IOSR Journal of Engineering (IOSRJEN) **www.iosrjen.org**
ISSN (e): 2250-3021, ISSN (p): 2278-8719
Vol. 08, Issue 5 (May. 2018), ||VIII|| PP 01-10

Flow and Heat Transfer of MHD Viscoelastic Fluid over a Stretching Sheet with Viscous Dissipation

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Abstract: An analysis is carried out to study the flow and heat transfer characteristics of MHD visco-elastic fluid flow over a stretching sheet with viscous dissipation. The governing partial differential equations are converted into ordinary differential equations by a similarity transformation. These equations are solved by a numerical method, Quasilinearization technique. The effects of various physical parameters such as Prandtl number, magnetic field, suction, visco-elasticity and viscous dissipation on flow and heat transfer are evaluated numerically and analyzed through graphs and tables.

Key words:visco-elastic, stretching sheet, Magneto Hydrodynamic, boundary layer, heat transfer, viscous dissipation

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Date of Submission: 05-05-2018 Date of acceptance: 21-05-2018

I. INTRODUCTION

The most important development in Fluid Mechanics during the 20th century was the concept of boundary layer flow introduced by Prandtl in [1]. In 1961, Sakiadis [2] initiated the study of boundary layer flow over a continuous solid surface moving with constant speed. Crane [3] considered the laminar boundary layer flow of a Newtonian fluid caused by a flat elastic sheet whose velocity varies linearly with the distance from the fixed point of the sheet. Rajagopal et al. [4] and Chang [5] presented an analysis on flow of viscoelastic fluid over stretching sheet.

The heat transfer analysis due to a continuously moving stretching surface through an ambient fluid has wide range of applications over a broad spectrum of Science and Engineering disciplines, especially in the field of chemical engineering. Heat transfer cases of these studies have been considered by Dandapat and Gupta [6], Vajravelu and Rollins [7], while flow of viscoelastic fluid over a stretching surface under the influence of uniform magnetic field has been investigated by Anderson [8]. Thereafter, a series of studies on heat transfer effects on viscoelastic fluid have been made by many authors [9–15] under different physical situations.

Magneto hydrodynamics (MHD) is a subject that studies the behavior of an electrically conducting fluid in the presence of an electromagnetic field. MHD boundary layers with heat and mass transfer over flat surfaces are found in many engineering and geophysical applications such as geothermal reservoirs, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors. Because of its wide range applications, many researchers tend to apply MHD flow into their problems [16–20].

Motivated by these studies, the study of flow and heat transfer of MHD viscoelastic fluid over a stretching sheet with viscous dissipation and suction on the wall is considered in present paper. The governing partial differential equations are converted into ordinary differential equations by a similarity transformation. These equations are solved by a numerical method, Quasilinearization technique.

This technique is directly applicable to computer aided solution of non-linear two-point boundary value problems in many engineering problems. Results are in good agreement with available literature. The effect of various parameters on fluid flow and heat transfer characteristics are evaluated and numerical results are presented through graphs and tables.

II. MATHEMATICAL FORMULATION:

A laminar steady flow of an incompressible viscoelastic (Walters' liquid B model) fluid over a wall coinciding with the plane $y = 0$ is considered, the flow being confined to $y > 0$.Two equal and opposite forces are applied along the x-axis, so that a sheet is stretched with a velocity proportional to the distance from the origin. The resulting motion of the quiescent fluid is thus caused solely by the moving surface. In deriving the equations, it is assumed that, in addition to the usual boundary layer approximations, the contribution due to normal stress is of the same order of magnitude as the shear stress. The flow satisfies the rheological equation of state derived by Beard and Walters [21].

The basic boundary layer equations for this fluid, in the usual notations, are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-k_0\left\{\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}+v\frac{\partial^3 u}{\partial y^3}\right\}-\frac{\sigma B_0^2 u}{\rho} \qquad (2)$$

$$where\ \ \upsilon=\frac{\mu}{\rho},\ k_0>0$$

where u and $v$are the velocity components along the x and y directions respectively, $\upsilon$is the kinematic viscosity, $k_0$is the co-efficient of elasticity, and $\rho$is the density.
The boundary conditions for the velocity field are given as

$$u=u_w=bx,\ \ v=-v_0\ \ at\ y=0,\ b>0$$

$$u\to 0,\ \frac{\partial u}{\partial x}\to 0\ as\ y\to\infty \qquad (3)$$

The condition $\frac{\partial u}{\partial x}\to 0\ as\ y\to\infty$ is the augmented condition since the flow is in an unbounded domain, which has been discussed by K.R. Rajgopal and Garg [22]. In this case, the flow is caused solely by the stretching of the sheet, since the free stream velocity is zero.

Defining new variables as:

$$u=bxf_\eta(\eta),\ v=-\sqrt{b\upsilon}\,f(\eta),\ \eta=\sqrt{b/\upsilon}\,y \qquad (4)$$

where $f_\eta\ (\eta)$denotes differentiation with respect to η. Clearly u and v defined above satisfy the equation (1), and the equation (2) is transformed as

$$f_\eta^2-ff_{\eta\eta}=f_{\eta\eta\eta}-k_1\left\{2f_\eta f_{\eta\eta\eta}-ff_{\eta\eta\eta\eta}-f_{\eta\eta}^2\right\}-Mnf_\eta \qquad (5)$$

where $k_1=k_0b/\upsilon$is the viscoelastic parameter, $Mn=\sigma B_0^2/\rho b$ is magnetic parameter.
The boundary conditions (3) become

$$f(0)=R,\ f_\eta(0)=1 \qquad (6a)$$

$$f_\eta(\eta)\to 0,\ f_{\eta\eta}(\eta)\to 0\ \ as\ \eta\to\infty \qquad (6b)$$

III. HEAT TRANSFER ANALYSIS

The equation of energy with viscous dissipation (or frictional heating), by usual boundary layer approximations, is given by

$$\rho\,c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \qquad (7)$$

where $k$is the thermal conductivity, $C_p$is the specific heat. The thermal boundary conditions depend on the type of heating process under consideration. Here it is considered as Prescribed Surface Temperature (PST case).

For this circumstance, the boundary conditions are

$$T=T_w[=T_\infty+A\left(\frac{x}{l}\right)^2\ \ \text{at } y=0 \qquad (8)$$

$$T\to T_\infty\qquad as\quad y\to\infty \qquad (9)$$

Where l is the characteristic length.

Using relation (4), equation (7) reduces to

$$\theta''+\Pr f\theta'-2\Pr f'\theta=-\Pr Ec(f'')^2 \qquad (10)$$

where $Ec = bl^2/C_p A$ Eckert number

with boundary conditions

$$\theta(0)=1, \theta(\infty)\to 0 \tag{11}$$

IV. NUMERICAL SOLUTION OF THE PROBLEM
V.

The flow equation (5) coupled with energy equation (10) constitute a set of highly nonlinear differential equations, so obtaining closed form solution for this set is cumbersome and time consuming. Hence quasilinearization method, given by Bellman & Kalaba [23] is used to solve this system.

For convenience equations (5) and (10) are rearranged as

$$f^{iv} = \frac{1}{k_1 f}\left[(f')^2 - ff'' - f''' + Mnf' + 2k_1 f'f''' - k_1(f'')^2\right] \tag{12}$$

$$\theta'' = -\Pr f\theta' + 2\Pr f'\theta - \Pr Ec(f'')^2 \tag{13}$$

In order to implement the quasilinearization method, the equations (12) and (13) are converted to a system of first order differential equations by substituting

$$(f, f', f'', f''', \theta, \theta') = (x_1, x_2, x_3, x_4, x_5, x_6)$$

then equations (12) and (13) reduce to:

$$\begin{aligned}
\frac{dx_1}{d\eta} &= x_2\\
\frac{dx_2}{d\eta} &= x_3\\
\frac{dx_3}{d\eta} &= x_4\\
\frac{dx_4}{d\eta} &= \frac{1}{k_1 x_1}\left\{x_2^2 - x_1x_3 - x_4 + Mnx_2 + 2k_1x_2x_4 - k_1x_3^2\right\}\\
\frac{dx_5}{d\eta} &= x_6\\
\frac{dx_6}{d\eta} &= -\Pr x_1x_6 + 2\Pr x_2x_5 - \Pr Ecx_3^2
\end{aligned} \tag{14}$$

Using Quasilinearization technique, the system (14) can be linearized as

$$\frac{dx_1^{r+1}}{d\eta} = x_2^{r+1}$$

$$\frac{dx_2^{r+1}}{d\eta} = x^{r+1}$$

$$\frac{dx_3^{r+1}}{d\eta} = x_4^{r+1}$$

$$\frac{dx_4^{r+1}}{d\eta} = \left(\frac{-1}{k_1 (x_1^r)^2} \left((x_2^r)^2 - x_4^r + Mnx_2^r + 2k_1 x_2^r x_4^r - k_1 (x_3^r)^2 \right) \right) x_1^{r+1}$$
$$+ \left(\frac{1}{k_1 x_1^r} \left(2x_2^r + 2k_1 x_4^r + Mn \right) \right) x_2^{r+1}$$
$$+ \left(\frac{1}{k_1 x_1^r} \left(-x_1^r - 2k_1 x_3^r \right) \right) x_3^{r+1}$$
$$+ \left(\frac{1}{k_1 x_1^r} \left(-1 + 2k_1 x_2^r \right) \right) x_4^{r+1} + \left(\frac{-x_4^r + Mnx_2^r}{k_1 x_1^r} \right) \tag{15}$$
$$\frac{dx_5^{r+1}}{d\eta} = x_6^{r+1}$$

$$\frac{dx_6^{r+1}}{d\eta} = \left(-\Pr x_6^r \right) x_1^{r+1} + (2\Pr x_5^r) x_2^{r+1} - \left(2Ec \Pr x_3^r \right) x_3^{r+1} + (2\Pr x_2^r) x_5^{r+1}$$
$$+ (-\Pr x_1^r) x_6^{r+1} + \left(Ec \Pr (x_3^r)^2 + \Pr x_1^r x_6^r - 2\Pr x_2^r x_5^r \right)$$

The above system of equations (15) is linear in $x_i^{r+1}, i = 1,2,\ 6$ and general solution can be obtained by using the principle of superposition.
The boundary conditions reduce to

$$x_1^{r+1}(0) = R,\ x_2^{r+1}(0) = 1,\ x_5^{r+1}(0) = 1 \tag{16}$$
$$x_2^{r+1}(\eta) \to 0, x_3^{r+1}(\eta) \to 0, x_5^{r+1}(\eta) \to 0 \qquad as\ \ \eta \to \infty$$

The initial values are chosen as follows:
For the homogeneous solution:

$$x_i^{h_1}(\eta) = [0\ 0\ 1\ 0\ 0\ 0]$$
$$x_i^{h_2}(\eta) = [0\ 0\ 0\ 1\ 0\ 0]$$
$$x_i^{h_3}(\eta) = [0\ 0\ 0\ 0\ 0\ 1]$$

For particular solution:

$$x_i^p(\eta) = [R\ 1\ 0\ 0\ 1\ 0]$$

The general solution of system of equations is given by

$$x_i^{r+1}(\eta) = c_1 x_i^{h_1}(\eta) + c_2 x_i^{h_2}(\eta) + c_3 x_i^{h_3}(\eta) + x_i^p(\eta) \tag{17}$$

Where C_1, C_2, C_3 are the unknown constants and are determined by considering the boundary conditions as $\eta \to \infty$. This solution $x_i^{r+1}, i = 1,2,\ 6$ is then compared with solution at the previous step $x_i^r, i = 1,2,\ 6$ and next iteration is performed if the convergence has not been achieved or greater accuracy is desired.

VI. RESULTS AND DISCUSSION:

Fig 1 depicts the effect of magnetic field parameter (Mn) on the horizontal velocity profile (f_η (η)). Horizontal velocity profile decreases with increase in Magnetic field parameter, since increase of Magnetic field parameter signifies the increase of Lorentz force, which opposes the horizontal flow in the reverse direction.

Fig 2(a) and Fig 2(b) depict the effect of viscoelastic parameter k_1 on longitudinal and transverse velocity components. It can be seen, for a fixed value of η, both f' (η) and f (η) decrease with increasing values of viscoelastic parameter k_1. This can be explained by the fact that, as the viscoelastic parameter k_1 increases, the boundary layer adheres strongly to the surface, which in turn retards the flow in longitudinal and transverse directions.

Fig 3 shows the effect of Magnetic field parameter on temperature distribution in PST case. Temperature profile increases with increase in Magnetic field. Since increase of magnetic field increases the thermal boundary layer thickness. The increasing frictional drag due to Lorentz force is responsible for increasing the thermal boundary layer thickness.

Fig.4 represents that Temperature $\theta(\eta)$ increases with increase in Eckert number (Ec). This is due to the fact that heat energy is stored in the fluid due to frictional heating.

Fig 5 reveals the effect of Prandtl number (Pr) on non-dimensional temperature $\theta(\eta)$ profiles are shown. Temperature θ (η) decreases with increase in the Prandtl number Pr. This is consistent with the fact that the thermal boundary layer thickness decreases with increasing values Prandtl number Pr.

In Fig 6 displays the values of temperature θ (η) for different values of viscoelastic parameter (k_1). It can be observed that, at a given point η, $\theta(\eta)$ decreases with increasing values of k_1, this is an important finding in MHD viscoelastic fluid , where opposite behavior can be seen in viscoelastic fluid flows.

Fig 7 depicts the effect of suction parameter(R) on the heat transfer$\theta(\eta)$, which decreases with increasing values of suction parameter(R). Due to suction parameter(R) there will be loss of fluid in the boundary layer region, hence there will be less scope for heat transfer from the sheet to the fluid. This causes the declination in the heat transfer for increasing values of suction parameter.

The heat transfer phenomenon is usually analyzed from the numerical values of wall temperature gradient $\theta'(0)$, which are recorded for various values of parameters in Table1. Analysis of this table reveals that the effect of increasing the values of Magnetic field parameter(Mn),viscous dissipation(Ec) is to increase the wall temperature gradient and the opposite trend is observed in case of Prandtl number (Pr), visco-elasticity($k_{1)}$,suction parameter(R).

VII.CONCLUSIONS:

From our numerical results, it can be concluded that:

i. Horizontal velocity profile decreases with increase in viscoelastic parameter and it also decreases withincrease in magnetic field parameter.
ii. Temperature profiles decreases with increase in viscoelastic parameter, increases with increase in magnetic field parameter. Hence the strength of external magnetic field should be as mild as possible for effective cooling of the stretching sheet.
iii. Thermal boundary layer thickness decreases with increase in Prandtl number.
iv. Temperature profiles decreases with increasing values of suction parameter (R).

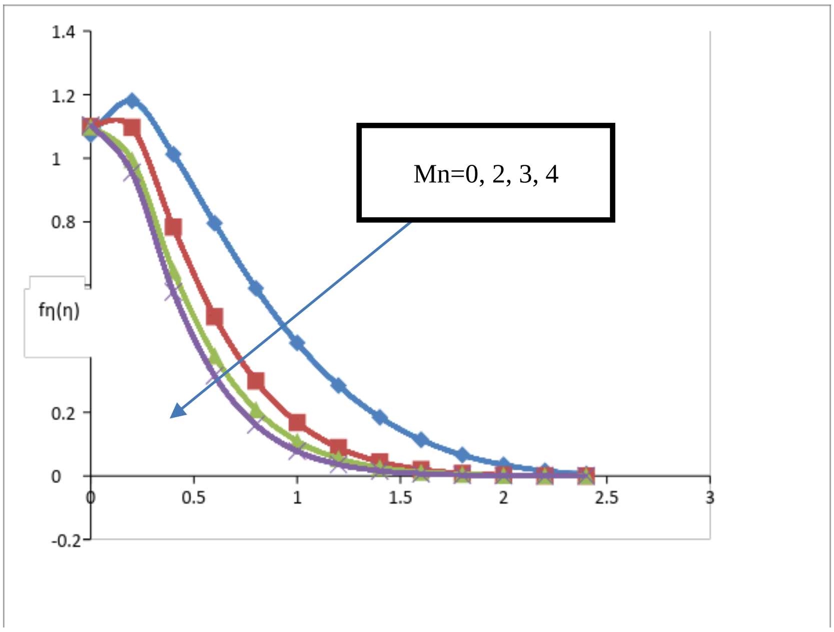

Fig1: Plot of velocity (f_η (η)) vs η for different values of Magnetic parameter (Mn)

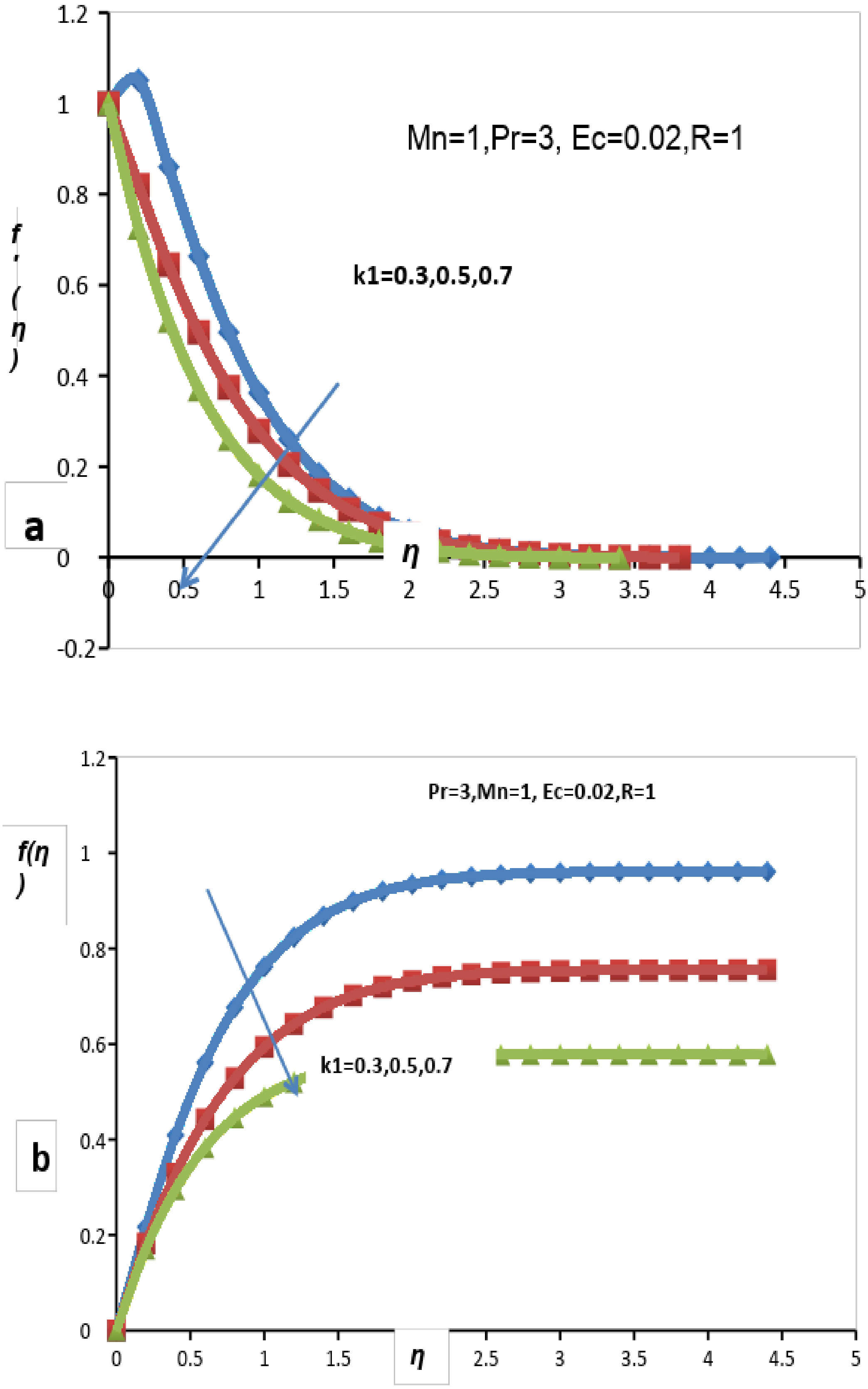

Fig2: Effect of viscoelasticity (k_1) on **(a)** transverse velocity component, **(b)** longitudinal velocity component

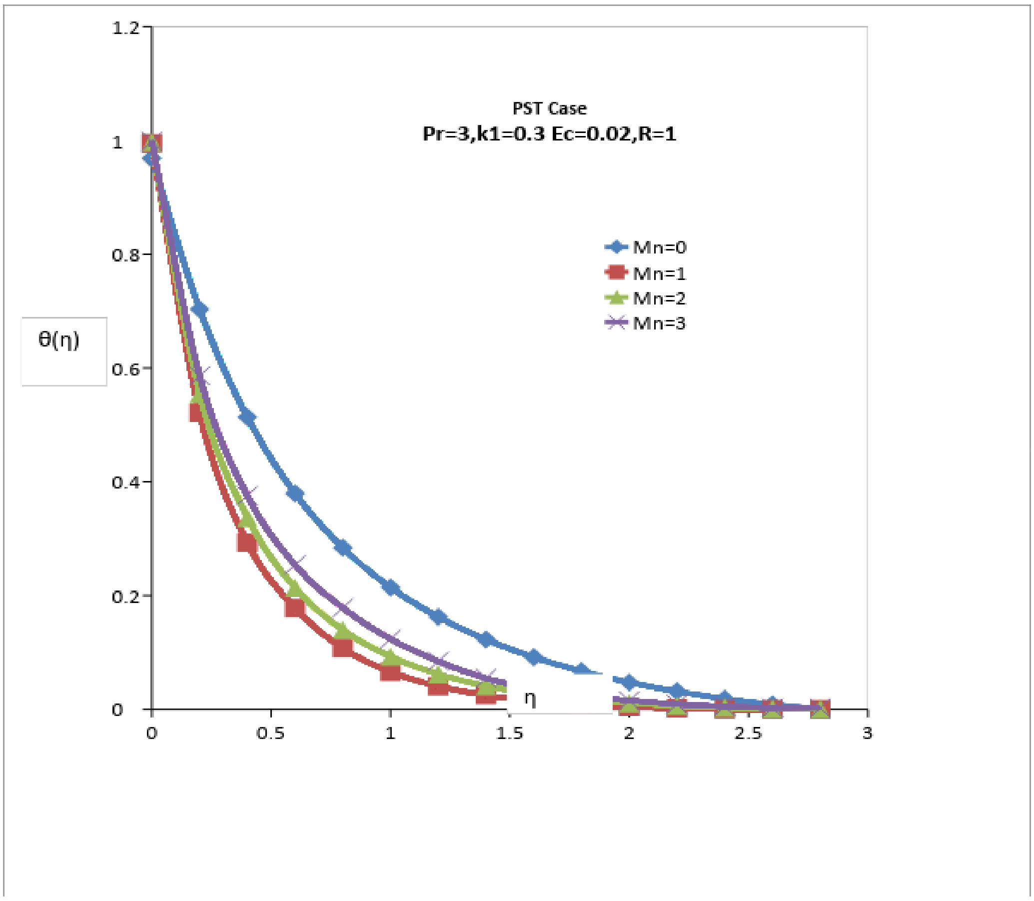

Fig 3. Effect of Magnetic field parameter (Mn) on temperature distribution θ (η)

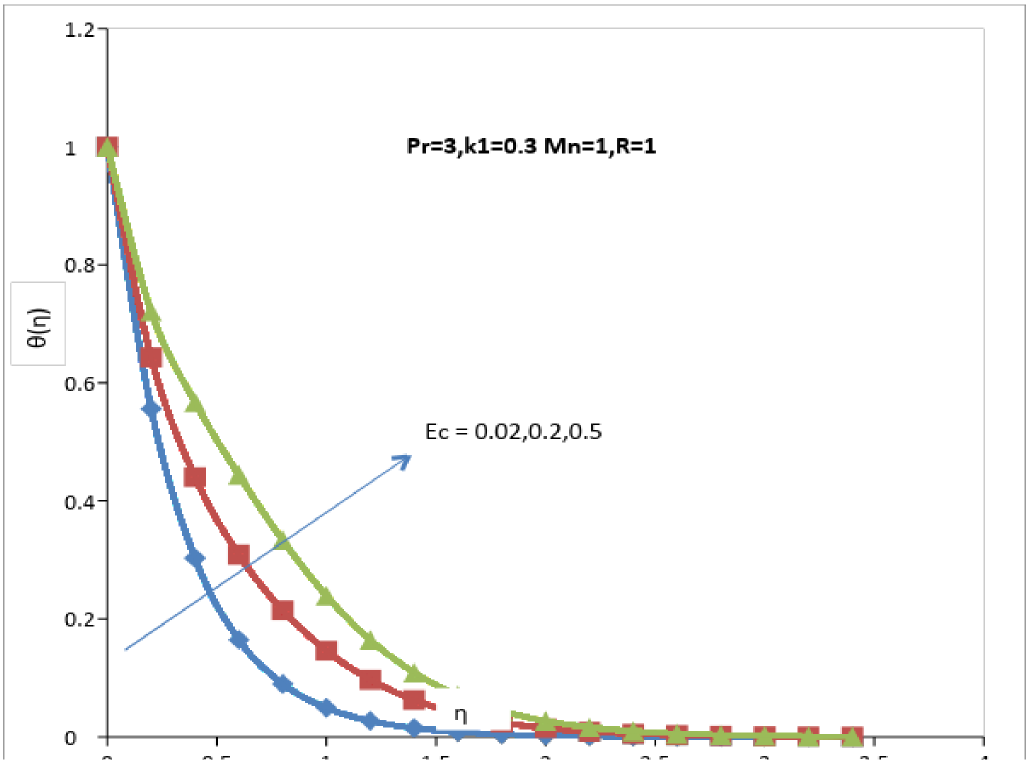

Fig4. Variation of non-dimensional temperature θ (η) Vs η for different values of Eckert number (Ec).

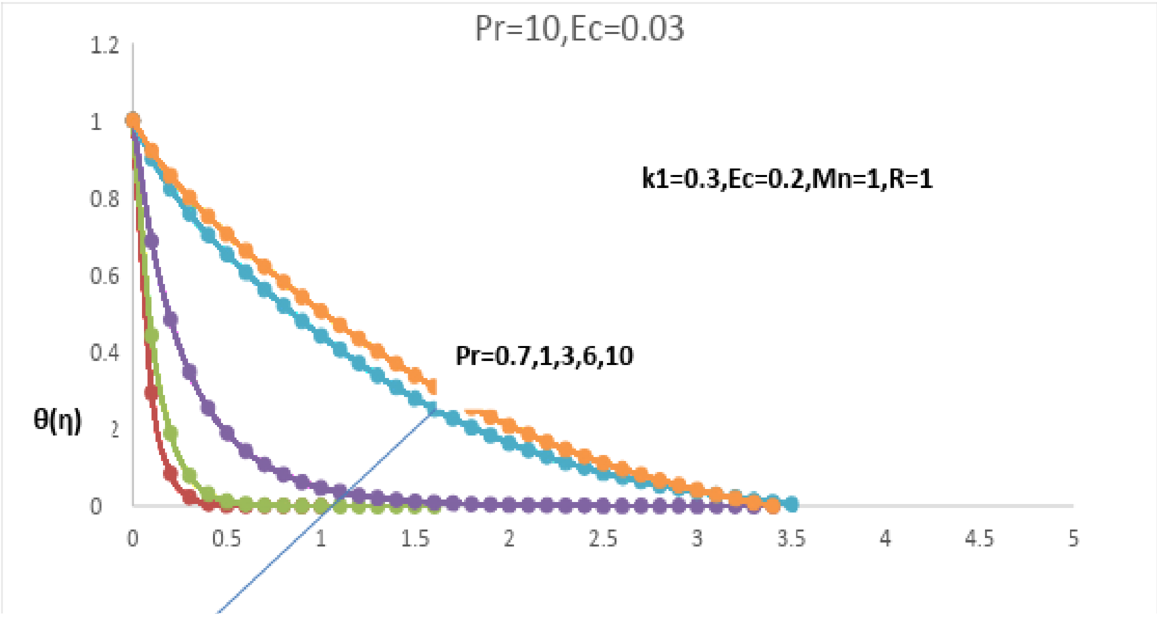

Fig 5. Variation of non-dimensional temperature θ (η) Vs η for different values of Prandtl number (Pr).

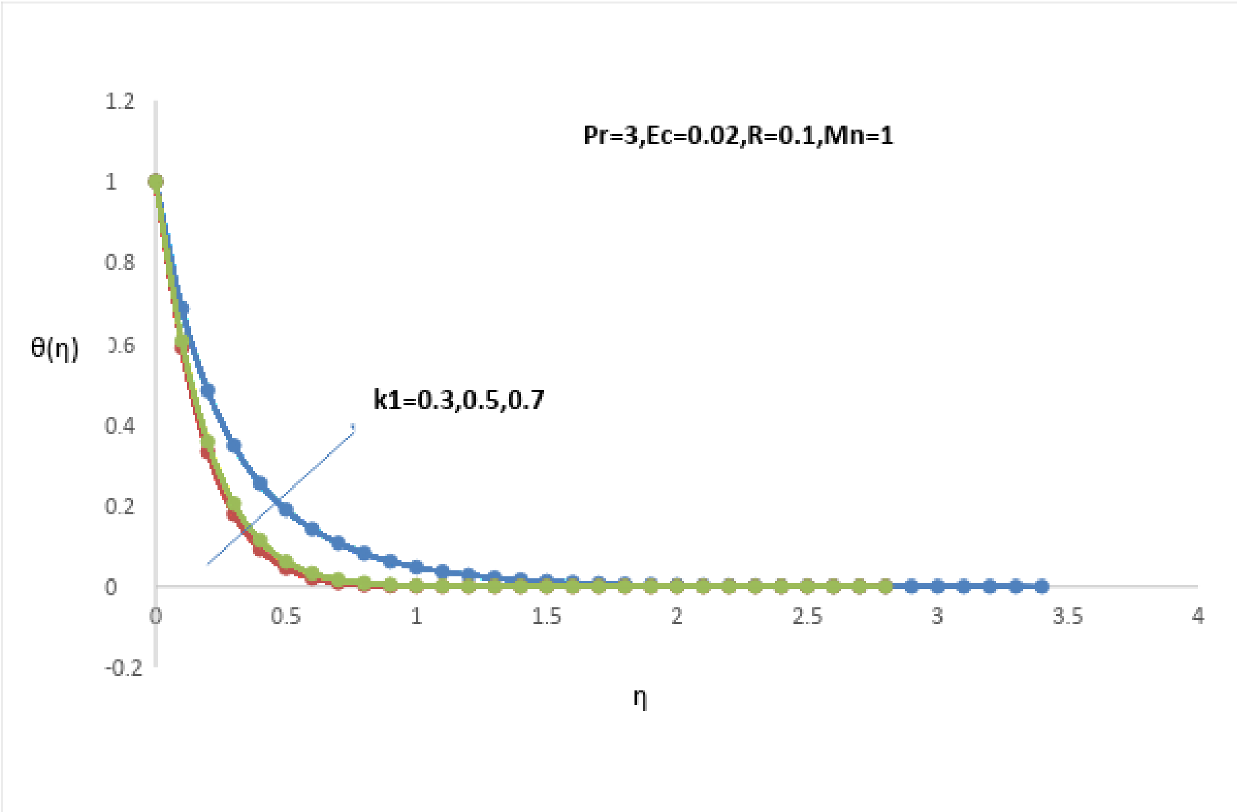

Fig 6. Variation of non-dimensional temperature θ (η) Vs η for different values of viscoelastic parameter (k_1) on θ (η)

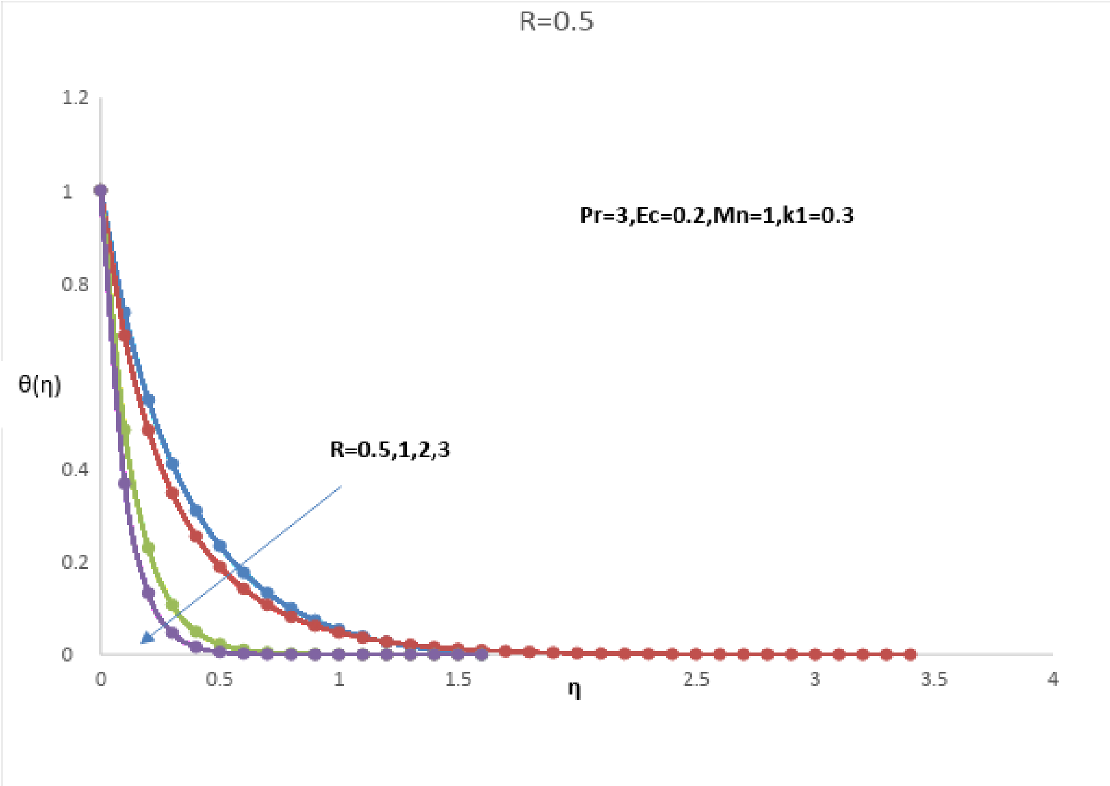

Fig7. Effect of suction parameter (R) on temperature distribution θ (η).

Table1:values of wall temperature gradient θ'(0) for various parameters

Ec	R	Mn	Pr	K_1	θ'(0)
0.02	1	1	3	k1=0.3	-5.05756
				k1=0.5	-4.84680
				k1=0.7	-3.88159
0.02	1	1	0.7		-0.87681
			1		-1.12533
			3		-2.99554
			6		-7.98400
			10		-10.7183
0.02	1	1	3	0.3	-4.93750
		2			-4.82564
		3			-3.88159
0.02	0.5	1	3	0.3	-3.09541
	1				-3.88159
	2				-7.14728
	3				-9.86315
0	1	1	3	0.3	-4.15077
0.02					-4.02504
0.5					-1.00770

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