z+B} - \tan^{-1}\frac{x-L}{z+D} - \tan^{-1}\frac{x-R}{z+B} + \tan^{-1}\frac{x-R}{z+D} \right]$$
(13b)

Substituting equations (11) - (13) into (8), the additional displacement components in the elastic half-space can be expressed as

$$u_{1}(z) = K \begin{bmatrix} (z+B) \log \frac{(x-L)^{2} + (z+B)^{2}}{(x-R)^{2} + (z+B)^{2}} + (z+D) \log \frac{(x-R)^{2} + (z+D)^{2}}{(x-L)^{2} + (z+D)^{2}} \\ + 2(x-L) \left\{ \tan^{-1} \frac{z+B}{x-L} - \tan^{-1} \frac{z+D}{x-L} \right\} - 2(x-R) \left\{ \tan^{-1} \frac{z+B}{x-R} - \tan^{-1} \frac{z+D}{x-R} \right\} \end{bmatrix} \\ + K \frac{2z}{3-4\nu} \left[\log \frac{\left\{ (x-L)^{2} + (z+B)^{2} \right\} \left\{ (x-R)^{2} + (z+D)^{2} \right\} }{\left\{ (x-L)^{2} + (z+D)^{2} \right\} \left\{ (x-R)^{2} + (z+B)^{2} \right\} } \right]$$
(14a)

$$w_{1}(z) = -K \begin{bmatrix} (x-L)\log\frac{(x-L)^{2} + (z+B)^{2}}{(x-L)^{2} + (z+D)^{2}} + (x-R)\log\frac{(x-R)^{2} + (z+D)^{2}}{(x-R)^{2} + (z+B)^{2}} \\ + 2(z+B)\left\{\tan^{-1}\frac{x-L}{z+B} - \tan^{-1}\frac{x-R}{z+B}\right\} - 2(z+D)\left\{\tan^{-1}\frac{x-L}{z+D} - \tan^{-1}\frac{x-R}{z+D}\right\} \end{bmatrix} \\ -K\frac{4z}{3-4\nu}\left[\tan^{-1}\frac{x-L}{z+D} - \tan^{-1}\frac{x-L}{z+B} - \tan^{-1}\frac{x-R}{z+D} + \tan^{-1}\frac{x-R}{z+B}\right]$$
(14b)

Using equations (7), (10) and (14), the non-zero components of thermoelastic displacement vector due to a rectangular inclusion in the interior of a homogeneous, isotropic elastic half-space with z=0 as displacement free boundary can be expressed as

$$\begin{split} u_{x} &= u_{0} + u_{1} \\ &= K \begin{bmatrix} (z-B) \log \frac{(x-L)^{2} + (z-B)^{2}}{(x-R)^{2} + (z-B)^{2}} + (z-D) \log \frac{(x-R)^{2} + (z-D)^{2}}{(x-L)^{2} + (z-D)^{2}} \\ &+ 2(x-L) \left\{ \tan^{-1} \frac{z-B}{x-L} - \tan^{-1} \frac{z-D}{x-L} \right\} - 2(x-R) \left\{ \tan^{-1} \frac{z-B}{x-R} - \tan^{-1} \frac{z-D}{x-R} \right\} \end{bmatrix} \\ &+ K \begin{bmatrix} (z+B) \log \frac{(x-L)^{2} + (z+B)^{2}}{(x-R)^{2} + (z+B)^{2}} + (z+D) \log \frac{(x-R)^{2} + (z+D)^{2}}{(x-L)^{2} + (z+D)^{2}} \\ &+ 2(x-L) \left\{ \tan^{-1} \frac{z+B}{x-L} - \tan^{-1} \frac{z+D}{x-L} \right\} - 2(x-R) \left\{ \tan^{-1} \frac{z+B}{x-R} - \tan^{-1} \frac{z+D}{x-R} \right\} \end{bmatrix} \\ &+ K \frac{2z}{3-4\nu} \left[\log \frac{\left\{ (x-L)^{2} + (z+B)^{2} \right\} \left\{ (x-R)^{2} + (z+D)^{2} \right\} \\ &(15a) \end{bmatrix} \right] \end{split}$$

$$\begin{split} u_{z} &= w_{0} + w_{1} \\ &= K \begin{bmatrix} (x-L) \log \frac{(x-L)^{2} + (z-B)^{2}}{(x-L)^{2} + (z-D)^{2}} + (x-R) \log \frac{(x-R)^{2} + (z-D)^{2}}{(x-R)^{2} + (z-B)^{2}} \\ &+ 2(z-B) \left\{ \tan^{-1} \frac{x-L}{z-B} - \tan^{-1} \frac{x-R}{z-B} \right\} - 2(z-D) \left\{ \tan^{-1} \frac{x-L}{z-D} - \tan^{-1} \frac{x-R}{z-D} \right\} \\ &- K \begin{bmatrix} (x-L) \log \frac{(x-L)^{2} + (z+B)^{2}}{(x-L)^{2} + (z+D)^{2}} + (x-R) \log \frac{(x-R)^{2} + (z+D)^{2}}{(x-R)^{2} + (z+B)^{2}} \\ &+ 2(z+B) \left\{ \tan^{-1} \frac{x-L}{z+B} - \tan^{-1} \frac{x-R}{z+B} \right\} - 2(z+D) \left\{ \tan^{-1} \frac{x-L}{z+D} - \tan^{-1} \frac{x-R}{z+D} \right\} \\ &- K \frac{4z}{3-4\nu} \left[\tan^{-1} \frac{x-L}{z+D} - \tan^{-1} \frac{x-L}{z+B} - \tan^{-1} \frac{x-R}{z+D} + \tan^{-1} \frac{x-R}{z+B} \right] \end{split}$$

Further, from equation (15), it is observed that $u_x = u_z = 0$ on the boundary z = 0 of half-space which is in accordance with the boundary conditions given in equation (6). The results obtained above are in good agreement to those of Min-zhong and Ke-fu (1991) due to a rectangular inclusion in the interior of a homogeneous, isotropic elastic half-space using solutions of elasticity based on the Boussinesq solutions.

III. STRAIN FIELD

Using the strain-displacement relations, the non-zero components of strain for a plane strain problem in xz – plane are written in the form:

$$e_{xx} = \frac{\partial u_x}{\partial x} \quad ; \quad e_{zz} = \frac{\partial u_z}{\partial z}$$

$$e_{xz} = \frac{1}{2} \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right)$$
(16)

Using this, the non-zero components of strain field due to a rectangular thermal inclusion in the interior of an isotropic elastic half-space is obtained as follows:

1

(15b)

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$$e_{xx} = 2K \left[\tan^{-1} \frac{z-B}{x-L} - \tan^{-1} \frac{z-D}{x-L} - \tan^{-1} \frac{z-B}{x-R} + \tan^{-1} \frac{z-D}{x-R} \right] + 2K \left[\tan^{-1} \frac{z+B}{x-L} - \tan^{-1} \frac{z+D}{x-L} - \tan^{-1} \frac{z+B}{x-R} + \tan^{-1} \frac{z+D}{x-R} \right] + K \frac{4z}{3-4v} \left[(x-L) \left\{ \frac{1}{(x-L)^2 + (z+B)^2} - \frac{1}{(x-L)^2 + (z+D)^2} \right\} + (x-R) \left\{ \frac{1}{(x-R)^2 + (z+D)^2} - \frac{1}{(x-R)^2 + (z+B)^2} \right\} \right]$$

(17a)

$$e_{zz} = 2K \left[\tan^{-1} \frac{x-L}{z-B} - \tan^{-1} \frac{x-R}{z-B} - \tan^{-1} \frac{x-L}{z-D} + \tan^{-1} \frac{x-R}{z-D} \right] + 2 \left(\frac{1-4\nu}{3-4\nu} \right) K \left[\tan^{-1} \frac{x-R}{z+B} - \tan^{-1} \frac{x-L}{z+B} - \tan^{-1} \frac{x-R}{z+D} + \tan^{-1} \frac{x-L}{z+D} + \right] + K \frac{4z}{3-4\nu} \left[(x-L) \left\{ \frac{1}{(x-L)^2 + (z+D)^2} - \frac{1}{(x-L)^2 + (z+B)^2} \right\} \\+ (x-R) \left\{ \frac{1}{(x-R)^2 + (z+B)^2} - \frac{1}{(x-R)^2 + (z+D)^2} \right\} \right]$$
(17b)

(17b)

$$e_{xz} = K \left[\log \frac{\{(x-L)^{2} + (z-B)^{2}\}\{(x-R)^{2} + (z-D)^{2}\}\{(x-L)^{2} + (z+B)^{2}\}\{(x-R)^{2} + (z+D)^{2}\}}{\{(x-R)^{2} + (z-B)^{2}\}\{(x-L)^{2} + (z-D)^{2}\}\{(x-R)^{2} + (z+B)^{2}\}\{(x-L)^{2} + (z+D)^{2}\}} \right] + 2 \left(\frac{1-2\nu}{3-4\nu}\right) K \left[\log \frac{\{(x-L)^{2} + (z+T)^{2}\}\{(x-R)^{2} + (z+B)^{2}\}}{\{(x-L)^{2} + (z+B)^{2}\}\{(x-R)^{2} + (z+D)^{2}\}} \right] + \frac{4z}{3-4\nu} \left[(z+B) \left\{ \frac{1}{(x-L)^{2} + (z+B)^{2}} - \frac{1}{(x-R)^{2} + (z+B)^{2}} \right\} \right] + (z+T) \left\{ \frac{1}{(x-R)^{2} + (z+T)^{2}} - \frac{1}{(x-L)^{2} + (z+D)^{2}} \right\} \right] + K \frac{2z}{3-4\nu} \left[\tan^{-1} \frac{x-L}{z+B} - \tan^{-1} \frac{x-L}{z+D} - \tan^{-1} \frac{x-R}{z+B} + \tan^{-1} \frac{x-R}{z+D} \right]$$
(17c)

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, the graphical representations of the strain components at the surface of a thermoelastic half-space due to a rectangular thermal inclusion are obtained using MATLAB software programming. The numerical computations are carried out for the value of Poisson's ratio V = 0.25. Figures 2 and 3 show the variation of strain components e_{zz} and e_{xz} , respectively, with the distance x/L. From Figure 2, it is observed that the strain e_{zz} assumes finite positive values. Initially it increases continuously and attains its maximum value; then it decreases gradually with increasing values of the distance x/L. Also it is noticed that the strain e_{xz} initially starts with its negative values for all value of ratio D/L. Then it crosses the value zero and thereafter assumes positive values. It is also observed that for the value of ratio D/L = 2, 1, 1/2, the

magnitude of strain e_{xz} first decreases gradually and then increases rapidly with increasing values of the distance x/L. For the ratio D/L = 1/10, the magnitude of strain e_{xz} first increases rapidly, then decreases gradually and again it increases rapidly with increasing values of the distance x/L.



Figure 2 Variation of Strain component e_{zz} with distance x/L



Figure 3 Variation of Strain component e_{xz} with distance x/L

V. CONCLUSIONS

In this study, the closed form analytical expressions for the displacement and strain fields due to a rectangular thermal inclusion in an isotropic elastic half-space have been obtained. In view of steady-state linear uncoupled thermoelasticity, the thermoelastic displacement potential functions are used for calculating the displacement components for the plane strain problem under displacement free boundary conditions. The strain components are also derived using strain-displacement relations.

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