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The Study of Effects of Rotation Modulation on Double Diffusive Convection in Oldroyd-B Liquids

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Abstract: The effect of rotation modulation is analysed in Oldroyd-B liquids subjected to double diffusive convection. Both linear and non-linear analysis has been done. A regular perturbation technique has been employed to arrive at the thermal Rayleigh number. The results show that stress relaxation destabilises the system whereas strain retardation parameter and Lewis number stabilises the system. Truncated Fourier series expansion gives a system of Lorentz equations which helps in performing the non-linear analysis. Mean Nusselt and Sherwood numbers are used to quantify the heat and mass transfer respectively. It is observed that Lewis number and strain retardation parameter decreases heat and mass transfer and stress relaxation parameter increases them. It is seen that modulation gives rise to sub-critical motion.

Key words: Double diffusive convection, Oldroyd-B liquids, Rayleigh-Bénard convection, Rotation modulation.

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I. INTRODUCTION

Convection in non-Newtonian liquids has been widely studied due to its application in various fields. Most of the fluids used in industries are non-Newtonian in nature. They show the characteristics of solids and liquids (elastic and viscous respectively). They find uses in diverse areas such as geothermal energy modeling, crystal growth, chemical industry, bioengineering, petroleum industry, solar receivers etc. The constitutive equations that define them include complex differential operators. As they exhibit both elastic and viscous behaviors an over stability is observed which is not seen in the case of Newtonian fluids. Oldroyd-B liquid is one such fluid which is viscoelastic in nature. This has led to many researchers studying Rayleigh–Bénardconvection in a viscoelastic fluid heated from below(Vest and Apaci[1], Sokolov and Tanner[2] , Green[3], Siddheshwar*et. al.*[4].

Li and Khayat[5] studied stationary and oscillatory convection in viscoelastic fluids which gave information about the formation of pattern in these fluids. An oscillatory convection was set up in a thin layer of fluid heated from below. The study of Rayleigh– Bénard convection in a viscoelastic fluid by Siddheshwar and Krishna[6]established that the ratio of strain retardation parameter to the stress relaxation parameter should be less than one for convection to set in.

Zhong *et. al.*[7]investigated the vortices arising in a rotating Rayleigh-Benard system and found that vortices are formed at higher values of the Rayleigh number. Also, time-dependent heat transport begins for Rayleigh numbers at or slightly above the first appearance of vortices. The effect of rotational speed modulation on heat transport in a fluid layer with temperature dependent viscosity and internal heat source was studied by Bhaduria and Kiran[8].They found that the effect of modulated rotation speed is found to have a stabilizing effect for different values of modulation frequency. The destabilizing and stabilizing effects of rotation on Oldroyd-Bliquids were found by Sharma[9]. In spite of these studies not many literaturesexists on non-linear convection in Oldroyd-B liquids.

The heat transport in Rayleigh - Bénard convection is caused due to a temperature difference between the two walls and the resulting temperature gradient. This is not the case in most practical cases. Convection may arise due to multiple factors. Double diffusive convection arises when there are two different gradients, mostly temperature and concentration (Charrier-Mojtabi[10]). Their varying diffusivities make the stability patterns unpredictable. The different rates of diffusion lead to the formation of salt fingers or oscillations in the fluid layer. Malashetty and Swamy[11]found that there is an internal competition between the processes of thermal diffusion, solute diffusion and viscoelasticity that causes the convection to set in through oscillatory mode rather than stationary. The most common example of this type of diffusion is found in the ocean.

Stommel[12]noticed that with the decrease in solute quantity causes a large amount of potential energy. Stern[13],[14] observed that if there are two diffusing components in a system, then the behavior of the system depends on whether the solute component is stabilizing or destabilizing.
Double diffusive magneto convection in viscoelastic fluids was investigated by Narayana *et. al.*[15]. A stability analysis of chaotic and oscillatory magneto-convection in binary viscoelastic fluids with gravity modulation was done by Bhadauria and Kiran[16]. A Ginzburg–Landau model was adopted to find the effects of the parameters. It was found that gravity modulation can be used to either advance or delay convection by varying its frequency. Siddheshwar*et. al.*[17] analyzed the heat transport by stationary magneto-convection in Newtonian liquids under g-gitter and temperature modulation and obtained similar results.

In this paper we use linear and non-linear stability analysis to investigate the effects ofrotation modulation on double diffusive convection in Oldroyd-B liquid.

II. MATHEMATICAL FORMULATION

Consider a layer of Oldroyd-B liquid held between two parallel plates at z = 0 and z = d. The two plates are maintained at two different temperatures with the difference in temperatures and solute concentrations ΔT and ΔS respectively. This causes variable heating of the fluid particles and hence, variable movements. That is, a temperature gradient arises and in turn gives rise to convection. The fluid density is assumed to be a linear function of temperature, T, and solute concentration, S. A Cartesian co-ordinate system is taken with origin in the lower boundary and z-axis vertically upwards (fig 1).

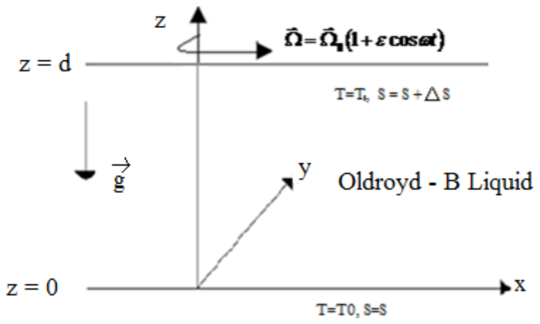

Fig. 1: Physical Configuration

Nomenclature	
d	thickness of the liquid
k	dimensionless wave number
pr	Prandtl number
q	velocity
Ra	thermal Rayleigh number
Rs	solutal Rayleigh number
t	time
T	temperature
T_o	constant temperature of the upper boundary
T_R	reference temperature
Le	Lewis number
Ta	Taylor number
Greek symbols	
α	thermal expansion coefficient
ε	amplitude of modulation
κ	thermal diffusivity
κ_s	solutal diffusivity
λ_1	stress relaxation coefficient
λ_2	strain retardation coefficient
Λ	elasticity ratio(λ_2/λ_1)
μ	viscosity
ω	frequency of modulation
ρ	density
ρ_0	reference density

Thus, the governing equationsof the problem are:
Continuity Equation:

$$\nabla.\vec{q} = 0 \tag{1}$$

Conservation of momentum:

$$\rho_0\left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} + 2(\vec{\Omega}\times\vec{q})\right] = -\nabla p + \rho\vec{g} + \nabla\tau' \tag{2}$$

Rheological Equation:

$$\left(1+\lambda_1\frac{\partial \vec{q}}{\partial t} + \lambda_1(\vec{q}.\nabla)\right)\tau' = \left(1+\lambda_2\frac{\partial \vec{q}}{\partial t} + \lambda_2(\vec{q}.\nabla)\right)\left(\nabla\vec{q} + \nabla\vec{q}^{tr}\right) \tag{3}$$

Operating $\left(1+\lambda_1\frac{\partial}{\partial t} + \lambda_1(\vec{q}.\nabla)\right)$ on *eq.* (2) and using *eq.*(3), the rheological equation becomes,

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\rho_0\frac{\partial \vec{q}}{\partial t} + \nabla p - \rho\vec{g}\right] = \mu\left(1+\lambda_2\frac{\partial}{\partial t}\right)\nabla^2\vec{q} \tag{4}$$

We neglect the convective acceleration term in $(\vec{q}.\nabla)\vec{q}$ comparison with the heat advection term since $(\vec{q}.\nabla)T$ we assume that thermally induced instabilities dominate hydrodynamic instabilities. This also means that we are considering small scale convective motions.

Conservation of Energy:

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \kappa\nabla^2 T \tag{5}$$

Conservation of Species:

$$\frac{\partial S}{\partial t} + (\vec{q}.\nabla)S = \kappa\nabla^2 S \tag{6}$$

Equation of State:

$$\rho = \rho_0\left[1-\alpha_t(T-T_0) + \alpha_s(S-S_0)\right] \tag{7}$$

III. BASIC STATE

Initially we consider the basic state wherein the fluid is at rest. In this situation the parameters are given by the following:

$$\vec{q} = \vec{q}_b = 0,\; p = p_b(z), \rho = \rho_b(z), S = S_b(z), T = T_b(z) \tag{8}$$

The temperature T_b, Concentration, S_b, pressure p_b and density ρ_b satisfy

$$\frac{dp_b}{dz} + \rho_b\vec{g} = 0 \tag{9}$$

$$\frac{\partial T_b}{\partial t} = \kappa\frac{\partial^2 T_b}{\partial z^2} \tag{10}$$

$$\frac{\partial S_b}{\partial t} = \kappa_s\frac{\partial^2 S_b}{\partial z^2} \tag{11}$$

$$\rho = \rho_0\left[1-\alpha_t(T-T_0) + \alpha_s(S-S_0)\right] \tag{12}$$

The basic state solution for temperature and concentration are

$$S_b = S_0\left(1-\frac{z}{d}\right)$$

$$T_b = T_0\left(1-\frac{z}{d}\right), \tag{13}$$

IV. STABILITY ANALYSIS

In order to study the stability of the system the infinitesimal perturbations on the quiescent basic state is superimposed. We assume that the basic state is slightly perturbed as follows, where the prime quantities represent infinitesimal perturbations. Therefore,

$$\vec{q} = \vec{q}', p = p_b + p', \rho = \rho_b + \rho', S = S_b + S', T = T_b + T' \tag{14}$$

Substituting the above *eq.* (14) in the governing equations and using the basic state solution wherever necessary, the following perturbation equations are obtained

$$\frac{\partial T'}{\partial t} + w'\frac{\partial T'}{\partial z} + w'\frac{\partial T_b}{\partial z} = \kappa\nabla^2 T', \tag{15}$$

$$\frac{\partial S'}{\partial t} + w'\frac{\partial S'}{\partial z} + w'\frac{\partial S_b}{\partial z} = \kappa_s\nabla^2 S', \tag{26}$$

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\rho_0\frac{\partial(\nabla^2\psi)}{\partial t} - 2\rho_0\vec{\Omega}(t)\frac{\partial V'}{\partial z} - \alpha_t\rho_0 g\nabla_1^2 T' + \alpha_s\rho_0 g\nabla_1^2 S'\right] = \mu\left(1+\lambda_2\frac{\partial}{\partial t}\right)\nabla^4\psi, \tag{17}$$

We consider only two-dimensional disturbances and restrict to xz plane, that is, all terms are independent of y.

Writing the y-component of *eq.* (17) where all the variations with respect to y are assumed to vanish, we get

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left[\rho_0\frac{\partial V}{\partial t} + 2\rho_0\vec{\Omega}(t)\frac{\partial \psi}{\partial z}\right] = \mu\left(1+\lambda_2\frac{\partial}{\partial t}\right)\nabla^2 V. \tag{18}$$

where $V = \nabla\times\vec{q}$

Using $\nabla_1^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right),$ the following dimensionless variables

$$w^* = \frac{w'}{\kappa/d},\ t^* = \frac{t}{d^2/\kappa}, T^* = \frac{T'}{\Delta T}, S^* = \frac{S}{\Delta S}, \nabla^* = d\nabla,\ (x^*, y^*, z^*) = \left(\frac{x}{d},\frac{y}{d},\frac{z}{d}\right) \tag{19}$$

The stream function ψ is introduced where $u = \frac{\partial\psi}{\partial z}, w = -\frac{\partial\psi}{\partial x}$ and all the terms are independent of y. This results in the following non-dimensional equations:

$$\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left[\frac{1}{\Pr}\frac{\partial(\nabla^2\psi)}{\partial t} - Ta^{1/2}(1+\varepsilon\cos\omega t)\frac{\partial V}{\partial z} + Ra\nabla_1^2 T - Rs\nabla_1^2 S\right] = \left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^4\psi, \tag{20}$$

$$\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left[\frac{1}{\Pr}\frac{\partial V}{\partial t} - Ta^{1/2}(1+\varepsilon\cos\omega t)\frac{\partial\psi}{\partial z}\right] = \left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^2 V, \tag{21}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)T + (\vec{q}.\nabla)T = -\frac{\partial\psi}{\partial x}, \tag{22}$$

$$\left(\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2\right)S + (\vec{q}.\nabla)S = -\frac{\partial\psi}{\partial x}. \tag{23}$$

The non-dimensional parameters that observed in *eqns.* (20) - (23) are stress relaxation parameter, strain retardation parameter, Lewis number, thermal Rayleigh number, solutal Rayleigh number and Prandtl number defined as given in *eq.* (24).

$$\left.\Lambda_1=\frac{\lambda_1\kappa}{d^2},\ \ \Lambda_2=\frac{\lambda_2\kappa}{d^2},\ Le=\frac{\kappa}{\kappa_s},\quad \Pr=\frac{\mu}{\rho_0\kappa},\ Ta=\left(\frac{2\Omega_0\rho_0 d^2}{\mu}\right)^2,\ Ra=\frac{\alpha_t\rho_0 g\Delta T d^3}{\mu\kappa},\ \ Rs=\frac{\alpha_s\rho_0 g\Delta S d^3}{\mu\kappa}\right\} \tag{24}$$

V. LINEAR STABILITY ANALYSIS

In this section, we neglect the Jacobians in *eqns.* (20) to (23) in order to discuss the linear stability analysis considering over-stable and marginal states. The linear equivalent of these equations are:

$$\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left[\frac{1}{\Pr}\frac{\partial(\nabla^2\psi)}{\partial t}-Ta^{1/2}(1+\varepsilon\cos\omega t)\frac{\partial V}{\partial z}+Ra\nabla_1^2T-Rs\nabla_1^2S\right]=\left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^4\psi, \tag{25}$$

$$\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left[\frac{1}{\Pr}\frac{\partial V}{\partial t}-Ta^{1/2}(1+\varepsilon\cos\omega t)\frac{\partial\psi}{\partial z}\right]=\left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^2 V, \tag{26}$$

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)T=\frac{\partial\psi}{\partial x}, \tag{27}$$

$$\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)S=-\frac{\partial\psi}{\partial x}. \tag{28}$$

Eliminating T, S, and V between equations (25) - (28), an equation for ψ is obtained in the form

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)\left[\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\frac{1}{pr}\frac{\partial}{\partial t}-\left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^2\right]^2\nabla^2\psi=$$

$$-Ta\left(\frac{\partial}{\partial t}-\nabla^2\right)\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)\left(1+\Lambda_1\frac{\partial}{\partial t}\right)^2(1+\varepsilon\cos\omega t)^2\frac{\partial^2\psi}{\partial z^2}$$

$$+\left[\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\frac{1}{pr}\frac{\partial}{\partial t}-\left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^2\right]\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left\{\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)Ra\frac{\partial^2\psi}{\partial x^2}-\left(\frac{\partial}{\partial t}-\nabla^2\right)Rs\frac{\partial^2\psi}{\partial x^2}\right\}$$

(29)

VI. PERTURBATION PROCEDURE

The Rayleigh number is obtained using a regular perturbation technique where the stream function and thermal Rayleigh number are expanded as given in *eq.* (30)

$$\psi=\psi_0+\varepsilon\psi_1+\varepsilon^2\psi_2+\ldots.$$

$$Ra=Ra_0+\varepsilon Ra_1+\varepsilon^2 Ra_2+\ldots. \tag{30}$$

This expression is substituted into *eq.* (29) and the coefficients of different powers of ε are equated and the following system of equations is obtained.

$$L\psi_0=0, \tag{31}$$

$$L\psi_1=-2Ta\left(\frac{\partial}{\partial t}-\nabla^2\right)\left(\frac{\partial}{\partial t}-\frac{\nabla^2}{Le}\right)\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\cos\omega t\frac{\partial^2\psi_0}{\partial z^2}$$

$$+\left[\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\frac{1}{\Pr}\frac{\partial}{\partial t}-\left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^2\right]\left(\frac{\partial}{\partial t}-\frac{\nabla^2}{Le}\right)\left(1+\Lambda_1\frac{\partial}{\partial t}\right)Ra\frac{\partial^2\psi}{\partial x^2} \tag{32}$$

$$L\psi_2 = -Ta\left(\frac{\partial}{\partial t}-\nabla^2\right)\left(\frac{\partial}{\partial t}-\frac{\nabla^2}{Le}\right)\left(1+\Lambda_1\frac{\partial}{\partial t}\right)^2\left[\cos\omega t\frac{\partial^2\psi_1}{\partial z^2}+2\cos\omega t\frac{\partial^2\psi_0}{\partial z^2}\right]$$
$$+\left[\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\frac{1}{\Pr}\frac{\partial}{\partial t}-\left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^2\right]\left(\frac{\partial}{\partial t}-\frac{\nabla^2}{Le}\right)\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left[Ra_0\frac{\partial^2\psi_1}{\partial x^2}+Ra_2\frac{\partial^2\psi_0}{\partial x^2}\right] \tag{33}$$

where

$$L\equiv\left\{\left[\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\frac{1}{pr}\frac{\partial}{\partial t}-\left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^2\right]\nabla^2\right\}^2\left(\frac{\partial}{\partial t}-\nabla^2\right)\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)\nabla^2$$
$$+Ta\left(\frac{\partial}{\partial t}-\nabla^2\right)\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)\left(1+\Lambda_1\frac{\partial}{\partial t}\right)^2\frac{\partial^2}{\partial z^2}+\left[\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\frac{1}{pr}\frac{\partial}{\partial t}-\left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^2\right] \tag{34}$$
$$\nabla^2\left[-\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)Ra_0\frac{\partial^2}{\partial x^2}+\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}-\nabla^2\right)Rs\frac{\partial^2}{\partial x^2}\right].$$

Each ψ_n is required to satisfy the boundary condition $\psi=\nabla^2\psi=\nabla^4\psi=0$ at z = 0, 1.

6.1. Solution to the zeroth order problem

The double diffusive problem of Oldroyd-B liquid with no modulation of temperature is the zeroth order problem. The general solution of *eq.* (31) obtained at o(ε^0) is the one used in the Oldroyd-B liquids convection under uniform temperature modulation. The marginal stable solutions are

$\psi_0 = \sin(\pi\alpha x)\sin(\pi z)$ (35)

with the corresponding eigenvalue, Ra_0, given by

$$Ra_0=\frac{k_1^2}{\pi^2\alpha^2}+\frac{Ta}{\pi^2}+RsLe \tag{36}$$

Where, $k_1^2=\pi^2\left(\alpha^2+1\right)$

6.2. Solution to the first order problem

Substituting *eq.* (35) in *eq.* (32), we get

$$L\psi_1=-2Ta\left(\frac{\partial}{\partial t}-\nabla^2\right)\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)\left(1+\Lambda_1\frac{\partial}{\partial t}\right)^2\cos\omega t\frac{\partial^2\psi_0}{\partial z^2}$$
$$+\left[\left(1+\Lambda_1\frac{\partial}{\partial t}\right)\frac{1}{pr}\frac{\partial}{\partial t}-\left(1+\Lambda_2\frac{\partial}{\partial t}\right)\nabla^2\right]\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)\left(1+\Lambda_1\frac{\partial}{\partial t}\right)Ra_1\frac{\partial^2\psi_0}{\partial x^2} \tag{37}$$

where $L(\omega,n)=Y_1+iY_2,$ (38)

$$Y_1=\frac{\omega^4k_n^2}{\Pr}-\frac{\omega^2k_n^6}{Le\Pr}+\frac{\omega^6k_n^2\Lambda_1}{\Pr}-\frac{\omega^4k_n^6\Lambda_1^2}{Le\Pr}-\left(\frac{2\omega^4k_n^4\Lambda_1}{Le\Pr}\right)\left(\frac{1+\Pr}{\Pr}\right)+\omega^2k_n^6-\frac{k_n^{10}}{Le}+\left(\frac{2\omega^4k_n^4\Lambda_1}{Le\Pr}\right)\left(\frac{1+\Pr}{\Pr}\right)$$
$$+\omega^2k_n^8\Lambda_2\left(\frac{1+Le\Pr}{Le\Pr}\right)-\omega^4k_n^6\Lambda_2^2-\frac{\omega^2K_n^{10}\Lambda_2^2}{Le}+\frac{4\omega^2k_n^6}{\Pr}-\frac{4\omega^4k_n^4\Lambda_2}{\Pr}-\frac{4\omega^4k_n^6\Lambda_1\Lambda_2}{\Pr}+\omega^2n^2\pi^2$$
$$-\left(\frac{k_n^4n^2\pi^2Ta}{Le}\right)(1+\Lambda_1^2)+2\omega^2k_n^2n^2\pi^2\Lambda_1Ta\left(\frac{1+Le}{Le}\right)-\omega^4n^2\pi^2\Lambda_1^2Ta+\left(\frac{\omega^2n^2\pi^2\alpha^2}{\Pr}\right)(Ra_0+Rs)$$
$$+2\Lambda_2\omega^2n^2\pi^2k_n^2Ra_0+\frac{\omega^4n^2\pi^2\alpha^2\Lambda_1\Lambda_2Ra_0}{\Pr}+\frac{\omega^2n^2\pi^2\alpha^2k_n^2Ra_0}{Le\Pr}-\frac{k_n^4n^2\pi^2\alpha^2Ra_0}{Le}$$
$$+\left(\frac{\omega^2n^2\pi^2\alpha^2k_n^2}{\Pr}\right)\left(\frac{1+\Pr}{\Pr}\right)(\Lambda_2Ra_0+\Lambda_1Rs)-n^2\pi^2\alpha^2k_n^4Rs+n^2\pi^2\alpha^2k_n^4\omega^2\Lambda_1\Lambda_2Rs$$

(39)

$$Y_2 = \left(\frac{\omega^3 k_n^4}{\Pr} + \frac{\omega^5 k_n^4 \Lambda_1^2}{\Pr} - \omega^3 k_n^8 \Lambda_2^2 + \omega k_n^2 n^2 \pi^2 Ta - \omega^3 k_n^2 n^2 \pi^2 \Lambda_1^2 Ta \right)\left(\frac{1+Le}{Le} \right) + \frac{\omega^5 k_n^4 \Lambda_1}{\Pr}$$
$$- \frac{2\omega^3 k_n^6 \Lambda_1}{Le\Pr} + \left(\frac{\omega k_n^8}{Le} \right)\left(\frac{1+\Pr}{\Pr} \right) + \omega k_n^8 - 2\omega^3 k_n^6 \Lambda_2 + \frac{\omega k_n^{10}\Lambda_2}{Le} - \frac{2\omega^3 k_n^4}{Le\Pr} + \frac{2\omega^5 k_n^4 \Lambda_1 \Lambda_2}{\Pr} - \frac{2\omega^3 k_n^8 \Lambda_1 \Lambda_2}{Le\Pr}$$
$$- 2\omega^3 n^2 \pi^2 \Lambda_1 Ta + \frac{\omega k_n^4 \Lambda_1 n^2 \pi^2 Ta}{Le} - \left(\frac{\omega^3 n^2 \pi^2 \alpha^2 Ra_0}{\Pr} - \frac{2\omega^3 k_n^6}{\Pr} \right)(\Lambda_2 + \Lambda_1) + -\omega k_n^2 n^2 \pi^2 \alpha^2 Ra_0$$
$$- \omega^3 k_n^2 n^2 \pi^2 \Lambda_2 Ra_0 + \frac{\omega n^2 \pi^2 \alpha^2}{Le\Pr}\left(k_n^2 Ra_0 - \omega^2 k_n^2 \Lambda_1 \Lambda_2 - \omega^3 \Lambda_1^2 Rs \right) + \frac{2\omega k_n^2 \Lambda_2 n^2 \pi^2 \alpha^2 Ra_0}{Le}$$

(40)

Equation (37) is inhomogeneous and contains a resonance term.
The time-independent part of the right hand side of *eq.* (37) is orthogonal to the null operator L and this implies that $Ra_1 = Ra_3 = Ra_5 .... = 0$. This is obtained from the solvability condition. Therefore, *eq.* (37) becomes

$$L\psi_1 = -2Ta\left(\frac{\partial}{\partial t} - \nabla^2 \right)\left(\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2 \right)\left(1 + \Lambda_1 \frac{\partial}{\partial t} \right)^2 \cos\omega t \frac{\partial^2 \psi_0}{\partial z^2} \tag{41}$$

Equation(33) is used to find R_{a2}, the first non-zero correction to R_0. Ra_2 subject to *eq.* (30) gives ψ_1 and using this in *eq.* (33) with ψ_0 given by *eq.* (33) and $Ra_1 = 0$, the Venezian approach (1966) yields Ra_2 in the form

$$Ra_2 = \frac{Le}{2k^2 \pi^2 \alpha^2 L(\omega, n)^2}\left[L_1 L_4 \omega Y_1 - L_1 L_3 \omega Y_2 + L_2 L_3 Y_1 + L_2 L_4 Y_2 \right] \tag{42}$$

where

$$L_1 = \left(2Tak^2 \pi^2 + 2Tak^2 \pi^2 \omega^2 \Lambda_1^2 \right)\left(\frac{1+Le}{Le} \right) - 4Ta\Lambda_1 \omega^2 \pi^2 + \frac{4Tak^4 \pi^2 \Lambda_1}{Le} \tag{43}$$

$$L_2 = -2Ta\omega^2 \pi^2 + \frac{2Tak^4 \pi^2}{Le} - \frac{2Ta\Lambda_1^2 \omega^2 k^4 \pi^2}{Le} + 2Ta\Lambda_1^2 \omega^4 \pi^2 + 4Ta\Lambda_1 \omega^2 k^2 \pi^2 \left(\frac{1+Le}{Le} \right) \tag{44}$$

$$L_3 = 2Ta\omega^2 \pi^2 + 2Ta\omega^4 \pi^2 \Lambda_1^2 + \frac{2k^6 Ta}{Le} \tag{45}$$

$$L_4 = 4Ta\pi^2 \omega^3 \Lambda_1 \tag{46}$$

VII.NON-LINEAR THEORY

A nonlinear analysis is made to know the amount of heat transfer and to find the effect of various viscoelastic parameters on the nature of onset of convection.

$$\left(1 + \Lambda_1 \frac{\partial}{\partial t} \right)\left[\frac{1}{\Pr}\frac{\partial(\nabla^2 \psi)}{\partial t} - Ta^{1/2}(1 + \varepsilon\cos\omega t)\frac{\partial V}{\partial z} + Ra\nabla_1^2 T - Rs\nabla_1^2 S \right] = \left(1 + \Lambda_2 \frac{\partial}{\partial t} \right)\nabla^4 \psi, \tag{47}$$

$$\left(1 + \Lambda_1 \frac{\partial}{\partial t} \right)\left[\frac{1}{\Pr}\frac{\partial V}{\partial t} - Ta^{1/2}(1 + \varepsilon\cos\omega t)\frac{\partial \psi}{\partial z} \right] = \left(1 + \Lambda_2 \frac{\partial}{\partial t} \right)\nabla^2 V, \tag{48}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2 \right)T + (\vec{q}.\nabla)T = -\frac{\partial \psi}{\partial x}, \tag{49}$$

$$\left(\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2\right)S+(\vec{q}.\nabla)S=-\frac{\partial\psi}{\partial x}. \tag{50}$$

The solution to *eqns.* (47)– (50) may be represented by an infinite Fourier series, with the amplitudes depending on time alone. Only one term in the Fourier representation for the stream function may be retained with two terms in the temperature expressions to retain some nonlinearity (Siddheshwar*et al*. (2013))
Equations (47) and (48) are second order equations which is decomposed into two first order equations:

$$\frac{1}{\Pr}\frac{\partial}{\partial t}(\nabla^2\psi)=-Ra\frac{\partial\theta}{\partial x}+Rs\frac{\partial\phi}{\partial x}+\sqrt{Ta}(1+\varepsilon\cos\omega t)\frac{\partial V}{\partial z}+\Lambda\nabla^4\psi+M, \tag{51}$$

$$\Lambda_1\frac{\partial M}{\partial t}=-M+(1-\Lambda)\nabla^4\psi, \tag{52}$$

$$\frac{1}{\Pr}\frac{\partial V}{\partial t}=-\sqrt{Ta}(1+\varepsilon\cos\omega t)\frac{\partial\psi}{\partial z}+\Lambda\nabla^2V+N, \tag{53}$$

$$\Lambda_1\frac{\partial N}{\partial t}=-N+(1-\Lambda)\nabla^2V, \tag{54}$$

$$\Lambda=\frac{\Lambda_2}{\Lambda_1}. \tag{55}$$

The solution to *eqns.* (49) – (54) may be represented by an infinite Fourier series with time-dependent amplitudes. As per Lorenz-Saltzman(1963) only one term for the stream function may be retained with two terms in temperature and concentration expressions to retain some nonlinearity. The work done by Chen and Price (2006)shows the relation between Rayleigh- Bénard convection and Lorenz system. The stream function, the temperature distribution, concentration distribution and M are represented as follows:

$$\psi=A(t)\sin(\pi\alpha x)\sin(\pi z), \tag{56}$$

$$\theta=B(t)\cos(\pi\alpha x)\sin(\pi z)+C(t)\sin(2\pi z), \tag{57}$$

$$\phi=E(t)\cos(\pi\alpha x)\sin(\pi z)+F(t)\sin(2\pi z), \tag{58}$$

$$V=G(t)\sin(\pi\alpha x)\cos(\pi z)+H(t)\sin(2\pi\alpha x) \tag{59}$$

$$M=J(t)\sin(\pi\alpha x)\sin(\pi z) \tag{60}$$

$$N=P(t)\sin(\pi\alpha x)\cos(\pi z) \tag{61}$$

The above form of M and N are determined by the form of ψ and V respectively. These forms were taken due to the fact that the mode *sin(παx)sin(πz)* always co-exist with the mode *cos(παx)sin(πz)* and they play similar roles in the description of convection The terms *C(t)sin(2πz)* and *F(t)sin(2πz)* represent modifications to temperature and concentration fields by a small scale convective motions.
Projecting *eqns.* (49) -(54) onto the modes (56) –(61) we get the following system:

$$\dot{A}(t)=\frac{-Ra\Pr\pi\alpha}{k^2}B(t)+\frac{Rs\Pr\pi\alpha}{k^2}E(t)+\frac{\pi\Pr\sqrt{Ta}(1+\varepsilon\cos\omega t)}{k^2}G(t)-\Lambda\Pr k^2A(t)-\frac{\Pr}{k^2}J(t), \tag{62}$$

$$\dot{B}(t)=-\pi\alpha A(t)-k^2B(t), \tag{63}$$

$$\dot{C}(t)=\frac{\pi^2\alpha}{2}A(t)B(t)-4\pi^2C(t), \tag{64}$$

$$\dot{E}(t)=-\pi\alpha A(t)-\frac{k^2}{Le}E(t), \tag{65}$$

$$\dot{F}(t)=\frac{\pi^2\alpha}{2}A(t)E(t)-\frac{4\pi^2}{Le}F(t), \tag{66}$$

$$\dot{G}(t)=-\Lambda k^2\Pr G(t)-\pi\Pr\sqrt{Ta}(1+\varepsilon\cos\omega t)A(t)+\Pr P(t) \tag{67}$$

$$\dot{H}(t)=-4\pi^2\alpha^2\Pr H(t)+\Pr P(t) \tag{68}$$

$$\dot{J}(t) = -\frac{1}{\Lambda_1}J(t) + \frac{(1-\Lambda)}{\Lambda_1}k^4 A(t) \tag{69}$$

$$\dot{P}(t) = -\frac{1}{\Lambda_1}P(t) + \frac{(1-\Lambda)}{\Lambda_1}k^2 G(t) \tag{70}$$

Equations (62) -(70) form a generalized Lorenz model.

VIII. HEAT TRANSPORT

In the study of convection problems, the determination of heat transport across the fluid layer is important. This is because; the onset of convection as Rayleigh number is increased is more readily detected by its effect on the heat transfer. In the basic state, the heat transfer is by conduction alone.

If H_T is the rate of heat transfer / unit area, then $H_T = -\chi \left\langle \frac{\partial T_{total}}{\partial z} \right\rangle_{z=0}$, (71)

where the angular bracket corresponds to a horizontal average and $T_{total} = \left[T_0 - \frac{\Delta T}{d} z \right] + T(x,z,t).$

The first term of the RHS of this equation is the temperature distribution of conduction state prevalent before convection sets in. The second term on the RHS represents the convective contribution to heat transport.

The Nusselt number *Nu* is defined by $Nu = \frac{H_T}{\kappa \Delta T / d}.$ (72)

Alternately, *Nu* may be directly defined in terms of the non-dimensional quantities as follows:

$$Nu = \frac{\left[\frac{k_c}{2\pi} \int_0^{2\pi/k_c} (1 - z + T)_z \, dx \right]}{\left[\frac{k_c}{2\pi} \int_0^{2\pi/k_c} (1 - z)_z \, dx \right]} \tag{73}$$

Similarly, the mass transfer is quantified using Sherwood number.

$$Sh = \left[\frac{\frac{k_c}{2\pi} \int_0^{2\pi/k_c} (1 - z + S)_z dx}{\frac{k_c}{2\pi} \int_0^{2\pi/k_c} (1 - z)_z dx} \right] \tag{74}$$

On simplifying, we get the following expressions for Nusselt and Sherwood numbers:

$$Nu = 1 - 2\pi C,$$
$$Sh = 1 - 2\pi F \tag{75}$$

The amplitudes can be determined from the generalized Lorenz model given in *eqns.* (62) – (70). It can be obtained by solving the system using Runge-Kutta-Fehlberg45 method that uses an adaptive step-size.

IX. RESULTS AND DISCUSSIONS

The problem addresses the linear and non-linear effects of rotation modulation on double diffusive convection in Oldroyd-B liquids for the relevant parameters. The linear stability problem is solved based on perturbation method. The parameters of the system are Le, Ra, Rs, Pr, Ta, $\Lambda_1, \Lambda_2, \delta, \omega$ which influence the convective heat and mass transfer. The first six parameters are related to the fluid layer and the remaining are the external measures of controlling the convection. The influence of various parameters on the correction Rayleigh number Ra_{2c} as a function of the frequency of modulation ω are discussed.

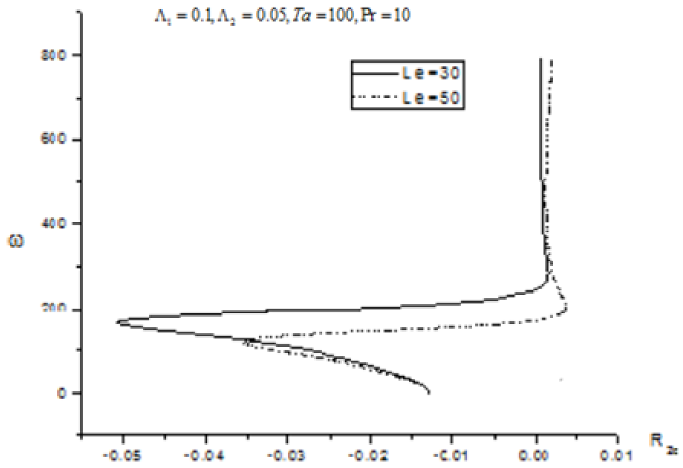

Fig. 2: Graph of Ra_{2c} vs ω for different values of Le

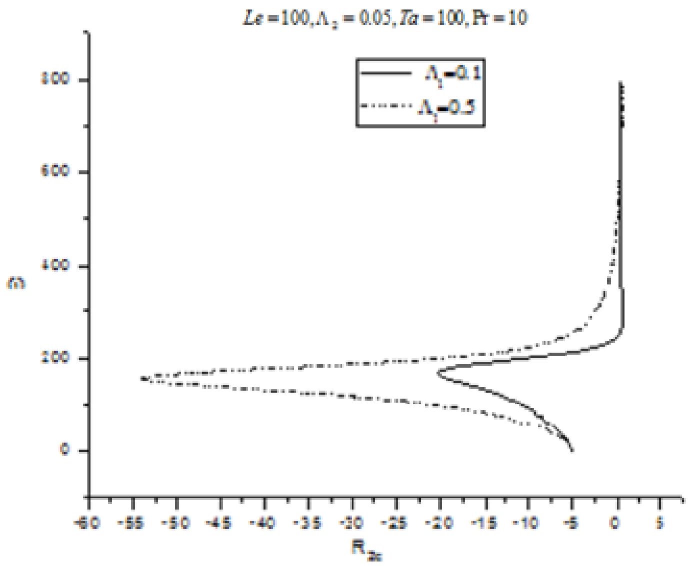

Fig. 3: Graph of R_{2c} versus ω for different values of Λ_1

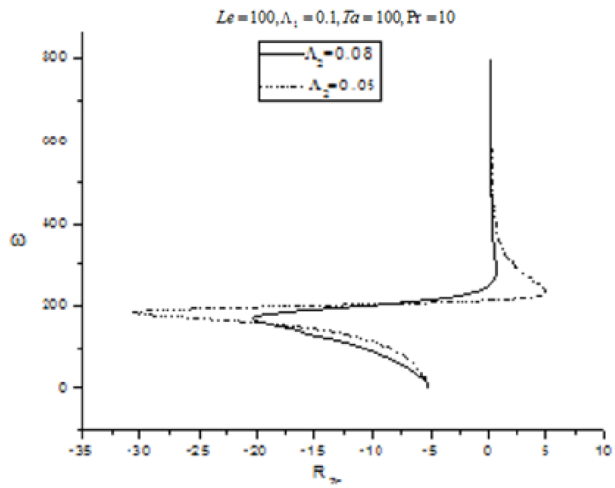

Fig. 4: Graph of R_{2c} versus ω for different values of Λ_2

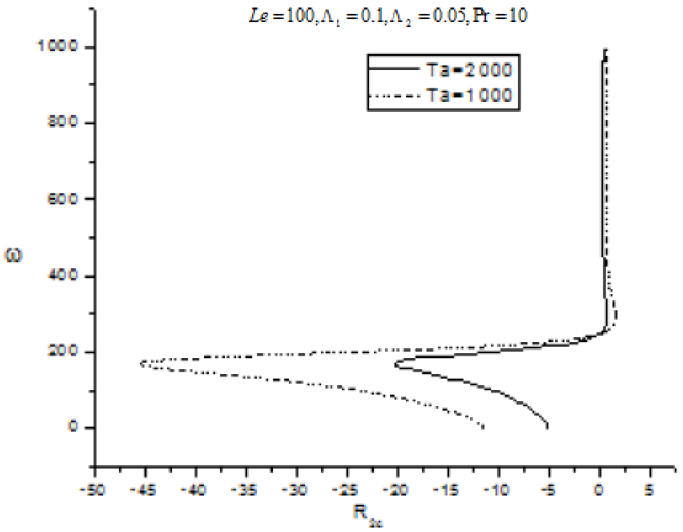

Fig. 5: Graph of R_{2c} versus ω for different values of Ta

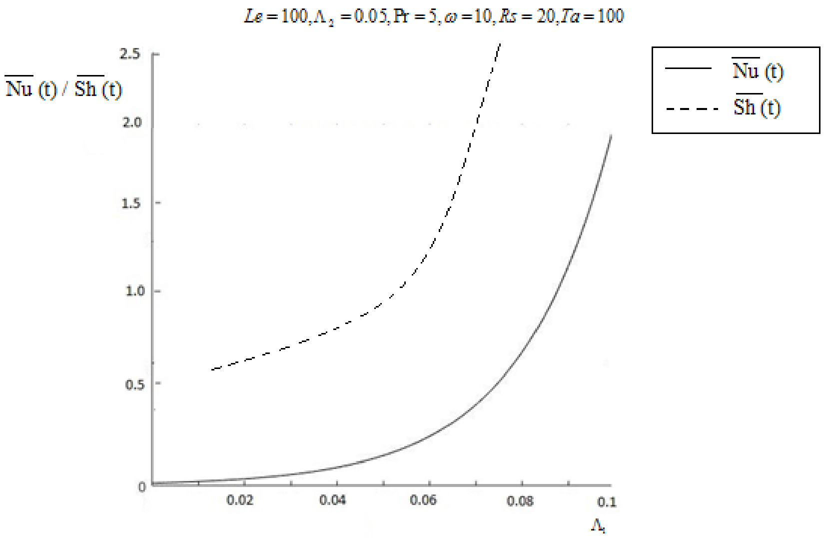

Fig. 6: Graph of $\overline{Nu}$ (t) and $\overline{Sh}$ (t) vs. Λ_1

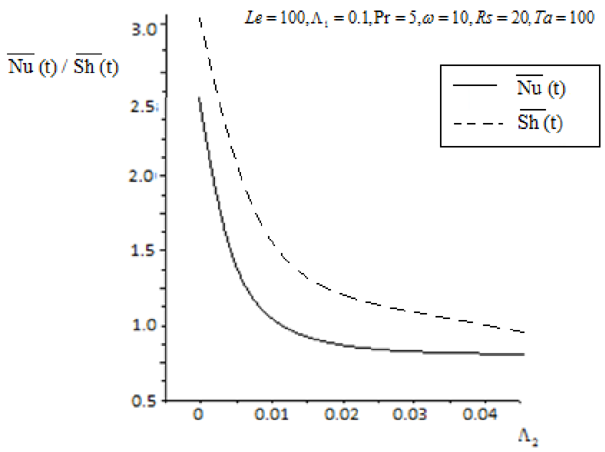

Fig. 7: Graph of $\overline{Nu}$ (t) and $\overline{Sh}$ (t) vs. Λ_2

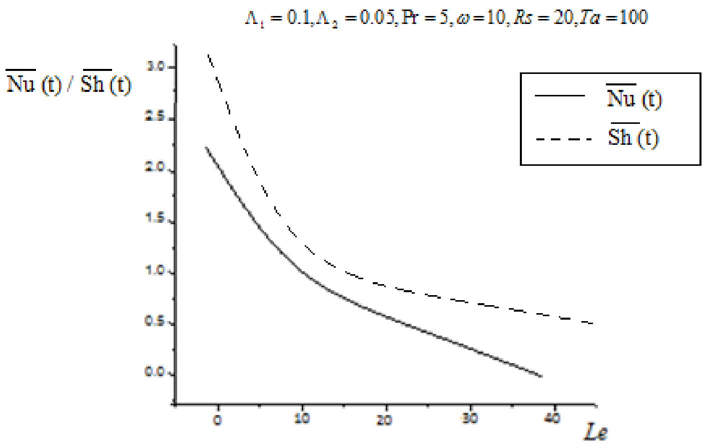

Fig. 8: Graph of $\overline{Nu}$ (t) and $\overline{Sh}$ (t) vs. Le

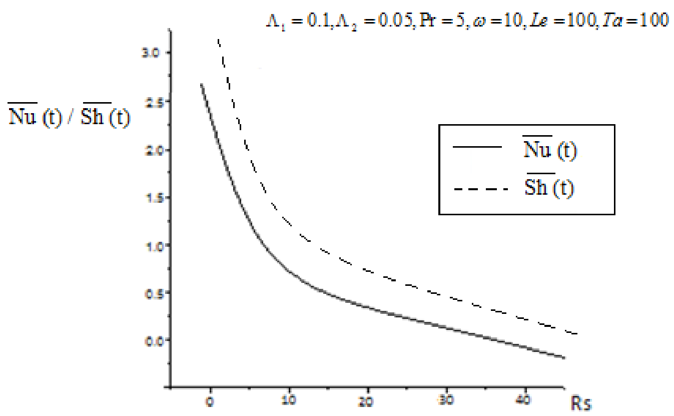

Fig. 9: Graph of $\overline{Nu}$ (t) and $\overline{Sh}$ (t) vs. Rs

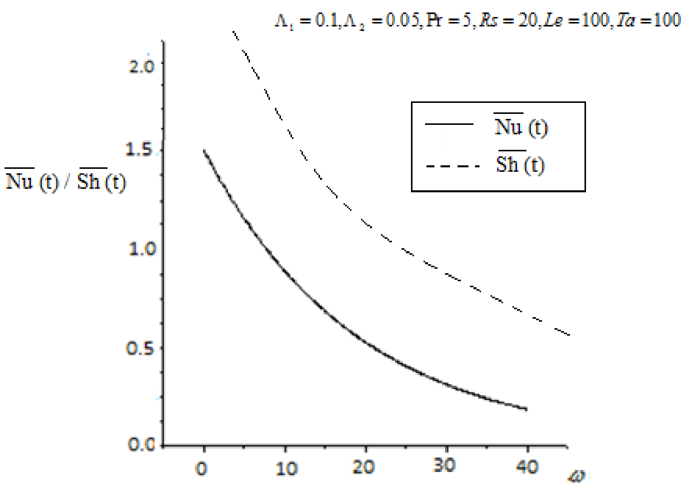

Fig. 10: Graph of $\overline{Nu}$ (t) and $\overline{Sh}$ (t) vs. ω

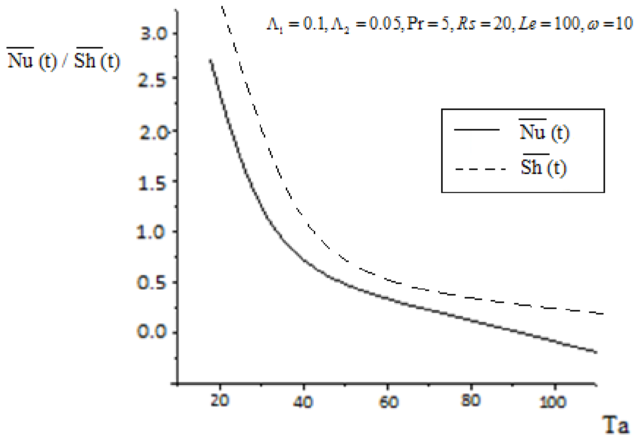

Fig. 11: Graph of $\overline{Nu}$ (t) and $\overline{Sh}$ (t) vs. Ta

Figure 2 is the plot of Ra_{2c}versus ω for different values of Lewis number, Le and the fixed values of Λ_1, Λ_2, Pr. It is observed that Ra_{2c} increases with Le for moderate values of ω. The profile consists of the steady straight line section and a parabolic profile which oscillates in time. As ω increases the parabolic part of the profile becomes more and more significant. It is known that the parabolic profile is subject to finite amplitude instabilities. Unlike Pr, Le influence Ra_0. We find that Ra_0 increases with increases in Le.

Figure 3 is the plot of Ra_{2c} versus ω for different values of Λ_1 and fixed values of Le, Λ_2 and Pr It is found that as Λ_1 increases Ra_{2c} decreases but remains negative. A subcritical motion is observed. Figure 4 establishes the fact that the effect of Λ_2 is opposite to that of Λ_1 when Le, Λ_1 and Pr are fixed. Fig. 5 shows the variation in Ra_{2c} with the values of the Taylor's number. It can be seen that increased values of Ta results in increase in Ra_{2c}. Therefore, it causes a stabilizing effect. The values of Ta taken are larger here since the variation is more evident for such values.

Figures 6 – 11 are the graphs of $\overline{Nu}$ and $\overline{Sh}$ versus time. Heat transfer is quantified using Nusselt number and mass transfer using Sherwood number. Figures 6 and 7 are graphs of $\overline{Nu}$ and $\overline{Sh}$ for different values of Λ_1 and Λ_2. These two parameters have opposing effects on the system. While Λ_1 increases the amount of heat and mass transfers, Λ_2 decreases it. This is as observed in figs 3-4, where Λ_1 destabilizes whereas Λ_2 stabilizes the system. From fig. 8 it is clear that when Le increases the mean Nusselt and Sherwood numbers decrease. This result is in accordance with fig. 2 wherein we find the Le stabilizes the system, therefore reducing heat transfer. Figure 9 shows the variation in $\overline{Nu}$ and $\overline{Sh}$ when the solutal Rayleigh number, Rs, increases. Rs is the ratio of solutal expansion to viscous forces. It is seen to cause a decrease in $\overline{Nu}$ and $\overline{Sh}$. Figures 10 and 11 shows that both the frequency of modulation, ω, and the Taylor number, Ta, reduces the heat and mass transfer, in effect stabilizing the system.

The results of rotation modulation on double diffusive convection in Newtonian fluids and Maxwell fluid can be obtained as special cases of this study (see table 1).

Table 1: Values of correction Rayleigh number, R_{2c}, Nusselt number, Nu, and Sherwood number, Sh, for Le = 100, Pr = 10, Rs = 20, ω=10, ε=0.1.

Newtonian fluid $\Lambda_1 = \Lambda_2$				Maxwell fluid $\Lambda_2 = 0$				Oldroyd-B fluid $\Lambda_1 \neq \Lambda_2$				
Λ_1	R_{2c}	Nu	Sh	Λ_1	R_{2c}	Nu	Sh	Λ_1	Λ_2	R_{2c}	Nu	Sh
0.1	657.0	1.3114	1.7754	0.1	148.4	1.8234	2.5960	0.1	0.05	180.39	1.6218	1.9963
0.5	643.25	1.6531	2.1342	0.5	145.32	2.2763	3.0015	0.1	0.08	172.53	1.8534	2.5531
0.8	630.59	1.9672	2.3641	0.8	137.31	2.6874	3.3419	0.1	0.09	168.38	2.2116	3.0167

From the above table we conclude the following:

1. $R_{2c}^{Maxwell\ fluid} < R_{2c}^{Oldroyd-B\ fluid} < R_{2c}^{Newtonian\ fluid}$
2. $Nu^{Maxwell\ fluid} > Nu^{Oldroyd-B\ fluid} > Nu^{Newtonian\ fluid}$
3. $Sh^{Maxwell\ fluid} > Sh^{Oldroyd-B\ fluid} > Sh^{Newtonian\ fluid}$

These results match with those obtained by Siddheshwar*et. al.*[17] and that of Vanishree and Anjana[18].

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