

# IF i $\tilde{B}$ G Homeomorphism in topological spaces.

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## Abstract

Throughout this paper we have introduced a new concept of intuitionistic fuzzy  $\tilde{B}$  generalized homeomorphism and intuitionistic fuzzy  $i\tilde{B}$  generalized homeomorphism in intuitionistic fuzzy topological spaces and some of their properties are discussed and also we have compared with existing homomorphism intuitionistic fuzzy topological spaces.

**Key words and phrases:** Intuitionistic fuzzy topology, intuitionistic fuzzy  $\tilde{B}$  generalized closed set, intuitionistic fuzzy  $\tilde{B}$  generalized continuous mapping, intuitionistic fuzzy  $\tilde{B}$  generalized homeomorphism, intuitionistic fuzzy  $i\tilde{B}$  generalized homeomorphism.

## I. Introduction

Zadeh [13] initiated the concepts fuzzy sets in 1965. Later, Atanassov [1] introduced the a new idea about intuitionistic fuzzy sets in 1986. After that Coker [3] has introduced intuitionistic fuzzy topological spaces in 1997. I have studied many research papers, later we get new idea of about intuitionistic fuzzy topological spaces. Further, we have introduced a new paper is intuitionistic fuzzy  $\tilde{B}$  generalized homeomorphism and intuitionistic fuzzy  $i\tilde{B}$  generalized homeomorphism in intuitionistic fuzzy topological spaces. Also we studied the relations with basic concepts of intuitionistic fuzzy homomorphisms, intuitionistic fuzzy continuous mappings.

## II. Preliminaries

**Definition 2.1:** [1] An intuitionistic fuzzy set (IFS in short)  $A$  in  $X$  is an object having the form  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ , where the functions  $\mu_A(x): X \rightarrow [0, 1]$  and  $\nu_A(x): X \rightarrow [0, 1]$  denote the degree of membership (namely  $\mu_A(x)$ ) and the degree of non-membership (namely  $\nu_A(x)$ ) of each element  $x \in X$  to the set  $A$ , respectively, and  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$  for each  $x \in X$ . Denote by  $\text{IFS}(X)$ , the set of all intuitionistic fuzzy sets in  $X$ .

**Definition 2.2:** [1] Let  $A$  and  $B$  be IFSs of the form

$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$  and  $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$ . Then

(a)  $A \subseteq B$  if and only if  $\mu_A(x) \leq \mu_B(x)$  and  $\nu_A(x) \geq \nu_B(x)$  for all  $x \in X$

(b)  $A = B$  if and only if  $A \subseteq B$  and  $B \subseteq A$

(c)  $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$

(d)  $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$

(e)  $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

**Definition 2.3:** [3] An intuitionistic fuzzy topology (IFT in short) on  $X$  is a family  $\tau$  of IFSs in  $X$  satisfying the following axioms.

(i)  $0_-, 1_- \in \tau$

(ii)  $G_1 \cap G_2 \in \tau$  for any  $G_1, G_2 \in \tau$

(iii)  $\cup G_i \in \tau$  for any family  $\{ G_i / i \in J \} \subseteq \tau$ .

**Definition 2.4:** [3] Let  $(X, \tau)$  be an IFTS and  $A = \langle x, \mu_A, \nu_A \rangle$  be an IFS in  $X$ . Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$\text{int}(A) = \cup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \}$ ,

$\text{cl}(A) = \cap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}$ .

**Definition 2.5:** [8] An IFS  $A = \langle x, \mu_A, \nu_A \rangle$  in an IFTS  $(X, \tau)$  is said to be an

(i) intuitionistic fuzzy semi closed set (IFSCS in short) if  $\text{int}(\text{cl}(A)) \subseteq A$ ,

(ii) intuitionistic fuzzy  $\alpha$ -closed set (IF $\alpha$ CS in short) if  $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$ .

**Definition 2.6:**[9] An IFS  $A$  in an IFTS  $(X, \tau)$  is said to be an intuitionistic fuzzy  $\pi$ - generalized closed set (IF $\pi$ GSCS in short) if  $scl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is an IF $\pi$ OS in  $(X, \tau)$ .

**Definition 2.7:**[8] An IFS  $A$  in an IFTS  $(X, \tau)$  is an

- (i) intuitionistic fuzzy generalized closed set (IFGCS) if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is an IFOS in  $X$ .
- (ii) intuitionistic fuzzy  $\alpha$  generalized closed set (IF $\alpha$ GCS) if  $\alpha cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is an IFOS in  $X$ .

**Definition 2.8:**[10] Let  $f$  be a mapping from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$ . Then  $f$  is said to be

- (i) intuitionistic fuzzy continuous (IF continuous) if  $f^{-1}(B) \in \text{IFO}(X)$  for every  $B \in \sigma$ .
- (ii) intuitionistic fuzzy semi continuous (IFS continuous) if  $f^{-1}(B) \in \text{IFSO}(X)$  for every  $B \in \sigma$ .
- (iii) intuitionistic fuzzy  $\alpha$ -continuous (IF $\alpha$  continuous) if  $f^{-1}(B) \in \text{IF}\alpha\text{O}(X)$  for every  $B \in \sigma$ .
- (iv) intuitionistic fuzzy generalized continuous (IFG continuous) if  $f^{-1}(B) \in \text{IFGC}(X)$  for every IFCS  $B$  in  $Y$ .
- (v) intuitionistic fuzzy generalized semi continuous (IFGS continuous) if  $f^{-1}(B) \in \text{IFGSC}(X)$  for every IFCS  $B$  in  $Y$ .
- (vi) intuitionistic fuzzy  $\alpha$ -generalized continuous (IF $\alpha$ G continuous) if  $f^{-1}(B) \in \text{IF}\alpha\text{GC}(X)$  for every IFCS  $B$  in  $Y$ .

**Definition 2.9:**[10] Let  $f$  be a mapping from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$ . Then  $f$  is said to be an intuitionistic fuzzy  $\hat{\beta}$  generalized open mapping (IF $\hat{\beta}$ G open mapping) if  $f(A) \in \text{IF}f\text{GOS}(X)$  for every IFOS  $A$  in  $X$ .

**Definition 2.10:**[11] A mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called an intuitionistic fuzzy  $\pi$  - generalized semi closed mapping (IF $\pi$ GS closed) if  $f(A)$  is an IF $\pi$ GSCS in  $(Y, \sigma)$  for every IFCS  $A$  of  $(X, \tau)$ .

**Definition 2.11:**[9] Let  $f$  be a mapping from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$ . Then the map  $f$  is said to be an intuitionistic fuzzy  $\pi$ - generalized semi irresolute (IF $\pi$ GS irresolute in short) if  $f^{-1}(B) \in \text{IF}\pi\text{GCS}(X)$  for every IF $\pi$ GCS  $B$  in  $Y$ .

**Definition 2.12:**[8] Let  $f$  be a bijection mapping from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$ . Then  $f$  is said to be

- (i) intuitionistic fuzzy homeomorphism (IF homeomorphism) if  $f$  and  $f^{-1}$  are IF continuous mappings.
- (ii) intuitionistic fuzzy semi homeomorphism (IFS homeomorphism) if  $f$  and  $f^{-1}$  are IFS continuous mappings.
- (iii) intuitionistic fuzzy  $\alpha$  homeomorphism (IF $\alpha$  homeomorphism in short) if  $f$  and  $f^{-1}$  are IF $\alpha$  continuous mappings.
- (iv) intuitionistic fuzzy generalized homeomorphism (IFG homeomorphism in short) if  $f$  and  $f^{-1}$  are IFG continuous mappings.
- (v) intuitionistic fuzzy generalized semi homeomorphism (IFGS homeomorphism in short) if  $f$  and  $f^{-1}$  are IFGS continuous mappings.
- (vi) intuitionistic fuzzy  $\alpha$  generalized homeomorphism (IF $\alpha$ G homeomorphism in short) if  $f$  and  $f^{-1}$  are IF $\alpha$ G continuous mappings.

### III. IF $\hat{\beta}$ G homeomorphism

**Definition 3.1:** A bijection mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called an intuitionistic fuzzy  $\hat{\beta}$  generalized homeomorphism (briefly IF $\hat{\beta}$ G homeomorphism) if  $f$  and  $f^{-1}$  are IF $\hat{\beta}$ G continuous mappings. We denote the group of all IF $\hat{\beta}$ G homeomorphisms of an IFTS  $(X, \tau)$  onto itself by **IF $\hat{\beta}$ G- $h(X, \tau)$** .

**Example 3.2:** Let  $X = \{a, b\}$ ,  $Y = \{u, v\}$  and  $\tau = \{0, G_1, 1\}$  and  $\sigma = \{0, G_2, 1\}$  where  $G_1 = \langle x, (0.21, 0.21), (0.6, 0.6) \rangle$  and  $G_2 = \langle y, (0.4, 0.7), (0.4, 0.2) \rangle$ .  $f$  is an IF $\hat{\beta}$ G continuous and  $f^{-1}$  is also an IF $\hat{\beta}$ G continuous.  $\therefore f$  is an IF $\hat{\beta}$ G homeomorphism.

**Proposition 3.3:** Every IF homeomorphism is an IF $\hat{\beta}$ G homeomorphism.

**Proof:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an IF homeomorphism. Then  $f$  and  $f^{-1}$  are IF continuous mappings and  $f$  is bijection. By Proposition, every IF continuous mapping is IF $\hat{\beta}$ G continuous mapping,  $f$  and  $f^{-1}$  are IF $\hat{\beta}$ G continuous.  $\therefore f$  is IF $\hat{\beta}$ G homeomorphism.

**Example 3.4:** Let  $X = \{a, b\}$ ,  $Y = \{u, v\}$  and  $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$  and  $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$  where  $G_1 = \langle x, (0.31, 0.21), (0.6, 0.6) \rangle$  and  $G_2 = \langle y, (0.51, 0.41), (0.4, 0.2) \rangle$ .  $f$  is an  $IF_{\tilde{B}}G$  homeomorphism except an IF homeomorphism since  $f$  and  $f^{-1}$  are not IF continuous.

**Proposition 3.5:** Every  $IF_{\alpha}$  homeomorphism is an  $IF_{\tilde{B}}G$  homeomorphism.

**Proof:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an  $IF_{\alpha}$  homeomorphism. Then  $f$  and  $f^{-1}$  are  $IF_{\alpha}$  continuous. By Proposition , every  $IF_{\alpha}$  continuous mapping is an  $IF_{\tilde{B}}G$  continuous,  $f$  and  $f^{-1}$  are  $IF_{\tilde{B}}G$  continuous mappings.  $\therefore f$  is an  $IF_{\tilde{B}}G$  homeomorphism.

**Example 3.6:** Let  $X = \{a, b\}$ ,  $Y = \{u, v\}$  and  $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$  and  $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$  where  $G_1 = \langle x, (0.51, 0.41), (0.5, 0.6) \rangle$  and  $G_2 = \langle y, (0.21, 0.21), (0.7, 0.7) \rangle$ .  $f$  is an  $IF_{\tilde{B}}G$  homeomorphism. For an IFCS  $A = \langle y, (0.7, 0.7), (0.21, 0.21) \rangle$  in  $(Y, \sigma)$ . Then  $f^{-1}(A) = \langle x, (0.7, 0.7), (0.21, 0.21) \rangle$  is not an  $IF_{\alpha}CS$  in  $(X, \tau)$ . Thus  $f$  is not an  $IF_{\alpha}$  homeomorphism.

**Theorem 3.7:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an  $IF_{\tilde{B}}G$  homeomorphism. Then  $f$  is an IF homeomorphism if  $(X, \tau)$  and  $(Y, \sigma)$  are  $IF_{\tilde{B}a}T_{1/2}$  space.

**Proof:** Let  $B$  be an IFCS in  $(Y, \sigma)$ . Since  $f$  is an  $IF_{\tilde{B}}G$  homeomorphism,  $f^{-1}(B)$  is an  $IF_{\tilde{B}}GCS$  in  $(X, \tau)$ . Since  $(X, \tau)$  is an  $IF_{\tilde{B}a}T_{1/2}$ space,  $f^{-1}(B)$  is an IFCS in  $(X, \tau)$ . Hence  $f$  is an IF continuous. Also,  $f^{-1}: (Y, \sigma) \rightarrow (X, \tau)$  is an  $IF_{\tilde{B}}G$  continuous. Let  $A$  be an IFCS in  $(X, \tau)$ . Then,  $(f^{-1})^{-1}(A) = f(A)$  is an  $IF_{\tilde{B}}GCS$  in  $(Y, \sigma)$ . Since  $(Y, \sigma)$  is an  $IF_{\tilde{B}a}T_{1/2}$ space,  $f(A)$  is an IFCS in  $(Y, \sigma)$ .  $f^{-1}$  is an IF continuous mapping.  $\therefore$  the mapping  $f$  is an IF homeomorphism.

**Theorem 3.8:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an  $IF_{\tilde{B}}G$  homeomorphism. Then  $f$  is an IFG homeomorphism if  $(X, \tau)$  and  $(Y, \sigma)$  are  $IF_{\tilde{B}b}T_{1/2}$  space.

**Proof:** Let  $B$  be an IFCS in  $(Y, \sigma)$ . Since  $f$  is an  $IF_{\tilde{B}}G$  homeomorphism,  $f^{-1}(B)$  is an  $IF_{\tilde{B}}GCS$  in  $(X, \tau)$ . Since  $(X, \tau)$  is an  $IF_{\tilde{B}b}T_{1/2}$ space,  $f^{-1}(B)$  is an IFGCS in  $(X, \tau)$ . Hence  $f$  is an IFG continuous. Also,  $f^{-1}: (Y, \sigma) \rightarrow (X, \tau)$  is an  $IF_{\tilde{B}}G$  continuous. Let  $A$  be an IFCS in  $(X, \tau)$ . Then,  $(f^{-1})^{-1}(A) = f(A)$  is an  $IF_{\tilde{B}}GCS$  in  $(Y, \sigma)$ . Since  $(Y, \sigma)$  is an  $IF_{\tilde{B}b}T_{1/2}$ space,  $f(A)$  is an IFGCS in  $(Y, \sigma)$ . Hence  $f^{-1}$  is an IFG continuous. Therefore,  $f$  is an IFG homeomorphism.

**Theorem 3.9:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a bijective mapping. If  $f$  is an  $IF_{\tilde{B}}G$  continuous, then the following are equivalent:

- (i)  $f$  is an  $IF_{\tilde{B}}G$  closed mapping
- (ii)  $f$  is an  $IF_{\tilde{B}}G$  open mapping
- (iii)  $f$  is an  $IF_{\tilde{B}}G$  homeomorphism.

**Proof:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a bijective mapping and  $f$  be an  $IF_{\tilde{B}}G$  continuous mapping.

(i)  $\Rightarrow$  (ii): let  $f$  be an  $IF_{\tilde{B}}G$  closed mapping. This implies that  $f^{-1}: (Y, \sigma) \rightarrow (X, \tau)$  is  $IF_{\tilde{B}}G$  continuous. By proposition 3.3, every IFOS in  $(X, \tau)$  is an  $IF_{\tilde{B}}GOS$  in  $(Y, \sigma)$ . Hence  $f^{-1}$  is an  $IF_{\tilde{B}}G$  open mapping.

(ii)  $\Rightarrow$  (iii): let  $f$  be an  $IF_{\tilde{B}}G$  open mapping. This implies that  $f^{-1}: (Y, \sigma) \rightarrow (X, \tau)$  is  $IF_{\tilde{B}}G$  continuous. Hence  $f$  and  $f^{-1}$  are  $IF_{\tilde{B}}G$  continuous. Therefore,  $f$  is an  $IF_{\tilde{B}}G$  homeomorphism.

(iii)  $\Rightarrow$  (i): Let  $f$  is an  $IF_{\tilde{B}}G$  homeomorphism. Then,  $f$  and  $f^{-1}$  are  $IF_{\tilde{B}}G$  continuous. By proposition 2.2.3, every IFCS in  $(X, \tau)$  is an  $IF_{\tilde{B}}GCS$  in  $(Y, \sigma)$ ,  $f$  is an  $IF_{\tilde{B}}G$  closed mapping.

**Remark 3.10:** The composition of two  $IF_{\tilde{B}}G$  homeomorphisms need not be an  $IF_{\tilde{B}}G$  homeomorphism in general. This can be shown from the following example.

**Example 3.11:** Let  $X = \{a, b\}$ ,  $Y = \{c, d\}$  and  $Z = \{u, v\}$ . Let  $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$ ,  $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$  and  $\eta = \{0_{\sim}, G_3, 1_{\sim}\}$  where  $G_1 = \langle x, (0.81, 0.61), (0.21, 0.41) \rangle$ ,  $G_2 = \langle y, (0.61, 0.11), (0.41, 0.31) \rangle$  and  $G_3 = \langle z, (0.41, 0.41), (0.61, 0.21) \rangle$ .  $f$  and  $f^{-1}$  are  $IF_{\tilde{B}}G$  continuous mappings. Also  $g$  and  $g^{-1}$  are  $IF_{\tilde{B}}G$  continuous mappings. Hence  $f$  and  $g$  are  $IF_{\tilde{B}}G$  homeomorphisms. Except the composition  $g \circ f: (X, \tau) \rightarrow (Z, \eta)$  is an  $IF_{\tilde{B}}G$  homeomorphism since  $g \circ f$  is not an  $IF_{\tilde{B}}G$  continuous mapping.

#### IV. $IF_{\tilde{B}}GS$ homeomorphism

**Definition 4.1:** A bijection mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called an intuitionistic fuzzy  $i_{\tilde{B}}$ generalized semi homeomorphism ( $IF_{\tilde{B}}GS$  homeomorphism in short) if  $f$  and  $f^{-1}$  are  $IF_{\tilde{B}}G$  irresolute mappings.

We denote the group of all  $IF_{\tilde{B}}G$ -homeomorphism of a topological space  $(X, \tau)$  onto itself by  $IF_{\tilde{B}}G-h(X, \tau)$ .

**Theorem 4.2:** Every  $IFi_{\tilde{B}}G$  homeomorphism is an  $IF_{\tilde{B}}G$  homeomorphism.

**Proof:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an  $IFi_{\tilde{B}}GS$  homeomorphism. Then,  $f$  and  $f^{-1}$  are  $IF_{\tilde{B}}G$  irresolute function. By Proposition, every  $IF_{\tilde{B}}G$  irresolute function is an  $IF_{\tilde{B}}G$  continuous function. Therefore,  $f$  and  $f^{-1}$  are  $IF_{\tilde{B}}G$  continuous function and hence  $f$  is an  $IF_{\tilde{B}}G$  homeomorphism.

**Example 4.3:** Let  $X = \{a, b\}$ ,  $Y = \{u, v\}$  and  $\tau = \{0., G_1, 1.\}$  and  $\sigma = \{0., G_2, 1.\}$  where  $G_1 = \langle x, (0.31, 0.31), (0.61, 0.61) \rangle$  and  $G_2 = \langle y, (0.21, 0.11), (0.41, 0.41) \rangle$ .  $f$  is an  $IF_{\tilde{B}}G$  homeomorphism. The IFS  $A = \langle y, (0.31, 0.21), (0.61, 0.61) \rangle$  in  $(Y, \sigma)$ , Clearly,  $A$  is an  $IF_{\tilde{B}}GCS$  in  $(Y, \sigma)$ , except  $f^{-1}(A)$  is an  $IF_{\tilde{B}}GCS$  in  $(X, \tau)$  and therefore,  $f$  is not an  $IF_{\tilde{B}}G$  irresolute mapping. Hence  $f$  is not an  $IFi_{\tilde{B}}G$  homeomorphism.

**Definition 4.4:** Let  $A$  be an IFS in an IFTS  $(X, \tau)$ . Then  $\tilde{B}gcl(A)$  is defined as  $\tilde{B}gcl(A) = \cap \{ B / B \text{ is an } IF_{\tilde{B}}GCS \text{ in } (X, \tau) \text{ and } A \subseteq B \}$ .

**Theorem 4.5:** If  $f: (X, \tau) \rightarrow (Y, \sigma)$  is an  $IFi_{\tilde{B}}G$  homeomorphism, then  $\tilde{B}gcl(f^{-1}(B)) = f^{-1}(\tilde{B}gcl(B))$  for every IFS  $B$  in  $(Y, \sigma)$ .

**Proof:** Since  $f$  is an  $IFi_{\tilde{B}}G$  homeomorphism,  $f$  is an  $IF_{\tilde{B}}G$  irresolute mapping. Consider an IFS  $B$  in  $(Y, \sigma)$ . Clearly  $\tilde{B}gcl(B)$  is an  $IF_{\tilde{B}}GCS$  in  $(Y, \sigma)$ . By hypothesis,  $f^{-1}(\tilde{B}gcl(B))$  is an  $IF_{\tilde{B}}GCS$  in  $(X, \tau)$ . Since  $f^{-1}(B) \subseteq f^{-1}(\tilde{B}gcl(B))$ ,  $\tilde{B}gcl(f^{-1}(B)) \subseteq \tilde{B}gcl(f^{-1}(\tilde{B}gcl(B))) = f^{-1}(\tilde{B}gcl(B))$ . This implies  $\tilde{B}gcl(f^{-1}(B)) \subseteq f^{-1}(\tilde{B}gcl(B))$ .

Since  $f$  is an  $IFi_{\tilde{B}}G$  homeomorphism,  $f^{-1}: (Y, \sigma) \rightarrow (X, \tau)$  is an  $IF_{\tilde{B}}G$  irresolute mapping. Consider an IFS  $f^{-1}(B)$  in  $(X, \tau)$ . Clearly  $\tilde{B}gcl(f^{-1}(B))$  is an  $IF_{\tilde{B}}GCS$  in  $(X, \tau)$ . This implies that  $(f^{-1})^{-1}(\tilde{B}gcl(f^{-1}(B))) = f(\tilde{B}gcl(f^{-1}(B)))$  is an  $IF_{\tilde{B}}GCS$  in  $(Y, \sigma)$ .

Clearly  $B = (f^{-1})^{-1}(f^{-1}(B)) \subseteq (f^{-1})^{-1}(\tilde{B}gcl(f^{-1}(B))) = f(\tilde{B}gcl(f^{-1}(B)))$ . Therefore,  $\tilde{B}gcl(B) \subseteq \tilde{B}gcl(f(\tilde{B}gcl(f^{-1}(B)))) = f(\tilde{B}gcl(f^{-1}(B)))$ , since  $f^{-1}$  is an  $IF_{\tilde{B}}G$  irresolute mapping. Hence  $f^{-1}(\tilde{B}gcl(B)) \subseteq f^{-1}(f(\tilde{B}gcl(f^{-1}(B)))) = \tilde{B}gcl(f^{-1}(B))$ . That is  $f^{-1}(\tilde{B}gcl(B)) \subseteq \tilde{B}gcl(f^{-1}(B))$ . This implies  $\tilde{B}gcl(f^{-1}(B)) = f^{-1}(\tilde{B}gcl(B))$ .

**Corollary 4.6:** If  $f: (X, \tau) \rightarrow (Y, \sigma)$  is an  $IFi_{\tilde{B}}G$  homeomorphism, then  $\tilde{B}gcl(f(B)) = f(\tilde{B}gcl(B))$  for every IFS  $B$  in  $(X, \tau)$ .

**Proof:** Since  $f$  is an  $IFi_{\tilde{B}}G$  homeomorphism,  $f^{-1}$  is an  $IFi_{\tilde{B}}G$  homeomorphism. Let  $B$  be an IFS in  $(X, \tau)$ . By theorem,  $\tilde{B}gcl(f(B)) = f(\tilde{B}gcl(B))$  for every IFS  $B$  in  $(X, \tau)$ .

**Corollary 4.7:** If  $f: (X, \tau) \rightarrow (Y, \sigma)$  is an  $IFi_{\tilde{B}}G$  homeomorphism, then  $f(\tilde{B}gint(B)) = \tilde{B}gint(f(B))$  for every IFS  $B$  in  $(X, \tau)$ .

**Proof :** For any IFS  $B$  in  $(X, \tau)$ ,  $\tilde{B}gint(B) = (\tilde{B}gcl(B^c))^c$ . By Corollary,  $f(\tilde{B}gint(B)) = f((\tilde{B}gcl(B^c))^c) = (f(\tilde{B}gcl(B^c)))^c = (\tilde{B}gcl(f(B^c)))^c$ . This implies that  $f(\tilde{B}gint(B)) = (\tilde{B}gcl(f(B)))^c = \tilde{B}gint(f(B))$ .

**Corollary 4.8:** If  $f: (X, \tau) \rightarrow (Y, \sigma)$  is an  $IFi_{\tilde{B}}G$  homeomorphism, then  $f^{-1}(\text{int}(B)) = \text{int}(f^{-1}(B))$  for every IFS  $B$  in  $(X, \tau)$ .

**Proof :** Since  $f^{-1}: (Y, \sigma) \rightarrow (X, \tau)$  is also an  $IFi_{\tilde{B}}GS$  homeomorphism, the proof follows from Corollary 4.7.

**Proposition 4.9:** If  $f: (X, \tau) \rightarrow (Y, \sigma)$  and  $g: (Y, \sigma) \rightarrow (Z, \eta)$  are  $IFi_{\tilde{B}}G$  homeomorphisms then their composition  $g \circ f: (X, \tau) \rightarrow (Z, \eta)$  is also an  $IFi_{\tilde{B}}G$  homeomorphisms.

**Proof:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  and  $g: (Y, \sigma) \rightarrow (Z, \eta)$  be any two  $IFi_{\tilde{B}}G$  homeomorphisms. Therefore,  $f, f^{-1}, g$  and  $g^{-1}$  are  $IF_{\tilde{B}}G$  irresolute functions. By theorem,  $g \circ f$  and  $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$  are  $IF_{\tilde{B}}GS$  irresolute functions and  $(g \circ f)$  is an  $IFi_{\tilde{B}}G$  homeomorphism.

**Proposition 4.10:** The set  $IFi_{\tilde{B}}G-h(X, \tau)$  is a group under the composition of maps.

**Proof :** Define a binary operation  $*$ :  $IFi_{\tilde{B}}G-h(X, \tau) \times IFi_{\tilde{B}}G-h(X, \tau) \rightarrow IFi_{\tilde{B}}G-h(X, \tau)$  by  $f * g = g \circ f$  for all  $f, g \in IFi_{\tilde{B}}G-h(X, \tau)$  and  $\circ$  is the usual operation of composition of maps.

- (i) **Closure Property:** Let  $f \in IFi_{\tilde{B}}G-h(X, \tau)$  and  $g \in IFi_{\tilde{B}}G-h(X, \tau)$ . By theorem 6.3.9,  $g \circ f \in IFi_{\tilde{B}}G-h(X, \tau)$ .
- (ii) **Associative property :** We know that the composition of mappings is associative.
- (iii) **Existence of identity :** The identity mappings  $I: (X, \tau) \rightarrow (X, \tau)$  belonging to  $IFi_{\tilde{B}}G-h(X, \tau)$  servers as the identity element.

- (iv) **Existence of inverse :** If  $f \in IFi_{\tilde{B}}G-h(X, \tau)$ , then  $f^{-1} \in IFi_{\tilde{B}}G-h(X, \tau)$  such that  $f^{-1} * f = f \circ f^{-1} = I$ . Therefore, inverse exists for each element of  $IFi_{\tilde{B}}G-h(X, \tau)$ .

Therefore,  $(IFi_{\tilde{B}}G-h(X, \tau), \circ)$  is a group under the operation of composition of maps.

**Theorem 4.11:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a  $IFi_{\beta}G$  homeomorphism. Then  $f$  induces an isomorphism from the group  $IFi_{\beta}G-h(X, \tau)$  onto the group  $IFi_{\beta}G-h(Y, \sigma)$ .

**Proof :** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a  $IFi_{\beta}G$  homeomorphism. We define a map  $\theta_f: IFi_{\beta}G-h(X, \tau) \rightarrow IFi_{\beta}G-h(Y, \sigma)$ , by  $\theta_f(h): f \circ h \circ f^{-1}$  for every  $h \in IFi_{\beta}G-h(X, \tau)$ , using the mapping  $f$ .

Obviously, to prove that  $\theta_f$  is a homeomorphism.  $\theta_f(h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{-1} = (f \circ h_1) \circ (h_2 \circ f^{-1}) = (f \circ h_1) \circ (f^{-1} \circ f) \circ (h_2 \circ f^{-1}) = (f \circ h_1 \circ f^{-1}) \circ (f \circ h_2 \circ f^{-1}) = \theta_f(h_1) \circ \theta_f(h_2)$ , for all  $h_1, h_2 \in IFi_{\beta}G-h(X, \tau)$ . Therefore,  $\theta_f$  is a homeomorphism. Hence  $f$  induces an isomorphism from the group  $IFi_{\beta}G-h(X, \tau)$  onto the group  $IFi_{\beta}G-h(Y, \sigma)$ .

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