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A New Aspect Of Double-Framed Normal Fuzzy Soft Ideal Structures Over Hemi Rings

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Abstract: *In this paper, we provide a general algebraic frame work for handling double-framed information by combining the theory of double-framed fuzzy soft sets with hemi rings. First, we present the concepts of double-framed fuzzy soft h-ideals and normal double-framed fuzzy soft h-ideals. Second, the characterizations of double-framed fuzzy soft h-ideals are investigated by means of positive t-cut, negative s-cut and homomorphism. Third, we give a general algorithm to solve decision making problems by using double-framed fuzzy soft set.*

Keywords: *Soft set, Hemi ring, Double-framed fuzzy soft set, Double-framed fuzzy soft h-ideal, comparison table,.* γ-inclusive, δ-exclusive
AMS Subject Classification [2000]: *03E70, 08E40*

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I. INTRODUCTION:

Aktas and Cagman[1] studied the basic concepts of soft set theory, and compared soft sets to fuzzy and rough sets, providing examples to clarify their differences. They also discussed the notion of soft groups. F.Feng, et.al [2] defined the concept of Soft semi rings. In 1999, Molodtsov introduced soft set theory [12] as an alternative approach to fuzzy set theory [16] defined by Zadeh in 1965. After Molodtsov's study, many researchers have studied on set theoretical approaches and decision making applications of soft sets. For example Majiet.al[11] defined some new operations of soft sets and gave a decision making method on soft sets. Jun et.al[[5],[6]] introduced the notion of double-framed soft sets (briefly, DFS-sets), and applied it to BCK/BCI-algebras. They discussed double-framed soft algebras (briefly, DFS-algebras) and investigated related properties. A.R.Hadipour [6] defined Double-framed soft BF-algebras and Yongukchoet.al [13] studied on double-framed soft Near-rings. Fuzzy set is a type of important mathematical structure to represent a collection of objects whose boundary is vague. There are several types of fuzzy set extensions in the fuzzy set theory, for example, intuitionistic fuzzy sets, interval fuzzy sets, vague sets etc. double-framed fuzzy set is another an extension of fuzzy set whose membership degree range is different from the above extensions. In 2000 , Lee [[9][10]] imitated an extension of of fuzzy set named bipolar valued fuzzy sets. He gave two kinds of representations of the notion of bipolar- valued fuzzy sets. In case of Bi-polar-Valued fuzzy sets membership degree range is enlarged from the interval [0,1] to [□ 1,0]. Ideals of hemi rings, as a kind of special hemi ring, play a crucial role in the algebraic structure theories since many properties of hemi rings are characterized by ideals. How-ever, in general, ideals in hemi rings do not coincide with the ideals in rings. Subsequently, La Torre [8] studied thoroughly the properties of the *h*-ideals and *k*-ideals of hemi rings. The rest of this paper organized as follows. Section-2 reviews some basic ideas related with this paper. In section-3, we propose main results of double-framed fuzzy soft *h*-ideals. Normal double-framed fuzzy soft *h*-ideals are studied in chapter-4. An algorithm approach is proposed in section-5 to present the application of double-framed fuzzy soft set in decision making followed by a numerical example. Finally the key conclusions are given in section-5.

II. PRELIMINARIES

In this section, we review some definitions, regarding hemi rings [8] and double-framed fuzzy soft sets [6].

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Suppose that $(S,+)$ and $(S,\cdot)$ are two semi groups, then the algebraic system $(S,+,\cdot)$ is called a semi ring, in which the two algebraic structures are connected by the distributive laws: $a.(b+c)=a.b+a.c$ and $(b+c).a=b.a+c.a$ for all $a,b,c\in S$.

The zero element of a semi ring $(S,+,\cdot)$ is an element $0\in S$ satisfying $0\cdot x=x\cdot 0=0$ and $x+0=0+x=x$ for all $x\in S$. A semi ring with zero and a commutative semi group $(S,+)$ is called a hemi ring.

A non-empty subset I of a hemi ring S is called a left (resp., right) ideal of S if I is closed with respect to addition and SI is subset of I (resp., IS is subset of I) I is called an ideal of S if it is both a left and a right ideal of S.

A left (resp., right) ideal of a hemi ring S is called a left (resp., right) h-ideal if any $x,z\in S$, any $a,b\in A$ and $x+a+z=b+z$ implies $x\in A$.

A mapping f from a hemi ring S to A semi ring T is said to be a homomorphism if for all $x,y\in S$. $f(x+y)=f(x)+f(y)$and $f(xy)=f(x).f(y)$.

Through out this paper, we only give the proof of results about left cases because the proof of results about right classes can be conducted by similar methods. In order to facilitate discussion, S and T are hemi rings unless otherwise specified.

Through out the paper, U refers to an initial universe, E is a set of parameters and $P(U)$ is the power set of U. $\subset$ and $\supset$ stand for proper subset and super set, respectively.

Definition 2.1 (12) *For any subset A of E, a soft set λ_A over U is a set, defined by a function λ_A, representing the mapping $\lambda_A:E\to P(U)$. A soft set over U can also be represented by the set of ordered pairs $\lambda_A=\left\{\left(x,\lambda_A(x)\right): x\in E, \lambda_A(x)\in P(U)\right\}$. Note that the set of all soft sets over U will be denoted by S(U).*

Definition 2.2 (12) *Let $\lambda,\mu\in S(U)$. Then*

(i) If $\lambda(e)=\varnothing$ for all $e\in E$, λ is said to be a null soft set, denoted by $\varnothing$.

(ii) If $\lambda(e)=U$ for all $e\in E$, λ is said to be an absolute soft set, denoted by U.

(iii) λ is a soft subset of μ, denoted $\lambda\subseteq\mu$, if $\lambda(e)\subseteq\mu(e)$ for all $e\in E$.

(iv) Soft union of λ and μ, denoted by $\lambda\cup\mu$, is a soft set over U and defined by $\lambda\cup\mu:E\to P(U)$ such that $(\lambda\cup\mu)(e)=\lambda(e)\cup\mu(e)$ for all $e\in E$.

(v) $\lambda=\mu$, if $\lambda\subseteq\mu$ and $\lambda\supseteq\mu$.

(vi) Soft intersection of λ and μ, denoted by $\lambda\cap\mu$, is a soft set over U and defined by $\lambda\cap\mu:E\to P(U)$ such that $(\lambda\cap\mu)(e)=\lambda(e)\cap\mu(e)$ for all $e\in E$.

(vii) Soft complement of λ is denoted by λ^C and defined by $\lambda^C:E\to P(U)$ such that $\lambda^C(e)=\cup/\lambda(e)$ for all $e\in E$.

Definition 2.3 *Let E be a parameter set, $S\subset E$ and $\lambda:S\to E$ be an injection function. Then $S\cup\lambda(s)$ is called extended parameter set of S and denoted by ξ_S.*

If S=E, then extended parameter set of S will be denoted by ξ.

Note 2.1 *Let $\lambda_S=\left(\alpha_S,\beta_S,E\right)$ be a double framed soft set over U. We will say that $\lambda_S(e)=\left(\alpha_S(e),\beta_S(e)\right)$ is image of parameter $e\in E$.*

Definition 2.4 *Let λ_A and $\lambda_B\in DFSE(U)$ then,*

(i) If $\alpha_A(e)=\varnothing$ and $\beta_A(e)=U$ for all $e\in E$, α_A is said to be a null double-framed soft set, denoted by $\varnothing_b=(\varnothing,U,E)$.

(ii) If $\alpha_A(e)=U$ and $\beta_A(e)=\varnothing$ for all $e\in E$, α_A is said to be an absolute double-framed soft set, denoted by $\varnothing_b=(U,\varnothing,E)$.

(iii) λ_A is double-framed soft subset of λ_B, denoted by $\lambda_A\subseteq\lambda_B$, if $\alpha_A(e)\subseteq\alpha_B(e)$ and $\beta_A(e)\supseteq\beta_B(e)$ for all $e\in E$.

(iv) Double – framed soft union and intersection of λ_A and λ_B, denoted by

$(\alpha_A \cup \alpha_B): A \cup B \to P(U)$ such that $(\alpha_A \cup \alpha_B)(e) = \alpha_A(e) \cup \alpha_B(e)$ and

$(\beta_A \cap \beta_B)(e) = \beta_A(e) \cap \beta_B(e)$ for all $e \in E$.

Also $(\alpha_A \cap \alpha_B): A \cap B \to P(U)$ such that $(\alpha_A \cap \alpha_B)(e) = \alpha_A(e) \cap \alpha_B(e)$ and

$(\beta_A \cup \beta_B)(e) = \beta_A(e) \cup \beta_B(e)$ for all $e \in E$.

(v) Double – framed soft complement of λ_A is denoted by λ_A^C and defined by

$\lambda_A^C : E \to P(U) \times P(U)$ such that $\lambda_A^C(e) = \{ (e, \alpha_A(e), \beta_A(e)) : e \in E \}$.

Definition 2.5 *A double-framed fuzzy soft set (F,A) is said to be a null double-framed fuzzy soft set denoted by empty set Φ, if for all $e \in A, F(e) = \Phi$.*

Definition 2.6 *A double-framed fuzzy soft set (F,A) is said to be an absolute double-framed fuzzy soft set, if for all $e \in A, F(e) = DFU$.*

Definition 2.7 *The complement of a double-framed fuzzy soft set (F,A) is denoted $(F,A)^c$ and is denoted by* $(F,A)^c = \{ (x, 1-\alpha_A^+(x), 1-\beta_A^-(x)) ; x \in U \}$.

Definition 2.8 *A double-framed fuzzy soft set $A(\alpha_A, \beta_A)$ of S is called a double-framed fuzzy soft left (resp., right) h-ideal of S provided that for all $x,y,z,a,b \in S$;*

(BFShI1) $\alpha_A(x+y) \geq \min\{\alpha_A(x), \alpha_A(y)\}$, $\beta_A(x+y) \leq \max\{\beta_A(x), \beta_A(y)\}$,

(BFShI2) $\alpha_A(xy) \geq \max\{\alpha_A(x), \alpha_A(y)\}$, $\beta_A(xy) \leq \min\{\beta_A(x), \beta_A(y)\}$,

(BFShI3) $x+a+z = b+z$ implies $\alpha_A(x) \geq \min\{\alpha_A(a), \alpha_A(b)\}$, $\beta_A(x) \leq \max\{\beta_A(a), \beta_A(b)\}$.

A double-framed fuzzy soft set which is a double-framed fuzzy left and right h-ideal of S is called a double-framed fuzzy soft h-ideal of S. In this paper, the collection of all double-framed fuzzy soft h-ideals of S is denoted by DFShI(S) in short.

Example 2.1 *Let S=0,1,2,3 be a set with the addition operation (+) and the multiplication (•) as follows;*

+	0	1	2	3
0	0	1	2	3
1	1	1	2	3
2	2	2	2	3
3	3	3	3	3

.	0	1	2	3
0	0	0	0	0
1	0	1	1	1
2	0	1	1	1
3	0	1	1	1

Then S is a hemi ring. Define a double-framed fuzzy soft set A as follows

	0	1	2	3
α_A	0.3	0.7	0.5	0.2
β_A	0.5	0.6	0.7	0.1

By routine calculations, we know that A is a double-framed fuzzy soft h-ideals of S.
An interesting consequence of double-framed fuzzy soft h-ideals of hemi rings is the following.

Proposition 2.1 *Let A be a non-empty subset of S. A double-framed fuzzy soft set* $A=(\alpha_A,\beta_A)$ *is defined by*

$$\alpha_A(x)=\begin{cases} m1; & if x\in A \\ m2 & otherwise\end{cases} \quad and \quad \beta_A(x)=\begin{cases} n1; & if x\in A \\ n2 & otherwise\end{cases}$$

where $0\le m2\le m1\le 1$, $-1\le n1\le n2\le 0$ *is a double-framed fuzzy soft h-ideal of S if and only if A is a left (resp., right) h-ideal of S.*

The research about the relationships of fuzzy sub algebras and crisp sub algebras by cut sets is usual. But important, as it is a tie which can connect abstract algebraic structures and fuzzy ones. However, now we encounter a significant challenge that the traditional cut sets are not suitable for the frame work of double-framed fuzzy soft *h*-ideals of hemi rings because the characterization of double-framed idea.
As a consequence, we defined γ-inclusive set and the δ-exclusive set

Definition 2.9 *(6) A double-framed pair* $((\alpha, \lambda):G)$ *is called a double-framed soft set (briefly DFS-set) over U where* α *and* λ *are mapping from A to P(U).*
For a DFS-set $<(\alpha, \lambda):G>$ *over U and two subsets* γ *and* δ *of U, the* γ*-inclusive set and the* δ*-exclusive set of* $<(\alpha, \lambda):G>$*, denoted by* $i_A(\alpha;\gamma)$ *and* $e_A(\lambda,\delta)$ *respectively, are defined as follows.*

Error! *and* **Error!** *respectively. The set* **Error!** *is called a double-framed including set of* <**Error!**,**Error!** *:G>. It is clear that* $DF_A(\alpha, \lambda)_{(\gamma,\delta)}=i_A(\alpha;\gamma)\cap e_A(\lambda,\delta)$.

From now on, we will take G, as set of parameters, which is a group unless otherwise specified. fuzzy soft set of S and $(s,t)\in[-1,0]\times[0,1]$*. we define* **Error!** *and* **Error!** *and call them* γ*-inclusive set and the* δ*-exclusive set of A respectively. For any* $k\in[0,1]$*, the set* $A_k\cap A_k$ *is called the k-cut of A. From the definition 2.8 , we can easily obtained the relation of double-framed fuzzy soft h-ideals of hemi rings.*

III. MAIN RESULTS

In this section we discuss the properties of the cut sets, image and pre-image of double-framed fuzzy soft h-ideals by homomorphism of hemi rings

Theorem 3.1 *Let A be a double-framed fuzzy soft set S. Then A is a double-framed fuzzy soft left (resp., right) h-ideal of S if and only if the following hold;*

(i) For all $\gamma\in[0,1]$, $A_\gamma\neq\Phi$ *implies* A_γ *is a left (resp., right) h-ideal of S.*

(ii) For all $\delta\in[0,1]$, $A_\delta\neq\Phi$ *implies* A_δ *is a left (resp., right) h-ideal of S.*

Proof: Let *A* be double-framed fuzzy soft *h*-ideal of *S* and $\gamma\in[0,1]$ with $A_\gamma\neq\Phi$. Then $\alpha_A(x)\geq\gamma$, $\alpha_A(y)\geq\gamma$ for all $x,y\in A_\gamma$, $\delta\in S$ It implies that

$\alpha_A(x+y)\geq min\{\alpha_A(x),\alpha_A(y)\}\geq\gamma$ and $\alpha_A(xy)\geq max\{\alpha_A(x),\alpha_A(y)\}\geq\gamma$, that is $x+y,xy\in\alpha_A$.
Moreover $x,z\in S$, $a,b\in A_\gamma$ with $x+a+z=b+z$. Then $\alpha_A(x)\geq min\{\alpha_A(a),\alpha_A(b)\}\geq\gamma$. This means that $x\in A_\gamma$.
Hence α_A is a left *h*-ideal of *S*.

Analogously, we can prove (ii).
Conversly, assume (i), (ii) are all valid.
For any $x\in S$, if $\alpha_A(x)=\gamma$, $\beta_A(x)=\delta$, then $A_\gamma\cap A_\delta$. Thus A_γ and A_δ are non empty. Suppose that *A* is not a double-framed fuzzy soft *h*-ideal of *S*, then there exists $x,z,a,b\in S$, such that $x+a+z=b+z$, $\alpha_A(x)<\gamma<min\{\alpha_A(a),\alpha_A(b)\}$ and $\beta_A(x)>\delta>max\{\beta_A(a),\beta_A(b)\}$. Therefore $a,b\in A_\gamma$ but $x\neq A_\gamma$ and $a,b\in A_\delta$ but x does not belong to A_δ. This is a contradiction. Therefore *A* is a double-framed fuzzy soft *h*-ideal of *S*.
As immediate consequence of theorem 3.1 , we have the following.

Corollary 3.1 *If A is a double-framed fuzzy soft h-ideal of S, then the k-cut of A is a double-framed soft h-ideal of S for all* $k\in[0,1]$*.*

For the sake of simplicity, we denote $S^{(t,s)}$ for the set $x\in S/\mu_A^+(x)\geq t\cap x\in S/\mu_A^-(x)\leq t$ where $A=(\alpha_A(x),\beta_A(x))$.

Corollary 3.2 *If A is a double-framed fuzzy soft left (resp., right) h-ideal of S, then* $S^{(\gamma,\delta)}$ *is a left (resp., right) h-ideal of S for all* $(\gamma,\delta)\in[0,1]\times[-1,0]$*. In particular, the non empty k-cut of A is an h-ideal of S for all* $k\in[0,1]$*.*

Theorem 3.2 *Assume that A DFShI(S) and* $\alpha_A(x)+\beta_A(x)\geq 0$ *for all* $x\in S$*, then* $A_k\cup A_{-k}$ *is a left (resp., right) h-ideal of S for all* $k\in[0,1]$*.*

Proof: Let $k\in[0,1]$, evidently $A_k\neq\Phi$, $A_{-k}\neq\Phi$ and they are all left *h*-ideals of *S* from theorem 3.1. Let $x_1,x_2\in A^{+}_k\cup A^{-}_{-k},x,z\in S$ with $x+x_1+z=x2+z$. To complete the proof, we just need to consider the following four cases;

(i) $x_1\in A_k,x_2\in A_k$

(ii) $x_1\in A_k,x2\in A_{-k}$

(iii) $x_1\in A_{-k},x_2\in A_k$

(iv) $x_1\in A_{-k},x_2\in A_{-k}$

case(i) implies $\alpha_A(x_1)\geq k$, $\alpha_A(x_2)\geq k$. since $A\in DFShI(S)$, we can obtain

$\alpha_A(x_1+x_2)\geq min\left\{\alpha_A(x_1),\alpha_A(x_2)\right\}\geq k$, $\alpha_A(x_1x_2)\geq max\left\{\alpha_A(x_1)\alpha_A(x_2)\right\}\geq k$ and $\alpha_A(x)\geq min\left\{\alpha^{+}_A(x_1),\alpha_A(x_2)\right\}\geq k$. Then $x_1+x_2,x_1x_2,x\in A_k\cup A_{-k}$.The proof of case (iv) is similar to case (i). For case (ii), we can easily acquire $\alpha_A(x_1)\geq k,\beta_A(x_2)\leq -k$. since $\alpha_A(x_2)+\bar{\alpha}_A(x_2)\geq 0,\alpha_A(x_2)\geq -\bar{\beta}_A(x_2)\geq k$, we have $\alpha_A(x_1+x_2)\geq min\left\{\alpha_A(x_1),\alpha_A(x_2)\right\}\geq min\left\{\alpha_A(x_1),\beta_A(x_2)\right\}\geq k.\alpha_A(x_1x_2)\geq max\left\{\alpha_A(x_1),\alpha_A(x_2)\right\}\geq k$ and $\alpha_A(x)\geq min\left\{\alpha_A(x_1),\alpha_A(x_2)\right\}\geq min\left\{\alpha_A(x_1),\alpha_A(x_2)\right\}\geq k$. Then $x_1+x_2x_1x_2\in A_\gamma$ is subset of $A_k\cup A_{-k}$. The proof of case (iii) is similar to (ii). Hence $A_k\cup A_{-k}$ is left *h*-ideal of *S*.

Definition 3.1 *(13) Let* $\Phi:S\to T$ *be a homomorphism of hemi rings, and B be a double-framed fuzzy soft set of T. Then the inverse image of* $B\Phi^{-1}(B)$ *is the bipolar fuzzy soft set of S given by* $\Phi(\alpha_B)(x)=\alpha_B(\Phi(x)),\Phi^{-1}(\beta_b)(x)=\beta_b(\Phi(x))$ *, for all* $x\in S$*. Conversly, let A be a double-framed fuzzy soft set of S. The image of A,* $\Phi(A)$ *is double-framed fuzzy soft set of T defined by*

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where $\Phi^{-1}(y)=x\in S/\Phi(x)=y$*.*

Theorem 3.3 *Let* $\Phi:S\to T$ *be a homomorphism of hemi rings and B be a double-framed fuzzy soft left (resp.,right) h-ideal of T, then the inverse image* $\Phi^{-1}(B)$ *is a double-framed fuzzy soft left (resp., right) h-ideal of S.*

Proof: Suppose that $B=(\mu^{+}_B,\mu_B)$ is a double-framed fuzzy soft left *h*-ideal of *T* and Φ is a homomorphism of hemi rings from *S* to *T*. Then for all $x,y\in S$, we have

(BFShI1)

Error! Thus, (i) is valid of definition 2.8. By the same way, we can show that (ii) is hold. Moreover, let $x,z,a,b\in S$ with $x+a+z=b+z$. we can acquire $\Phi(x)+\Phi(a)+\Phi(z)=\Phi(b)+\Phi(z)$ and $\Phi^{-1}(\beta_B)(x)=\beta_B(\Phi(x_1))\geq min\left\{\beta_B(\Phi(a)),\beta_B(\Phi(b))\right. =min\left.\right\}\left\{\Phi^{-1}(\alpha_B)(a),\Phi^{-1}(\beta_B)(b)\right\}$. Analogously, we have $\Phi^{-1}(\beta_B)(x)\leq max\left\{\Phi^{-1}(\beta_B)(a),\Phi^{-1}(\beta_B)(b)\right\}$. Hence $\Phi^{-1}(B)$ is a double-framed fuzzy soft *h*-ideal of *S*.

Theorem 3.4 *Assume that* $\Phi:S\to T$ *be an epimorphism of hemi rings. If A is a double-framedfuzzy soft left (reso., right) h-ideal of S, then the image* $\Phi(A)$ *is a bipolar fuzzy soft left (resp., right) h-ideal of T.*

Proof: Since Φ is an epimorphism, by theorem 3.1, it is sufficient to show that $\Phi(A)^{\gamma}$ and $\Phi(A)^{\delta}$ are h-ideals of T for all $(\gamma,\delta)\in[0,1]\times[-1,0]$ satisfying $\Phi(A)^{\gamma}\neq\Phi$, $\Phi(A)^{\delta}\neq\Phi$.
Let $\gamma\in[0,1]$ and $\Phi(A)\neq\Phi$. Then for all $y_1,y_2\in\Phi(A)^{\gamma}$, we can obtain

$$\Phi(\alpha_A)(y_1)=\underset{x\varepsilon\Phi^{-1}(y_1)}{V\alpha_A(x)\geq t} \quad \text{and} \quad \Phi(\alpha_A)(y_2)=\underset{x\varepsilon\Phi^{-1}(y_2)}{V\alpha_A(x)\geq\gamma;}$$

This means that there exist $x_1\in\Phi^{-1}(y_1)$, $x_2\in\Phi^{-1}(y_2)$ such that $\alpha_A(x_1)\geq\gamma$, $\overset{+}{\alpha}_A(x_2)\geq\gamma$, Then

$$\Phi\alpha_A(y_1+y_2)=\underset{x\varepsilon\Phi^{-1}(y_1)}{V\alpha_A(x)}\geq\alpha_A(x_1+x_2)\geq min\left\{\alpha_A(x_1),\alpha_A(x_2)\right\}\geq\gamma .$$

Therefore $y_1+y_2\in\Phi(A)^{\gamma}$.

For all $y_0\in\Phi(A)^{\gamma}$, we have $\Phi(\alpha_A)(y_0)=\underset{x\varepsilon\Phi^{-1}(y)}{V\alpha_A(x)}\geq\gamma$, which implies that there exists $x_0\in\Phi^{-1}(y_0)$ such that $\alpha_A(x_0)\geq\gamma$.

For each $y\in T$, since Φ is an epimorphism and A is a double-framed fuzzy soft left h-ideal of S , there exists $x\in S$ such that $\Phi(x)=y_1$, $\Phi_A(xx_0)\leq max\left\{\Phi_A(x),\Phi_A(x_0)\right\}\leq\gamma$. Then $\Phi(\alpha_A)(yy_0)=\underset{x\varepsilon\Phi^{-1}(y)}{Vmax\left\{\alpha_A(x);\alpha_A(x_0)\right\}=\gamma}$. Thus $yy_0\in\Phi(A)^{\gamma}$. More over, let any $y,z\in T$ and any $m,n\in\Phi(A)_t^{+}$ such that $y+m+z=n+z$. Then we can acquire

$$\Phi(\alpha_A)(m)=\underset{x\varepsilon\Phi^{-1}(m)}{V\alpha_A(x)\geq\gamma} \quad \text{and} \quad \Phi(\overset{+}{\alpha}_A)(n)=\underset{x\varepsilon\Phi^{-1}(n)}{V\alpha_A(x)\geq\gamma;}$$. Thus $y\in\Phi(A)^{\gamma}$.

This means that $\Phi(A)^{\gamma}$ is a left h-ideal of T. Analogously, we can prove that $\Phi(A)^{\delta}$ is a left h-ideal of T. This completes the proof.

IV. DOUBLE-FRAMED NORMAL FUZZY SOFT *H*-IDEALS

In this section, we introduce and characterize normal double-framed fuzzy soft h-ideals of hemi rings. By definition 2.8, it is clear that a double-framed fuzzy soft set A is an double-framed fuzzy soft h-ideals of S providing that $\overset{+}{\mu}_A=1$ and $\overset{-}{\mu}_A=-1$ for $x\in S$. However, as a general rule, $\overset{+}{\mu}_A=1$ and $\overset{-}{\mu}_A=-1$ may not always hold. Therefore, it is necessary for us to define the following definition.

Definition 4.1 *A double-framed fuzzy soft h-ideal A of S is said to be normal if there exits an element* $x\in S$ *such that* $A(x)=(1,-1)$ *that means* $\overset{+}{\mu}_A=1$ *and* $\overset{-}{\mu}_A=-1$

Example 4.1 *Consider S=0,1,2,3 which is described in example 2.1. Let A be a double-framed fuzzy soft set S defined by*

	0	1	2	3
α_A	1	1	1	0.6
β_A	1	1	1	0.5

Clearly, A is a normal double-framed fuzzy soft h-ideal of S.

Definition 4.2 *A element* $x_0\in S$ *is called extremal for a double-framed fuzzy soft set A if* $\mu\alpha_A(x_0)\geq\mu\alpha_A(x)$ *and* $\mu\beta_A(x_0)\leq\beta_A(x)$*, for all* $x\in S$.

From the above definitions, we can easily derived the following properties.

Proposition 4.1 *A double-framed fuzzy soft set A of S is a normal double-framed fuzzy soft h-ideal if and only if A(x)=(−1,1) for its all extremal elements.*

Theorem 4.1 *If x_0 is an element of a double-framed fuzzy soft left (resp., rtght) h-ideal, then a double-framed fuzzy soft set A defined by $\mu\alpha_A(x)=\mu\alpha_A(x)+1-\alpha_A(x_0)$ and $\mu\beta_A(x)=\mu\beta_A(x)-1-\beta_A(x_0)$ for all $x\in S$ is a normal double-framed fuzzy soft left (resp., right) h-ideal of S containing A.*

Proof: First, we claim that Ã is normal. In fact, since $\tilde{A}\alpha_A(x)+1-\alpha_A(x_0)$, $\tilde{A}=\beta_A(x)-1-\beta_A(x_0)$ and x_0 is an extremal element of A. we have $\alpha_A(x_0)=1$, $\beta_A(x_0)=-1$. $\alpha_A(x)\in[0,1]$ and $\beta_A(x)=[-1,0]$ for all $x\in S$, Thus Ã is normal.

Next we show that Ã is double-framed fuzzy soft h-ideal of S. For all $x,y\in S$, we have

Error! and

$\tilde{A}(x+y)=\alpha_A(x+y)+1-\alpha_A(x_0)\leq max\ \{\alpha_A(x),\beta_A(y)\ \ +\}1-\beta_A(x_0)=$

EQ *max* \b \bc\{ (β\s\do5(*A*)(*x*)+1−β\s\do5(*A*)(*x*\s\do5(0))\,β\s\do5(*A*)(*y*)+1−β\s\up(+,*A*)(*x*\s\do5(0)))=*max* \b \bc\{ (\O(*A*, ˜)(*x*)\,\O(*A*, ˜)(*y*))

. Thus (DFShI1) is valid. Similarly, we can prove that (DFShI2) holds. More over, let any $x,z,a,b\in S$ such that $x+a+z=b+z$, we have

$\tilde{A}(x)=\alpha_A(x)+1-\alpha_A(x_0)\geq min\ \{\alpha_A(a),\alpha_A(b)\ \ +\}1-\overset{+}{\alpha}_A(x_0)=$

EQ *min* \b \bc\{ (α\s\do5(*A*)(*a*)+1−α\s\do5(*A*)(*x*\s\do5(0))\,α\s\do5(*A*)(*b*)+1−α\s\up(+,*A*)(*x*\s\do5(0)))=*min* \b \bc\{ (\O(*A*, ˜)(*a*)\,\O(*A*, ˜)(*b*))

Analogously, we have $\tilde{A}(x)\leq max\ \{\tilde{A}(a),\tilde{A}(b)\ \ \}$. Thus Ã is normal double-framed fuzzy soft h-ideal of S. Clearly A is contained in Ã.

Corollary 4.1 *From the definition of A in theorem 4.1, we get $\dot{A}=\tilde{A}$ for all $A\in DFShI(S)$. In particular, if A is normal, then $\tilde{A}=A$.*

Definition 4.3 *A non empty double-framed fuzzy soft h-ideal of S is called completely normal if there exists $x\in S$ such that A(x)=(0,0).*

Let all the completely normal double-framed fuzzy soft h-ideals of S be denoted by $C(S)$.

Theorem 4.2 *Let $f:[0,1]\to[0,1$ and $g:[-1,0]\to[-1,0]$ be two increasing functions and A be a bipolar fuzzy soft set of S. Then $A_{(f,g)}=(\alpha_{Af},\beta_{Ag})$ where $\alpha_{Af}(x)=f(\alpha_A(x))$ and $\beta_{Ag}(x)=g(\beta_A(x))$ for all $x\in S$ is a double-framed fuzzy soft h-ideal of S if and only if $g(\alpha_A(0))=-1$, then $A_{(f,g)}$ is normal.*

Proof: Let $A_{(f,g)}\in DFShI(S)$, then for all $x,y\in S$. we have

Error!.

Since f is increasing, it follows that $\alpha_A(x+y)\geq min\ \{\alpha_A(x)\},\alpha_A(y))$. Conversely, if $A\in DFShI(S)$, then for all $x,y\in S$, we have **Error!** Similarly, we have **Error!**. Thus **Error!** satisfies (*DFShI*1) if and only if *A* satisfies (*DFShI*1).The analogous connection between $A_{(f,g)}$ and A can be obtained in the case of axioms (*DFShI*2) and (*DFShI*3). This completes the proof.

V. DECISION MAKING APPROACH FOR DOUBLE-FRAMED FUZZY SOFT SET

Double-framed fuzzy soft set has several application to deal with uncertainties from our different kinds of daily life problems. Here we discuss such an application for solving a socialistic decision making problem.

5.1 Comparison table:

It is a square table in which number of rows and number of colums are equal and both are labeled by the object name of the universe such as $c_1,c_2,\ldots,c_n$ and the entries d_{ij} where d_{ij}=the number of parameters for which the value of d_i exceeds or equal to the value of d_j.

5.2 Algorithm

(i) Input the *ACE* of choice of parameters of the *X*.
(ii) Consider the double-framed fuzzy soft set in tabular form.
(iii)Compute the comparision table of positive values function and negative values function.
(iv)Compute the α-values and β- values score.
(v) Compute the final score by averaging α-membership values score and β-membership values score.

5.3 Double-framed decision making problem.

Assume that a real estate agent has a set of different types of houses $U=\{u_1,u_2,u_3,u_4,u_5\}$ which may be characterized by a set of parameters $E=\{x_1,x_2,x_3,x_4\}$ for j=1,2,3,4 the parameters x_j stand for in "good location", "cheap", "modern", "larg", respectively. Suppose that a married couple , Mr.X and Mrs. X, come to the real estate agent to buy a house. If each partner has to consider their own set of parameters, then we select a house on the basis of the sets of partners' parameters by using double-framed fuzzy soft sets as follows
Assume that $U=\{u_1,u_2,u_3,u_4,u_5\}$ is a universal set and $E=\{x_1,x_2,x_3,x_4\}$ set of all parameters. Our aim is to find the attractive houses for Mr. X. Suppose the wishing parameters of Mr.X be *A* is subset of *E*, where $A=\{e_1,e_2,e_5\}$
.

$F(e_1)=\left\{(c_1;0.6;-0.7);(c_2;0.3;-0.2);(c_3;0.7;-0.3);(c_4;0.8;-0.4)\right\}$
$F(e_2)=\left\{(c_1;0.4;-0.6);(c_2;0.7;-0.5);(c_3;0.9;-0.4);(c_4;0.5;-0.3)\right\}$
$F(e_5)=\left\{(c_1;0.9;-0.6);(c_2;0.3;-0.1);(c_3;0.8;-0.9);(c_4;0.7;-0.4)\right\}$

For the maximum score, if it occurs in *i*-th row, then Mr.X buy to di, 1≤*i*≤4.
Step-1 α-values function and β- values function of the given data

•	e_1	e_2	e_5
c_1	0.6	0.4	0.9
c_2	0.3	0.7	0.3
c_3	0.7	0.9	0.8
c_4	0.8	0.5	0.7

•	e_1	e_2	e_5
c_1	0.7	0.6	0.6
c_2	0.2	0.5	0.1
c_3	0.3	0.4	0.9
c_4	0.4	0.3	0.4

Step-2: Comparison tables of step-1

•	c_1	c_2	c_3	c_4
c_1	3	2	1	1
c_2	1	3	1	1
c_3	2	3	3	1
c_4	2	2	2	3

•	c_1	c_2	c_3	c_4
c_1	3	2	3	3

c_2	0	3	1	1
c_3	2	1	3	2
c_4	0	2	2	3

Step-3: Membership score tables

•	Row sum (a)	Column sum (b)	Membership score (a□b)
c_1	7	8	-1
c_2	6	10	-4
c_3	9	7	2
c_4	9	6	3

Step-4 Compute the α-values and β-values score.

•		Column sum (B)	Non-Membership score (A□B)
c_1	11	5	6
c_2	5	8	-3
c_3	8	9	-1
c_4	7	9	-2

Step-5 Final score table

•	α-Membershipvalue score (P)	β-Membershipvalue score (N)	Final score (P+N/2)
c_1	-1	6	2.5
c_2	-4	-3	-3.5
c_3	2	-1	0.5
c_4	3	-2	0.5

Clearly the maximum score is 2.5 scored by the house c_1.

Decision: Mr.X will by c_1. If he does not want to buy due to certain reason, his second choice will be c_3 or c_4.

VI. CONCLUSION

Double-framed sets plays a very important role in many branches of pure and applied mathematics.. In this paper, we have applied double-framed fuzzy sets theories to hemi rings and have discussed some basic properties on the subject of double-framed fuzzy h-ideals of hemirings, which is, in fact, just a incomplete beginning of the study of the hemi ring theory, so it is necessary to carry out more theoretical researches to establish a general framework for the practical application. We believe that the research in this direction can invoke more new topics and can provide more applications in some fields such as mathematical morphology, logic and inform ation science, engineering, medical diagnosis.

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