# Hierarchical Algorithm for MIMO Uncertain Plant

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**Abstract:** It is a well-known fact that every plant has an occurrence matrix and every matrix has a well known diagonal singular value matrix. If this singular value matrix is dimensionally reduced, it is as good as eliminating the disturbances and hence the uncertainties. Once these uncertainties are eliminated a robust stability can be achieved as this information can be fed back to improve error performance It is a derived fact that stability is associated with eigen values of the matrix and maximum gain corresponds to the eigenvectors associated to the maximum eigen value. This idea has been brought forward in this technical paper and it illustrates the QFT-ICST-SVD-PCA based novel hierarchical algorithm to control MIMO uncertain plant. The controller is designed through this algorithm which is executed in the matlab environment.

**Keywords**: ICSP, ICST, MIMO, QFT, SVD, STM, PCA, Occurrence Matrix.

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## I. INTRODUCTION

The control engineering now deals with logically combining process design and controller design. Tracking of a reference signal and transferring the plant operating point from one to another are some of the basic control purposes. The goal is to bring in the best features of the plant such that task is performed as desired. An optimized controller is one which achieves effective reduction in disturbances, noise filtering and fast and reliable tracking of set points.

Study is on with respect to plants with more than one manipulated and control input variables. It has been illustrated that the inputs and outputs are vectors which define a multivariable system or multi input and multi output (MIMO) system. To make such a system stable Isaac Horowitz (1963) [4] developed a quantitative feedback theory (QFT) based on frequency domain and Nichols charts. The concept so developed achieves robust design over a specified region of plant uncertainty. The robust design depends on loop shaping which results in reduction of disturbance and noise. The concept involves reduction in dimensions of the generated matrix which symbolizes reduction in load disturbance and measurement noise Loop shaping results in objectives of tracking, disturbance rejection, robust stability etc. In QFT plant uncertainty is depicted by templates, which is a measure of controller design possibility. The system specification is nothing but the bounds at each design frequency and is such that open loop transfer function must satisfy them. QFT bounds are treated as guiding factors for loop shaping. QFT bounds are quadratic inequalities for the robust stability and performance specifications. The controller deals with specifications such as stability, disturbance rejection whereas Pre-filter deals with tracking. Pre-filter synthesis problem is an interval constraint satisfaction problem (ICSP) [18] which is efficiently solved using singular valve decomposition and dimensionality reduction technique.

## II. LITERATURE OVERVIEW

Many methodologies in the time as well as in the frequency domains have been proposed previously such as Linear Quadratic Regulator (LQR) [1], It has good robustness properties in terms of stability margins and is well suited for numerical computations. It is extended well up to multivariable case but, the approach lacks implementation since it requires a constant gain controller for each state. The drawbacks are overcome by Linear Quadratic Gaussian (LTG) [2] approach which is seen as an extension of LQR. Design of optimal controller and observer are the design basics of LQG. This approach fails for real systems and for frequency domain analysis. The drawback of LQG prompted the development of $H_\infty$-synthesis [3]. The method is well suited for structured uncertainty but approach finds limitations for unstructured uncertainty and a latest approach is seen as quantitative feedback theory (QFT). Traditionally QFT synthesis is manual but, many automatic methods proposed by Gera and Horowitz (4), analytical methods [7], genetic algorithm [8], linear programming [9], interval analysis [10], evolutionary algorithms [11], combined feedforward-feedback design [12] and optimization algorithms [13]. The Horowitz's approach of quantitative feedback theory (QFT) is the noted one in the QFT domain. It is a frequency domain technique with Nichols plots as a tool. The QFT handles single-input single-output (SISO) and multi-input multi-output (MIMO), linear and non-linear, time-varying and time-invariant, and lumped and distributed parameter systems. The automatic synthesis of a QFT controller is still an open problem. The most successful method for such a design takes into consideration the nonlinear/non-convex QFT bounds without any approximation. It thereby ensures closed loop stability of the system and becomes largely independent of the initial controller solution. Existing QFT design approaches [14] fall short on at least one of these counts. A central problem in QFT consists of proving the existence (or non-existence) of a QFT controller solution to a given design problem. In certain cases, such as in LTI SISO problems, one can sometimes analytically verify the non-existence of a controller solution to a given problem, as demonstrated by Horowitz [4]. In the two degree-of-freedom structures used in QFT, a prefilter is required to meet the desired tracking specifications. In the QFT literature, there is no method for automation of the prefilter design, except the one given in [15]. However, even the method in [15] does not find all possible prefilters of a given structure. Recent methods for automated synthesis of QFT controllers [16] have mainly used interval global optimization techniques, which are, however, inherently slow. Interval constraint satisfaction techniques (ICST) based on the various refinements of box and hull consistency methods [17, 18, and 19] are known to greatly speed up pure interval techniques.

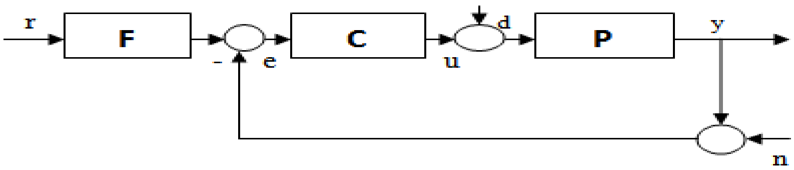
In spite of the undisputable performance advantage of centralized multivariable controllers, many complex multivariable plants employ decentralized controllers [20]. In the face of large plant parameter variations, unknown or uncertain multivariable plants, the input-output structure of the plant may endure fundamental changes, which will severely degrade the decentralized controller performance [21], The well-known input-output pairing techniques are unable to analyze the effect of uncertainty on input output pairing and only recently, pairing methods are proposed for uncertain multivariable plants [21], To design a satisfactory control plant in the presence of large modeling uncertainties, noise, and disturbances, a hierarchical control structure could be used. The control architecture consists of a bank of candidate controllers supervised by a logic-based switching [19]. The problem of robust adaptive control via combining QFT and switching supervisory control is introduced in [16] for Single Input-Single Output (SISO) plants.

The numerical and mathematical modeling drawbacks of the previous systems are taken care in this paper and a novel approach is presented with the advent of programming tools such as SVD, PCA [24, 25, and 26] powerful computational machinery and platforms such as MATLAB.

## III. CONTROLLER DESIGN PROBLEM

The design problem is stated in two stages. The first stage designs a feedback controller and in the second stage the pre-filter is designed. Both designs depend on load disturbance and measurement noise. Both the disturbances are optimized by using SVD and PCA technique wherein the resulting parameters are reduced disturbance and noise. The aspects of the design are:

- The load disturbance effect must be minimized
- The system must maintain closed loop stability against uncertainty.
- The measurement noise fed into the plant must be minimized
- The output of the plant must follow the reference input.




By referring to the figure above the governing equations can be written as:

(1)

From equations (1) & (2) the various parametric equations can be reproduced as

Sensitivity function: (3)

Complementary sensitivity function: (4)

Load disturbance sensitivity function: (5)

Noise sensitivity function: (6)

The parameters represented by equations 3, 4 & 5 are dealt by feedback controller C and the last parameter represented by equation 6 is concerned with Pre-filter F.

## IV. PROPOSED METHODOLOGY

According to Horowitz "Feedback is not needed if there is no plant model uncertainty; disturbances are small and if the control gain is large over a wide frequency range". Feedback is necessary for disturbance rejection and error optimization. There are severe drawbacks with high gain controller which demands high bandwidth for the feedback network. Load disturbance have low frequency and measurement noise has high frequency. Stability is achieved by shaping the loop such that it should have high gain at low frequency and low gain at high frequency. To have a robust stability the loop transfer function must have good phase margin at the crossover frequency and hence good phase margin and good loop shaping is a tradeoff between performance and robustness. The paper proposes an optimized control design approach to round off load disturbances and measurement noise so that robust stability can be achieved. Every plant is best described by its state space model which in turn described by occurrence matrix. This matrix is evaluated into singular values by decomposing it. The decomposed matrix in turn reduced dimensionally so that load and noise disturbances are eliminated. A norm is suggested which will round off the values. These rounded off values which are further propagated to get robust stability.

The plant design consists of 2-DOF problem in which a controller is designed first and then a pre-filter. The controller deals with stability and disturbance rejection specifications and a pre-filter works on tracking specifications. Both the designs make the plant stable. The pre-filter design is a constraint satisfaction problem (system of equations and inequalities) consists of a finite set of constraints specifying which value combinations from given variable domains are admitted. Hence it is required to find one or more value combinations satisfying the constraints. This solution is achieved by SVD-PCA approach along with Euclidian Norm. This norm gives out value combination which can be fed back to achieve total stability.

SVD decomposes a rectangular matrix **A** with *m* rows and *n* columns into a product of three matrices, $\mathbf{A} = \mathbf{USV}^T$, where **U** ($m \times m$) and $\mathbf{V}^T$ ($n \times n$) are the left and right orthogonal matrices and **S** ($m \times n$) is a rectangular matrix with non-negative singular values on the diagonal in order of decreasing magnitude. The number of singular values r, determines the rank of matrix **A**. Selection of orthonormal bases $\mathbf{V} = (v_1, v_2, \ldots v_r)$ for the row space, and $\mathbf{U} = (u_1, u_2, \ldots u_r)$ for the column space is done such that $\mathbf{A}v_i$ is in the direction of $u_i$, with $s_i$ providing the scaling factor, i.e. $\mathbf{A}v_i = s_i u_i$. In matrix form, this becomes $\mathbf{AV} = \mathbf{US}$ or $\mathbf{A} = \mathbf{USV}^T$. **A** is thus the linear transformation that carries orthonormal basis $v_i$ from space $\boldsymbol{R}^n$ to orthonormal basis $u_i$ in space $\boldsymbol{R}^m$ and are related to each other by the magnitudes of the singular values. The singular values capture the dominant associative relationships in the original matrix **A**. The linear combinations (in terms of singular values) of the vectors in **U** and **V** describe the variables, parameters and functions uniquely as linearly independent vectors in an *r*-dimensional space. This description takes into account how each of them is “implicitly” related to all the others whether or not they were explicitly related in **A**. Each variable/parameter or function vector can now be “plotted” in an *r*-dimensional space as a single point. **US** and $\mathbf{SV}^T$ are special linear combinations because the $s_i$ capture the dominant patterns in the data in decreasing order of magnitude. One entry change in the occurrence matrix brings about changes in all components. The dimensionality reduction on the decomposed matrices, **U**, **S** and **V** is done to produce a linear least square truncated approximation of **A**. From the **U**, **S** and **V** matrices first largest *k* singular values in S are retained as $A' = \mathbf{U}\,(m\times k) * \mathbf{S}\,(k\times k) * \mathbf{V}^T\,(k\times n)$, it will be an optimal *k*-rank least squares approximation of **A**. Dimensionality reduction implies that, instead of using *r* dimensions or linear combinations of abstract vectors to describe a variable, parameter or function, a lower number *k* is used as a least squares approximation of the associative relationships between variables, parameters or functions.

## V. ALGORITHMIC STEPS

1. Get a Plant Model
2. Get its state space model
3. Get its occurrence Matrix
4. Decompose the matrix by applying SVD Technique
5. Reduce the Matrix dimension by using PCA
6. Set a Euclidian Norm
7. Get nearest or round off optimized values
8. Fed these values back to get robust stability.

## VI. CONCLUSION

A good controller would be one which achieves significant reduction in disturbances on the plant output, elimination of sensor noise and quick tracking of set points. The objective of the QFT design is to synthesize controller and pre-filter such that the various stability and performance specifications are met. QFT is based on the idea of converting closed loop specifications into specifications on the open loop. The paper concludes that the concept of Quantitative Feedback Theory (QFT) based on Interval Constraint Satisfaction Problem which solved by Interval Constraint Satisfaction Technique involving singular value decomposition and dimension reduction by principal component analysis can effectively solve the plant stability problem.

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