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Non-Uniform Motion of the Three-Body Problem When the Primaries Are Oblate Spheroids

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Abstract: In this article, we have investigated the complete synchronization of two identical non-linear dynamical systems of the three-body problem by taking into consideration the primaries as oblate bodies. We assumed that the primaries are moving in the circular motion around there centre of mass in the non-uniform motion. Here we have designed a non linear controller based on the Lyapunov stability theory. The proposed controller ensures that the states of the controlled chaotic slave system asymptotically synchronizes the states of the master system. For validation of results by numerical simulations we used the Mathematica10 when the primaries are Jupiter and Mars.

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I. INTRODUCTION

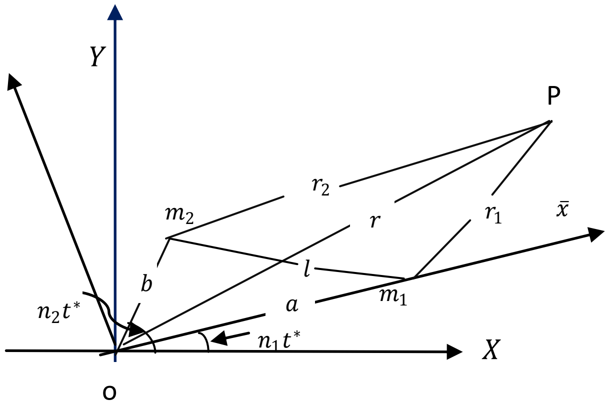
A deterministic system is chaotic whenever its evolution sensitively depends on the initial conditions. This property implies that two trajectories emerging from near by initial conditions separate exponentially in the course of time. Pecora and Carroll [1] gave idea of synchronization of chaotic systems using the concept of master and slave system and they demonstrated that chaotic synchronization could be achieved by driving or replacing one of the variables of a chaotic system with a variable of another similar chaotic device. Many methods and techniques for chaos control and synchronization of various chaotic systems have been developed, such as non linear feedback control method discussed by Lu L. Zhang and C. Guo Z. A. [2] sliding mode control technique studied by Haeri. M. and Emadzadeh. A. [3]. adaptive control technique to stabilize and synchronize a hyperchaotic system with uncertain parameters have been discussed by Israr Ahmad et al. [4]. In an another paper Israr Ahmad et al have been studied a synchronization problem of a three-dimensional (3-D) Coullete chaotic system using the active- and adaptive-based synchronization control techniques [5]. Chaos synchronization using active control has recently been widely accepted as an efficient technique for synchronizing chaotic systems. This method has been applied to many practical systems such as spatiotemporal dynamical systems [6], the Rikitake two-disc dynamo-a geographical system [7], Non-linear Bloch equations modeling "jerk" equation (Ucar et al. [8]), Complex dynamos (Mahmoud [9]), drive-response chaotic system Israr Ahmad et al. [10]. The synchronization problem via nonlinear control scheme is studied by Chen and Han [11] , Chen [12], Ju H. Park.[13] and Mossa et. All [14].

Many mathematicians have made the huge contributions to the analytical, qualitative and numerical studies of the restricted three-body problem when the primaries are moving in the circular motion around there centre of mass in the uniform motion. A detailed analysis of this problem is illustrated in the work of American mathematician Szebehely [15]. In (1975) Sharma, R. K. and Subbarao[16] have discussed the collinear equilibria and their characteristic exponents in the restricted three body problem when the primaries are oblate spheroids. The Lagrangian triangular equilibria in the planar restricted three body problem where the primaries are oblate homogeneous spheroids discussed by Arredondo, J.A. et al [17]. Khan and Shahzad [18] investigated the synchronization behavior of the two identical circular restricted three body problem influenced by radiation evolving from different initial conditions via the active control. In an another paper the Complete synchronization, anti-synchronization and hybrid synchronization of two identical parabolic restricted three body problem have been studied by Khan and Rimpi pal [19]. Arif [20] studied the complete synchronization, anti-synchronization and hybrid synchronization in the planar restricted three problem by taking into consideration the small primary is ellipsoid and bigger primary an oblate spheroid via active control technique.

Being motivated by the above discussion, in section 2 we have formulated the equation of motion of the three-body problem when the primaries as oblate bodies are moving in a circular motion around there centre of mass in the non-uniform motion. Section 3 deals with the complete synchronization behavior of two identical systems via nonlinear control technique. In this section we have designed a non linear controller based on the Lyapunov stability theory. Numerical simulations are performed to plot time series analysis graphs of the master system and the slave system which further illustrate the effectiveness of the proposed control technique Finally, we conclude the paper in section 4.

II. EQUATION OF MOTION

To write the equation of motion we assume that the two oblate bodies of masses m_1 and m_2 are moving in the circular motion around their centre of mass O with angular velocities, the mean motions n_1 and n_2 respectively Fig(1). The motion of a particle P of mass m defined by its radius vector $\bar{r}$ will be referred to a frame of reference $O\bar{x}\bar{y}$ that rotates in the same direction and the same angular velocity n_1 as the primary of mass m_1, which in this frame are taken to stay at rest on x-axis. Balance between the gravitational and centrifugal forces requires that



Fig(1)

Figure (1) : Non-uniform motion of charged particle P around there centre of mass.

$$\frac{k^2 m_1^2\ m_2^2}{l^2} = a\, m_1 n_1^2 = b\, m_2 n_2^2 \qquad (1)$$

$k =$ Gaussian constant, $l^2 = a^2 + b^2 - 2ab\cos(n_1 - n_2)\,t^* =$ variable mutual distance between the primaries.

Then the equation of motion of the particle P in the fixed system may be written as:

$$\frac{d^2X}{dt^{*2}} = \frac{\partial F(X,Y,t^*)}{\partial X} \qquad (2)$$

$$\frac{d^2Y}{dt^{*2}} = \frac{\partial F(X,Y,t^*)}{\partial Y} \qquad (3)$$

Where

$$F = k^2\left[\frac{m_1}{R_1} + \frac{I_1}{2R_1^3} + \frac{m_2}{R_2} + \frac{I_2}{2R_2^3}\right] \qquad (4)$$

$R_1^2 = (X - X_1)^2 + (Y - Y_1)^2$, $R_2^2 = (X - X_2)^2 + (Y - Y_2)^2$, $X_1 = a\cos n_1 t^*$, $Y_1 = a\sin n_1 t^*$, $X_2 = b\cos n_2 t^*$, $Y_2 = b\sin n_2 t^*$. Where t^* is the dimensional time, $I_i = m_i\left(\frac{R_{ie}^2 - R_{ip}^2}{5}\right)$, R_{ie}, R_{ip} equatorial and polar radii of the primaries. $i = 1{,}2$.

Now introduce a rotating co-ordinate system $(\bar{x}, \bar{y})$ by substituting

$$Z = z\, e^{in_1 t^*} \qquad (5)$$

Where

$$Z = X + iY, \qquad z = \bar{x} + i\bar{y}$$

After, using the complex vector Z, the equation of motion 2 and 3 takes the form:

$$\frac{d^2z}{dt^{*2}} + 2in_1\frac{dz}{dt^*} - {n_1}^2 z = -k^2\left[\frac{m_1(z-a)}{R_1^3} + \frac{3\,I_1(z-a)}{2R_1^5} + \frac{m_2\{z - be^{(n_2-n_1)t^*}\}}{R_2^3} + \frac{3\,I_2\{z - be^{(n_2-n_1)t^*}\}}{2R_2^5}\right] \qquad (6)$$

Where

$$R_1 = |z - z_1|, \quad R_2 = |z - z_2|, \quad z_1 = ae^{in_1t^*} \text{ and } z_2 = be^{in_2t^*}$$

The equations of motion in the rotating coordinate system are:

$$\frac{d^2\bar{x}}{dt^{*2}} - 2n_1\frac{d\bar{y}}{dt^*} - {n_1}^2\bar{x} = -k^2\left[\frac{m_1(\bar{x}-a)}{r_1^3} + \frac{3\,I_1(\bar{x}-a)}{2r_1^5} + \frac{m_2\{\bar{x}-b\,cos\,(n_2-n_1)t^*\}}{r_2^3} + \frac{3\,I_2\{\bar{x}-b\,cos\,(n_2-n_1)t^*\}}{2r_2^5}\right] \tag{7}$$

$$\frac{d^2\bar{y}}{dt^{*2}} + 2n_1\frac{d\bar{x}}{dt^*} - {n_1}^2\bar{y} = -k^2\left[\frac{m_1\bar{y}}{\bar{r}_1^3} + \frac{3\,I_1\bar{y}}{2r_1^5} + \frac{m_2\{\bar{y}-b\,sin\,(n_2-n_1)t^*\}}{\bar{r}_2^3} + \frac{3\,I_2\{\bar{y}-b\,sin\,(n_2-n_1)t^*\}}{2r_2^5}\right] \tag{8}$$

Where

$$F = \frac{{n_1}^2}{2}(\bar{x}^2+\bar{y}^2) + k^2\left[\frac{m_1}{\bar{r}_1} + \frac{I_1}{2\bar{r}_1^3} + \frac{m_2}{\bar{r}_2} + \frac{I_2}{2\bar{r}_2^3}\right].\ \ \bar{r}_1 = [(\bar{x}-a)^2 + \bar{y}^2]^{\frac{1}{2}},$$

$$\bar{r}_2 = [\bar{x}^2 + \bar{y}^2 + b^2 - 2\bar{y}b\sin(n_2-n_1)t^* - 2\bar{x}b\,cos(n_2-n_1)t^*]^{\frac{1}{2}}$$

We shall introduce the dimensionless pulsating co-ordinates system given by:

$$x = \frac{\bar{x}}{l},\quad y = \frac{\bar{y}}{l},\quad \mu_1 = \frac{m_1}{M},\quad \mu_2 = \frac{m_2}{M},\quad r_1 = \frac{\bar{r}_1}{l},\quad r_2 = \frac{\bar{r}_2}{l}.$$

Let a particular case

$$\frac{n_1}{n_2} = \frac{1}{6}.$$

Then $(n_2-n_1)t^* = 5n_1t^* = t.$

(Primaries are in a 1:6 resonance, the inner primary will complete six periods for every one period of outer primary, i.e. orbital period of Mars ≈ 687 days where orbital period of Jupiter ≈ 4380 days).

Now substituting the values in equations (7) and (8), we get

$$\ddot{x} + \frac{2}{l}\dot{l}\dot{x} - 2n_1\,\dot{y} = \frac{1}{l}\left[U_x + 2\,n_1 y\dot{l}\right] \tag{9}$$

$$\ddot{y} + \frac{2}{l}\dot{l}\dot{y} + 2n_1\,\dot{x} = \frac{1}{l}\left[U_y - 2\,n_1 x\dot{l}\right] \tag{10}$$

Where dots denote the derivatives with respect to the dimensionless time (t) and subscripts signify partial derivatives and

$$U = {n_1}^2\left[\frac{(x^2+y^2)}{2} + l^2\left\{\frac{36\mu_2}{r_1} + \frac{\mu_1}{r_2}\right\}\right] - \frac{(x^2+y^2)}{2}\ddot{l} + \frac{{n_1}^2(a+36b)}{2l}\left(\frac{I_1}{{r_1}^3} + \frac{I_2}{{r_2}^3}\right),$$

$${r_1}^2 = [(x-\mu_1)^2 + y^2],\ \ {r_2}^2 = [x^2 + y^2 + {\mu_2}^2 - 2y\mu_2\sin t - 2x\mu_2\,cost].$$

III. COMPLETE SYNCHRONIZATION

To design a non linear controller let

$$x = x_1,\ \dot{x} = x_2,\ y = x_3,\ \dot{y} = x_4$$

Then the equation (9) and (10) can be written as:

$$\dot{x}_1 = x_2 \tag{11}$$

$$\dot{x}_2 = x_1\left({n_1}^2 - \frac{\ddot{l}}{l}\right) - \frac{2}{l}\dot{l}x_2 + \frac{2\,n_1x_3\dot{l}}{l} + 2n_1x_4 + A_1 \tag{12}$$

$$\dot{x}_3 = x_4 \tag{13}$$

$$\dot{x}_4 = -\frac{2\,n_1x_1\dot{l}}{l} - 2n_1x_2 + x_3\left({n_1}^2 - \frac{\ddot{l}}{l}\right) - \frac{2}{l}\dot{l}x_4 + B_1 \tag{14}$$

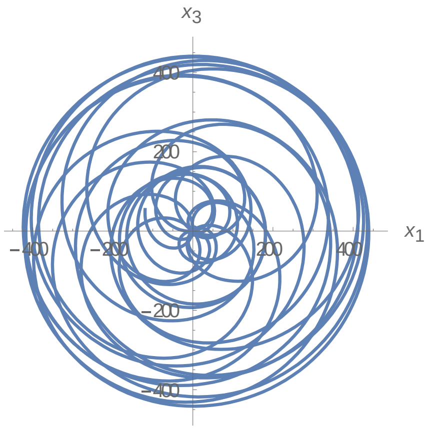
Where

$$A_1 = -{n_1}^2 l\left\{\frac{36(x_1-\mu_1)\mu_2}{{r_1}^3} + \frac{(x_1-\mu_2\,cost)\mu_1}{{r_2}^3}\right\} + \frac{3\,{n_1}^2(a+36b)}{2l}\left\{\frac{(x_1-\mu_1)I_1}{{r_1}^5} + \frac{(x_1-\mu_2\,cost)I_2}{{r_2}^5}\right\},$$

$$B_1 = -{n_1}^2 l\left\{\frac{36x_3\mu_2}{{r_1}^3} + \frac{(x_3-\mu_2\,sint)\mu_1}{{r_2}^3}\right\} + \frac{3\,{n_1}^2(a+36b)}{2l}\left\{\frac{x_3I_1}{{r_1}^5} + \frac{(x_3-\mu_2\,sint)I_2}{{r_2}^5}\right\}$$

$$r_1^2 = (x_1-\mu_1)^2 + {x_3}^2,\ \ {r_2}^2 = [{x_1}^2 + {x_3}^2 + {\mu_2}^2 - 2x_3\mu_2\sin t - 2x_1\mu_2\,cost],$$

The system (11,12,13 and 14) is the master system. The state orbits of this master system are shown in Figure (2) and this figure shows that the system is chaotic.



Fig(2)

Figure (2) : state orbits of the master system

Let us define identical slave system

$$\dot{y_1} = y_2 + u_1(t) \qquad (15)$$

$$\dot{y_2} = y_1\left(n_1{}^2 - \frac{\ddot{l}}{l}\right) - \frac{2}{l}\dot{l}y_2 + \frac{2\,n_1 y_3 \dot{l}}{l} + 2n_1 y_4 + A_2 + u_2(t) \qquad (16)$$

$$\dot{y_3} = y_4 + u_3(t) \qquad (17)$$

$$\dot{y_4} = -\frac{2\,n_1 y_1 \dot{l}}{l} - 2n_1 y_2 + y_3\left(n_1{}^2 - \frac{\ddot{l}}{l}\right) - \frac{2}{l}\dot{l}y_4 + B_2 + u_4(t) \qquad (18)$$

Where

$$A_2 = -n_1{}^2 l\left\{\frac{36(y_1-\square_1)\square_2}{r_1{}^3} + \frac{(y_1-\square_2\,cost)\square_1}{r_2{}^3}\right\} + \frac{3\,n_1{}^2(a+36b)}{2l}\left\{\frac{(y_1-\square_1)I_1}{r_1{}^5} + \frac{(y_1-\square_2\,cost)I_2}{r_2{}^5}\right\},$$

$$B_2 = -n_1{}^2 l\left\{\frac{36y_3\square_2}{r_1{}^3} + \frac{(y_3-\square_2\,sint)\square_1}{r_2{}^3}\right\} + \frac{3\,n_1{}^2(a+36b)}{2l}\left\{\frac{y_3 I_1}{r_1{}^5} + \frac{(y_3-\square_2\,sint)I_2}{r_2{}^5}\right\}$$

$$r_1^2 = (y_1-\mu_1)^2 + y_3{}^2,\quad r_2{}^2 = [y_1{}^2 + y_3{}^2 + \mu_2{}^2 - 2y_3\mu_2\sin t - 2y_1\mu_2\,cost],$$

and $u_i(t)$; i =1 ,2,3,4 are control functions to be determined.

Let $e_i = y_i - x_i$; i = 1, 2, 3, 4 be the synchronization errors.

From (11) to (18), we have

$$\dot{e_1} = e_2 + u_1 \qquad (19)$$

$$\dot{e_2} = \left(n_1{}^2 - \frac{\ddot{l}}{l}\right)e_1 - \frac{2}{l}\dot{l}e_2 + \frac{2\,n_1\dot{l}}{l}e_3 + 2n_1 e_4 + A_2 - A_1 + u_2 \qquad (20)$$

$$\dot{e_3} = e_4 + u_3 \qquad (21)$$

$$\dot{e_4} = -\frac{2\,n_1\dot{l}}{l}e_1 - 2n_1 e_2 + \left(n_1{}^2 - \frac{\ddot{l}}{l}\right)e_3 - \frac{2}{l}\dot{l}e_4 + B_2 - B_1 + u_4 \qquad (22)$$

Lyapunov stability theory state that when controller satisfies the assumption with $V(e) = \frac{1}{2}e^t e$ a positive definite function and the first derivative of this function $V' < 0$, the chaos synchronization of two identical systems (master and slave) for different initial conditions is achieved.

So the first derivative of $V(e)$ for the system will be

$$V' = e_1(e_2+u_1) + e_2\left\{\left(n_1{}^2 - \frac{\ddot{l}}{l}\right)e_1 - \frac{2}{l}\dot{l}e_2 + \frac{2\,n_1\dot{l}}{l}e_3 + 2n_1e_4 + A_2 - A_1 + u_2\right\} + e_3(e_4+u_3)$$

$$+e_4\left\{-\frac{2\,n_1\dot{l}}{l}e_1 - 2n_1e_2 + \left(n_1{}^2 - \frac{\ddot{l}}{l}\right)e_3 - \frac{2}{l}\dot{l}e_4 + B_2 - B_1 + u_4\right\}.$$

Hence, if we choose the controller u as follows,

$$u_1 = -e_1 - e_2 \qquad (23)$$

$$u_2 = -\left(n_1{}^2 - \frac{\ddot{l}}{l}\right)e_1 + \frac{2}{l}\dot{l}e_2 - \frac{2\,n_1\dot{l}}{l}e_3 - A_2 + A_1 - e_2 \qquad (24)$$

$$u_3 = -e_3 - e_4 \qquad (25)$$

$$u_4 = \frac{2\,n_1\dot{l}}{l}e_1 - \left(n_1{}^2 - \frac{\ddot{l}}{l}\right)e_3 + \frac{2}{l}\dot{l}e_4 - B_2 + B_1 - e_4 \qquad (26)$$

Then

$$V' = -e_1{}^2 - e_2{}^2 - e_3{}^2 - e_4{}^2 < 0 \qquad (27)$$

Hence the error state

$$\lim_{t\to\infty} \|e(t)\| = 0$$

which gives asymptotic stability of the system. This means that the controlled chaotic systems (master and slave) are synchronized for deferent initial conditions.

IV. NUMERICAL SIMULATION

Let us consider an example of Jupiter-Mars system in the three body problem in which the inner primary m_2 is taken as the Mars and outer primary is m_1 as the Jupiter and small body as a space- craft. From the astrophysical data we have

Mass of the Mars $m_2 = 6.407 \times 10^{23}$ kg.
Mass of the Jupiter $m_1 = 1.89712 \times 10^{27}$kg.
Mean distance of Mars from the origin = 227.800000 km.
Mean distance of Jupiter from the origin = 778.547200 km.
In dimensionless system $m_1 + m_2 = 1$ unit. Variable distance between the primaries l is 1 unit.
The initial conditions of master and slave systems $[x_1(0), x_2(0), x_3(0), x_4(0)] = [-2, 2, 0, 4]$ and $[y_1(0), y_2(0), y_3(0), y_4(0)] = [2, 3, 2, 7]$ respectively. We have simulated the system under consideration using mathematica 10. Results for uncoupled system are given in figures 3,5,7,9 and that of controlled system are shown in figures 4,6,8 and 10 for respectively. These figures shows that the state $[x_1(t), x_2(t), x_3(t), x_4(t)]$ of master system [11 to 14] asymptotically synchronize with the state $[y_1(t), y_2(t), y_3(t), y_4(t)]$ of slave system [15 to 18].

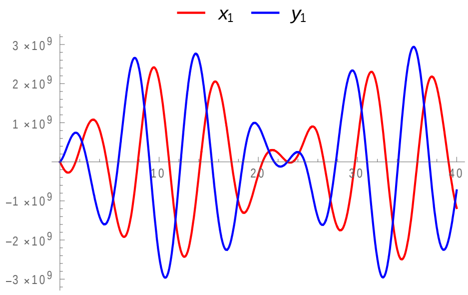
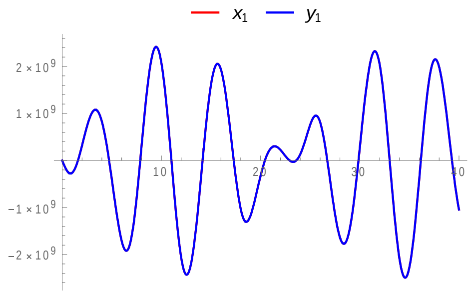
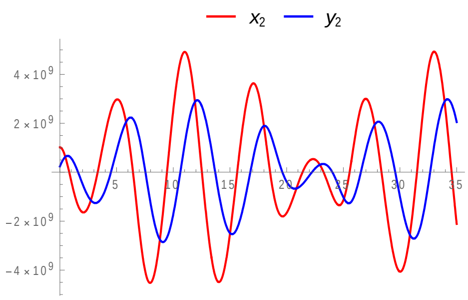


Fig (3)

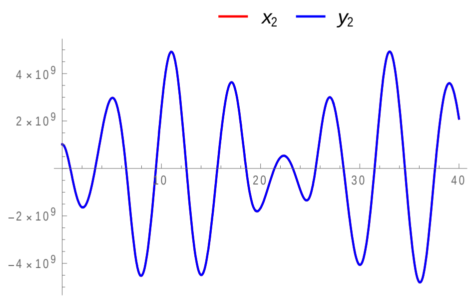


Fig(4)

Figure (3) :Time series of the Uncontrolled states x_1, y_1. Figure (4):Time series of the synchronized states x_1 , y_1.

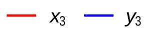


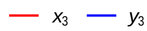
Fig(5)

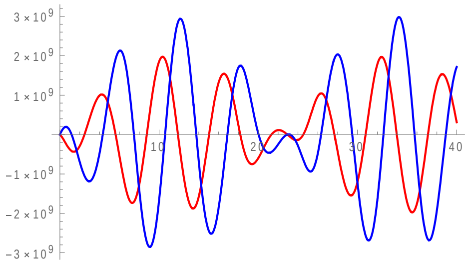


Fig(6)

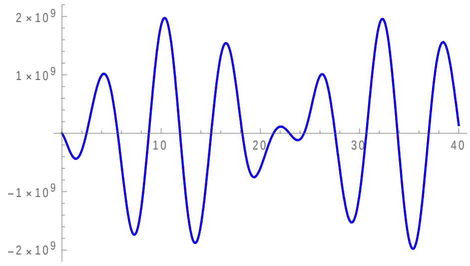
Figure (5) :Time series of the Uncontrolled states x_2, y_2. Figure (6):Time series of the synchronized states x_2 , y_2.





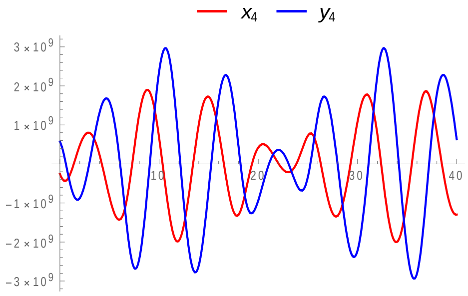


Fig(7)

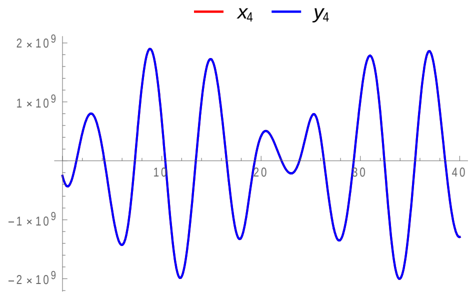


Fig(8)

Figure (7) :Time series of the Uncontrolled states x_3, y_3. Figure (8):Time series of the synchronized states x_3 , y_3.



Fig(9)



Fig(10)

Figure (9) :Time series of the Uncontrolled states x_4, y_4. Figure (10):Time series of the synchronized states x_4 , y_4.

V. CONCLUSION

The equation of motion of the three -body problem when the primaries are moving in a circular orbit around there centre of mass in the non-uniform motion by taking into consideration the primaries as oblate bodies formulated. We have also investigated the complete synchronization behavior of two identical non-linear dynamical systems of the three-body restricted problem by taking into consideration the primaries as oblate bodies, via non linear controller based on the Lyapunov stability theory. Here two systems (master and slave) are compete synchronized when start with deferent initial conditions. Hence the slave chaotic system completely traces the dynamics of the master system in the course of time. For validation of results by numerical simulations we used the Mathematica 10 when the primaries are Jupiter and Mars

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