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Flow and Heat Transfer of MHD Viscoelastic Fluid over a Stretching Sheet with Viscous Dissipation and Non-uniform Heat Source/Sink

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Abstract: In this paper, the problem of steady laminar two-dimensional boundary layer flow and heat transfer of an incompressible magneto hydrodynamic viscoelastic fluid with a presence of Viscous Dissipation and Non-uniform Heat Source/Sink over linearly stretching sheet is investigated numerically. The governing boundary layer equations are reduced into ordinary differential equations by a similarity transformation. The transformed equations are solved using Quasilinearisation technique. The effect of various physical parameters on flow and heat transfer are computed and analyzed through graphs.

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I. INTRODUCTION

The study of viscoelastic fluid flow has gained lot of importance due to its various applications such as petroleum drilling, manufacturing of foods and paper and many other similar activities. The boundary layer concept of viscoelastic fluids is of special importance due to its applications to many engineering problems. Continuous surfaces are surfaces such as those of polymer sheets or filaments continuously drawn from a die. Boundary layer behavior over a moving continuous solid surface is an important type of flow occurring in several engineering processes like heat-treated materials travelling between a feed roll and a wind-up roll or materials manufactured by extrusion.

Sakiadis [1]-[2] analyzed the boundary layer flow induced on a continuous moving sheet. Erickson et al [3] extended this problem to the case for which suction or blowing existed at the moving surface. Crane [4] extended the problem of Sakiadis to the stretching sheet whose velocity is proportional to the distance from the slit. Non-Newtonian fluids have gained considerable importance, because the power required in stretching a sheet in a viscoelastic fluid and heat transfer for a viscoelastic fluid is found to be less than that of Newtonian fluid. Therefore several authors [5]-[11] studied viscoelastic boundary layer flow along a stretching sheet for Non-Newtonian fluids.

Magneto hydrodynamics (MHD) is a subject that studies the behavior of an electrically conducting fluid in the presence of an electromagnetic field. MHD boundary layers with heat and mass transfer over flat surfaces are found in many engineering and geophysical applications such as geothermal reservoirs, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors. Because of its wide range applications, many researchers tend to apply MHD flow into their problems [12–16].

In the present study, an incompressible MHD viscoelastic (Walters' liquid B model) fluid over a stretching sheet with viscous dissipation and non-uniform heat source is considered. Governing boundary layer equations are solved by using numerical approach, quasilinearization technique. Results are in good agreement with the available literature.

II. MATHEMATICAL FORMULATION

Consider the flow of an incompressible MHD viscoelastic (Walters' liquid B model) fluid over a wall coinciding with the plane y = 0, the flow being confined to y > 0.Two equal and opposite forces are applied along the x-axis, so that the wall is stretched keeping the origin fixed. Under the usual boundary layer assumptions, governing boundary layer equations, derived by Beard and Walters [17], are given as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-k_0\left\{\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}+v\frac{\partial^3 u}{\partial y^3}\right\}-\frac{\sigma B_0^2 u}{\rho} \tag{2}$$

$$where \;\; \upsilon = \frac{\mu}{\rho}, k_0 > 0$$

Where u and v are the velocity components respectively along the x and y directions, υ is the kinematic viscosity, k_0 is the co-efficient of elasticity, k is the thermal conductivity, ρ is the density,
The boundary conditions for the velocity field are:

$$u = u_w = bx, v = -v_0 \quad \text{at } y = 0, b > 0 \tag{3a}$$

$$u \to 0, \partial u / \partial y \to 0 \;\; as \;\; y \to \infty \tag{3b}$$

where $\partial u / \partial y \to 0 \;\; as \;\; y \to \infty$ is the augmented condition given by K. R. Rajagopal [18]. Here the flow is caused solely by the stretching of the sheet, since the free stream velocity is zero.
Defining new variables as

$$u = bxf_\eta(\eta), v = -\sqrt{b\upsilon}\, f(\eta), \eta = \sqrt{b/\upsilon}\, y \tag{4}$$

Substituting (4) in (2) gives

$$f_\eta^2 - ff_{\eta\eta} = f_{\eta\eta\eta} - k_1\left\{2f_\eta f_{\eta\eta\eta} - ff_{\eta\eta\eta\eta} - f_{\eta\eta}^2\right\} - Mnf_\eta \tag{5}$$

where $k_1 = k_0 b / \upsilon$

Similarly corresponding boundary conditions takes the form

$$f(0) = R, f_\eta(0) = 1 \tag{6a}$$

$$f_\eta(\eta) \to 0, f_{\eta\eta}(\eta) \to 0 \;\; as \;\; \eta \to \infty \tag{6b}$$

III. HEAT TRANSFER ANALYSIS

The governing boundary layer equation with viscous dissipation and internal heat generation or absorption is given by

$$\rho C_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + q''' \tag{7}$$

Where k is the thermal diffusivity and c_p is the specific heat of a fluid at constant pressure and q''' is the space and temperature dependent internal heat generation / absorption. This can be modeled in simplest terms as

$$q''' = \left(\frac{ku_w(x)}{x\upsilon} \right) \left[A^*(T_w - T_\infty) f'(\eta) + B^*(T - T_\infty) \right] \tag{8}$$

Here A* and B* are parameters of space and temperature dependent internal heat generation/ absorption. It is to be noted that A* > 0 and B* > 0 correspond to internal heat generation while A* < 0 and B* < 0 correspond to internal heat absorption.

Heat transfer analysis is considered for two types of heating processes, namely,
(i) Prescribed Surface Temperature (ii) Prescribed Heat Flux, which will be discussed below.

3.1 Prescribed Power Law Surface Temperature (PST case)

For this heating process, the prescribed surface temperature is assumed to be quadratic function of x and is given by

$$T = T_w [= T_\infty + A\left(\frac{x}{l}\right)^2] \; at \; y = 0$$

$$T \to T_\infty \;\; as \;\; y \to \infty \tag{9}$$

Here l is the characteristic length. Define non-dimensional temperature as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \qquad (10)$$

Using (4), equation (7) reduces to

$$\theta_{\eta\eta} + \Pr f\theta_\eta - \left(\Pr f_\eta - B^*\right)\theta + \left(A^* - \Pr\theta\right)f_\eta = -Ec\Pr f_{\eta\eta}^2 \qquad (11)$$

where $\Pr = \frac{\mu C_p}{k}$, the Prandtl number

$k_1 = \left(k_0 b / \upsilon\right)$, the viscoelastic parameter

$Ec = \left(b^2 l^2\right) / \left(A c_p\right)$, Eckert number for PST case

with boundary conditions

$$\theta(0) = 1,\ \theta(\infty) \to 0 \qquad (12)$$

3.2 Prescribed Power law surface Heat Flux (PHF Case)

Prescribed power law surface heat flux (PHF), where surface is subjected to a power law heat flux q_w on the wall surface is considered to be a quadratic power of x in the form

$$-k\frac{\partial T}{\partial y} = q_w = D\left(\frac{x}{l}\right)^2 \quad \text{at } y = 0 \qquad (13)$$

T □ T□ as y □ □

where D is a constant, k is the thermal conductivity

Define a dimensionless, scaled temperature as

$$g(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \qquad (14)$$

where $T_w - T_\infty = \frac{D}{k}\left(\frac{x}{l}\right)^2 \sqrt{\frac{\upsilon}{b}}$

Using (4.4), equation (4.7) reduces to

$$g_{\eta\eta} + \Pr f g_\eta - \left(\Pr f_\eta - B^*\right)g + \left(A^* - \Pr g\right)f_\eta = -Ec\Pr f_{\eta\eta}^2 \qquad (15)$$

$Ec = \left(k b^2 l^2 \sqrt{b/\upsilon}\right) / \left(D c_p\right)$, Eckert number for PHF case.

With corresponding boundary conditions as

$$g_\eta(\eta) = -1 \ at\ \eta = 0$$
$$g(\eta) \to 0 \ as\ \eta \to \infty \qquad (16)$$

IV. NUMERICAL SOLUTION OF THE PROBLEM

The flow equation (5) coupled with energy equation (11) **or** (15) constitute a set of highly nonlinear differential equations for which obtaining closed form solution is difficult. Hence a quasilinearization technique, given by Bellman & Kalaba [19] is used to solve this system. This method converts the nonlinear two-point boundary value problem into an iterative scheme of solution, which involves the step-by-step integration of linearised differential equations, with two point boundary conditions. This method is quadratically convergent, starting from the initial guess value and solution obtained is valid for a large range of parameters. Even when the required number of initial conditions are not given, this method converges at a fast speed. In order to implement the quasilinearization technique, the system of equations (5) and ((11) **or** (15)) are converted to a system of first order differential equations as follows:

Substitute $\left(f, f', f'', f''', \theta, \theta'\right) = \left(x_1, x_2, x_3, x_4, x_5, x_6\right)$

(OR)

$$\left(f, f', f'', f''', g, g'\right) = \left(x_1, x_2, x_3, x_4, x_5, x_6\right)$$

Then equations (5) and (11) / (15) reduce to:

$$\frac{dx_1}{d\eta} = x_2$$
$$\frac{dx_2}{d\eta} = x_3$$
$$\frac{dx_3}{d\eta} = x_4$$
$$\frac{dx_4}{d\eta} = \frac{1}{k_1 x_1}\left\{x_2^2 - x_1 x_3 - x_4 + Mn x_2 + 2k_1 x_2 x_4 - k_1 x_3^2\right\} \tag{17}$$
$$\frac{d(x_5)}{d\eta} = x_6$$
$$\frac{dx_6}{d\eta} = \left(2\Pr x_2 - B^*\right)x_5 - \Pr x_1 x_6 - \left(Ec \Pr x_3^2 + A^* x_2\right)$$

Let x_i^r (i =1,2,…6) be an approximate current solution and x_i^{r+1} (i = 1,2,…6) be an improved solution of (17). By taking tailor’s series expansion around the current solution and neglecting the second and higher order derivatives, the coupled first order system (17) is linearized as:

$$\frac{dx_1^{r+1}}{d\eta} = x_2^{r+1}$$
$$\frac{dx_2^{r+1}}{d\eta} = x^{r+1}$$
$$\frac{dx_3^{r+1}}{d\eta} = x_4^{r+1}$$
$$\begin{aligned}\frac{dx_4^{r+1}}{d\eta} = &\left(\frac{-1}{k_1\left(x_1^r\right)^2}\left(\left(x_2^r\right)^2 - x_4^r + Mn x_2^r + 2k_1 x_2^r x_4^r - k_1\left(x_3^r\right)^2\right)\right)x_1^{r+1} \\ &+ \left(\frac{1}{k_1 x_1^r}\left(2x_2^r + 2k_1 x_4^r + Mn\right)\right)x_2^{r+1} + \left(\frac{1}{k_1 x_1^r}\left(-x_1^r - 2k_1 x_3^r\right)\right)x_3^{r+1} \\ &+ \left(\frac{1}{k_1 x_1^r}\left(-1 + 2k_1 x_2^r\right)\right)x_4^{r+1} + \left(\frac{-x_4^r + Mn x_2^r}{k_1 x_1^r}\right)\end{aligned}$$
$$\frac{dx_5^{r+1}}{d\eta} = x_6^{r+1} \tag{18}$$
$$\begin{aligned}\frac{dx_6^{r+1}}{d\eta} = &\left(-\Pr x_6^r\right)x_1^{r+1} + \left(2\Pr x_5^r - A^*\right)x_2^{r+1} - \left(2Ec \Pr x_3^r\right)x_3^{r+1} + \left(2\Pr x_2^r - B^*\right)x_5^{r+1} - \Pr x_1^r x_6^{r+1} \\ &+ \left(Ec \Pr\left(x_3^r\right)^2 - 2\Pr x_2^r x_5^r + \Pr x_1^r x_6^r\right)\end{aligned}$$

The above system of equations is linear in x_i^{r+1} and general solution can be obtained by using the principle of superposition.
The boundary conditions reduce to

$x_1^{r+1}(0)=0,\ x_2^{r+1}(0)=1,\ x_5^{r+1}(0)=1$ for PST case (19a)

$x_1^{r+1}(0)=0,\ x_2^{r+1}(0)=1,\ x_6^{r+1}(0)=-1$ for PHF case (19b)

$$x_2^{r+1}(\eta)\to 0, x_3^{r+1}(\eta)\to 0, x_5^{r+1}(\eta)\to 0 \quad as\ \eta\to\infty$$

The initial values are chosen as follows:

For the homogeneous solution:

$$x_i^{h_1}(\eta)=[0\ 0\ 1\ 0\ 0\ 0]$$
$$x_i^{h_2}(\eta)=[0\ 0\ 0\ 1\ 0\ 0] \qquad (20)$$
$$x_i^{h_3}(\eta)=[0\ 0\ 0\ 0\ 0\ 1] \qquad \text{(PST case)}$$

(OR)

$$x_i^{h_3}(\eta)=[0\ 0\ 0\ 0\ 1\ 0] \qquad \text{(PHF case)}$$

For particular solution:

$$x_i^{p}(\eta)=[R\ 1\ 0\ 0\ 1\ 0] \qquad \text{(PST case)} \qquad (21a)$$

(OR)

$$x_i^{p}(\eta)=[R\ 1\ 0\ 0\ 0\ -1] \qquad \text{(PHF case)} \qquad (21\mathbf{b})$$

The general solution of system of equations is given by

$$x_i^{r+1}(\eta)=C_1x_i^{h_1}(\eta)+C_2x_i^{h_2}(\eta)+C_3x_i^{h_3}(\eta)+x_i^{p}(\eta) \qquad (22)$$

whereC_1, C_2, C_3 are the unknown constants and are determined by considering the boundary conditions as $\eta\to\infty$. This solution $\left(x_i^{r+1}, i=1,2,\ldots 6\right)$ is then compared with solution at the previous step $x_i^{r}, i=1,2,\ldots 6$ and next iteration is performed if the convergence has not been achieved or greater accuracy is desired.

V. RESULTS AND DISCUSSION

Flow and Heat transfer in the steady laminar flow of an incompressible MHD viscoelastic fluid over a stretching sheet with prescribed surface temperature and prescribed heat flux, including viscous dissipation and the non-uniform heat source has been examined. Numerical computations of the results are graphed and discussed.

Fig. 1 depicts temperature profiles θ (η) versus η in PST and PHF cases respectively. It shows that temperature θ (η) increases with the increase in the value of viscoelastic parameter k_1 in both the cases. This is due to the fact that an increase of viscoelastic normal stress gives rise to thickening of thermal boundary layer.

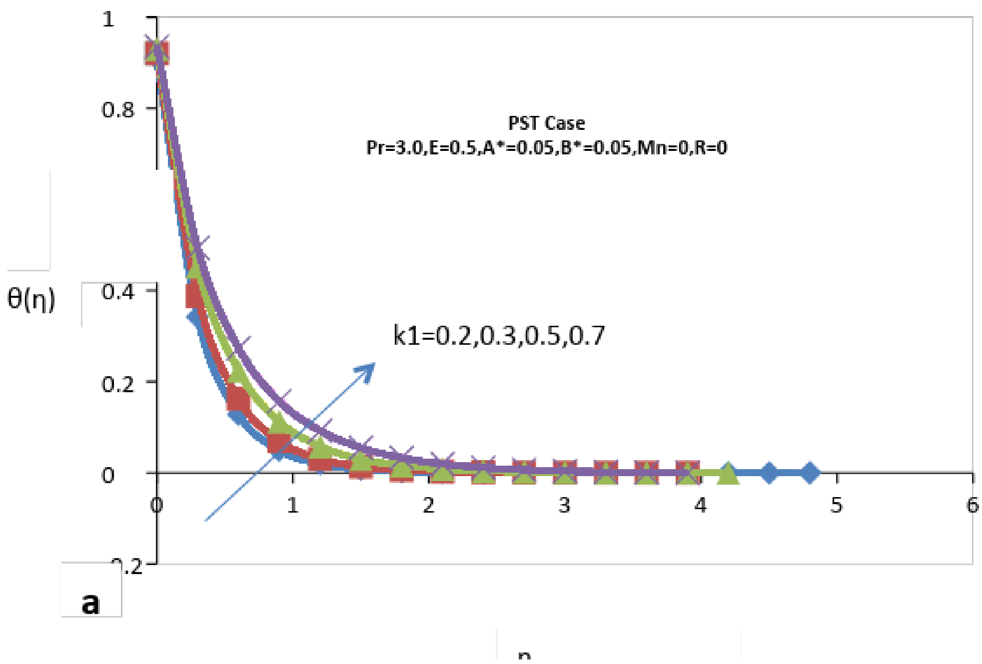
Fig 2(a) and 2(b) reveal that temperature θ (η) decreases with increase in Prandtl number (Pr), which implies viscous boundary layer is thicker than the thermal boundary layer.

Fig 3(a) and 3(b) depict that temperature θ (η) increases with Eckert number. This is due to the fact that heat energy is stored in the liquid due to the frictional heating. The effect of increasing Eckert number is to enhance the temperature at any point.

Figs 4 and 5 depicts that temperature θ (η) increases, when A* and B* are positive (heat source), since thermal boundary layer generates energy. Temperature θ (η) decreases, when A* and B* are negative (absorption).

Fig 6 depicts the effect of magnetic field parameter (Mn) on the horizontal velocity profile (f_η (η)). Horizontal velocity profile decreases with increase in Magnetic field parameter, since increase of Magnetic field parameter signifies the increase of Lorentz force, which opposes the horizontal flow in the reverse direction.
Fig 7(a) and Fig 7(b) depict the effect of viscoelastic parameter k_1 on longitudinal and transverse velocity components. It can be seen, for a fixed value of η, both f' (η) and f (η) decrease with increasing values of viscoelastic parameter k_1. This can be explained by the fact that, as the viscoelastic parameter k_1 increases, the boundary layer adheres strongly to the surface, which in turn retards the flow in longitudinal and transverse directions.
Fig 8 shows the effect of Magnetic field parameter on temperature distribution. Temperature profile increases with increase in Magnetic field. Since increase of magnetic field increases the thermal boundary layer thickness. The increasing frictional drag due to Lorentz force is responsible for increasing the thermal boundary layer thickness.
Fig 9 depicts the effect of suction parameter(R) on the heat transfer θ (η). Temperature profiles decreases with increasing values of suction parameter(R). Due to suction parameter(R) there will be loss of fluid in the boundary layer region, hence there will be less scope for heat transfer from the sheet to the fluid. This causes the declination in the heat transfer for increasing values of suction parameter.
Fig 10 displays that temperature profiles decreases with increasing values of viscoelastic parameter (k_1), when magnetic field is non-zero. This is an important finding in MHD viscoelastic fluid, where opposite behavior can be seen in viscoelastic fluid flows.
From our numerical results, it can be concluded that:

i. Horizontal velocity profile decreases with increase in viscoelastic parameter and magnetic field parameter.
ii. Temperature profiles increases with increase in magnetic field parameter.
iii. Thermal boundary layer thickness decreases with increase in Prandtl number.
iv. Temperature profiles decreases with increasing values of suction parameter (R).
v. Temperature profiles decreases with increasing values viscoelastic parameter k_1 in presence of Magnetic field, whereas opposite behavior can be seen viscoelastic fluids.



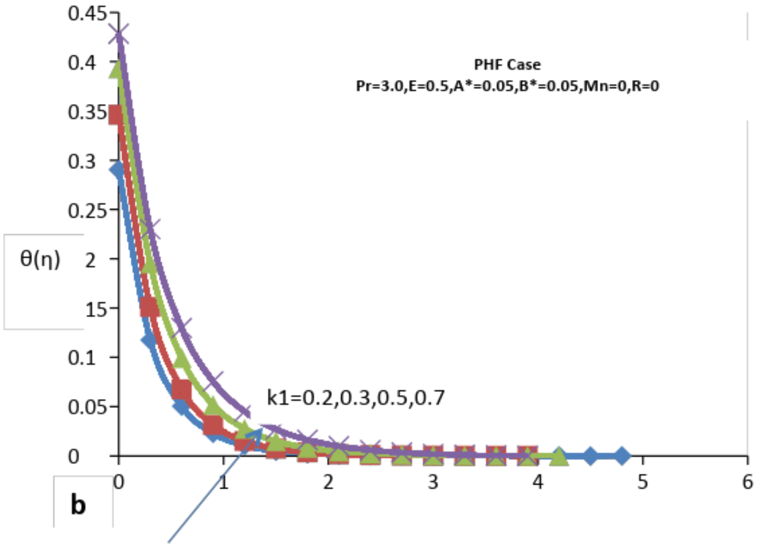


Fig 1. Effect of visco-elasticity (k_1) on temperature distribution in **(a)** PST case **(b)** PHF case

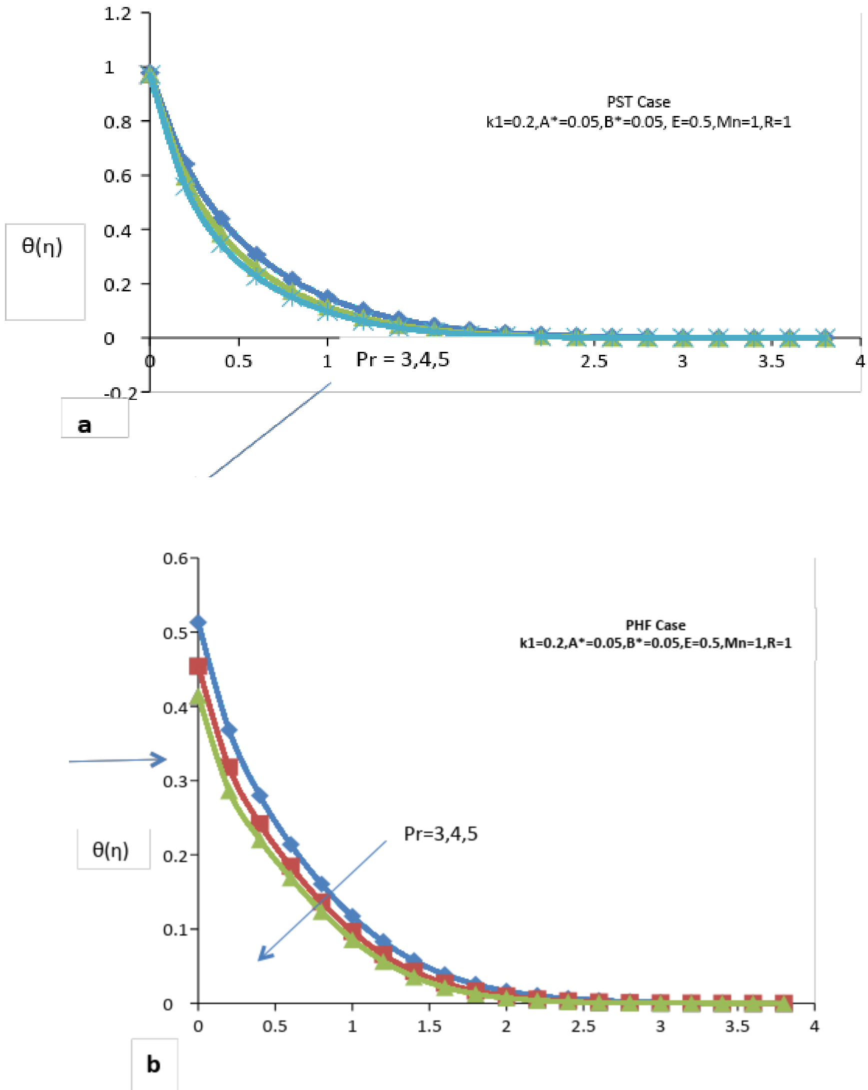


Fig. 2. Effect of Prandtl number (Pr) on temperature distribution in **(a)** PST case **(b)** PHF case

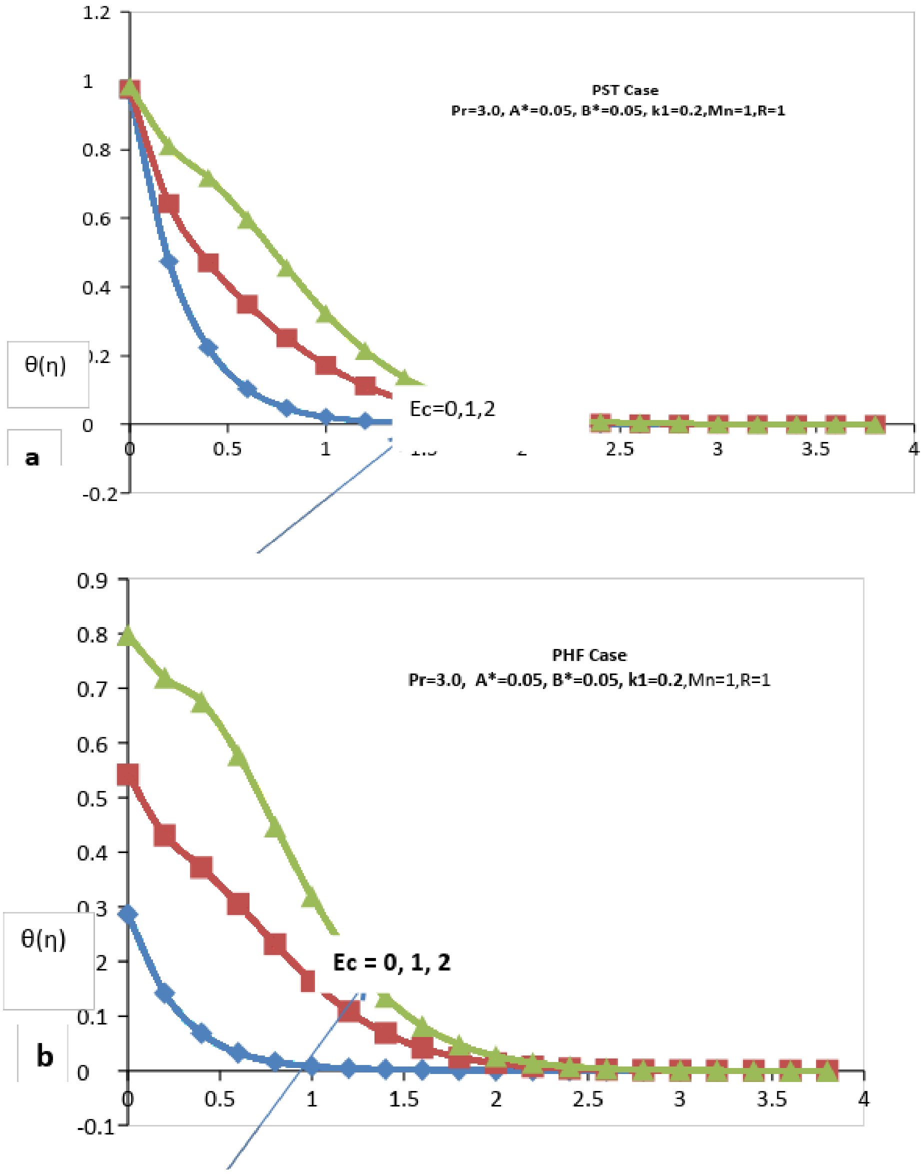
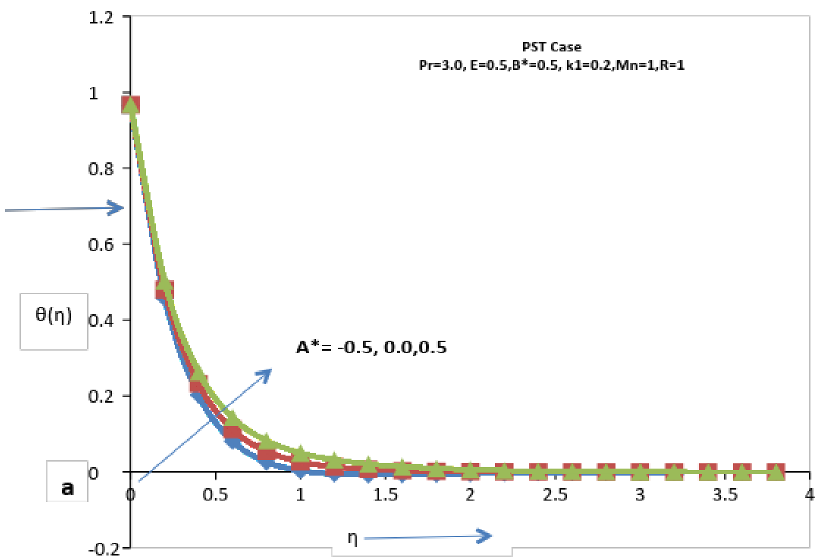


Fig. 3. Effect of Eckert number (Ec) on temperature distribution in **(a)** PST case **(b)** PHF case



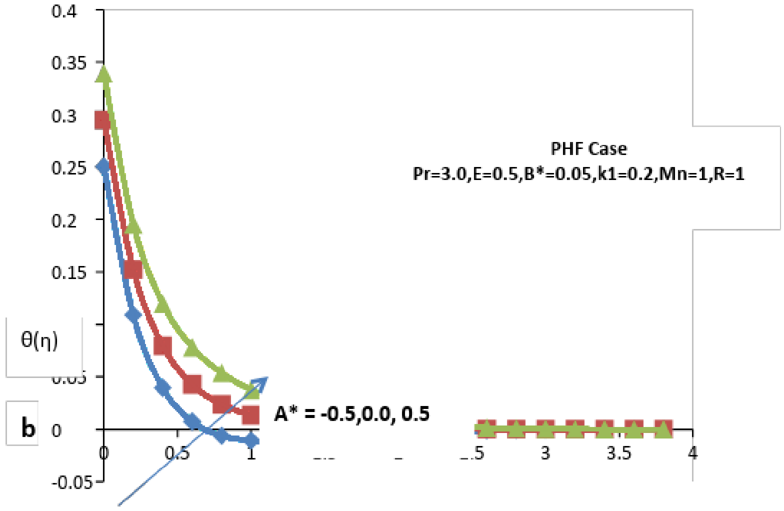


Fig. 4. Effect of non-uniform heat source/sink parameter (A*) on temperature distribution in **(a)** PST case **(b)** PHF case

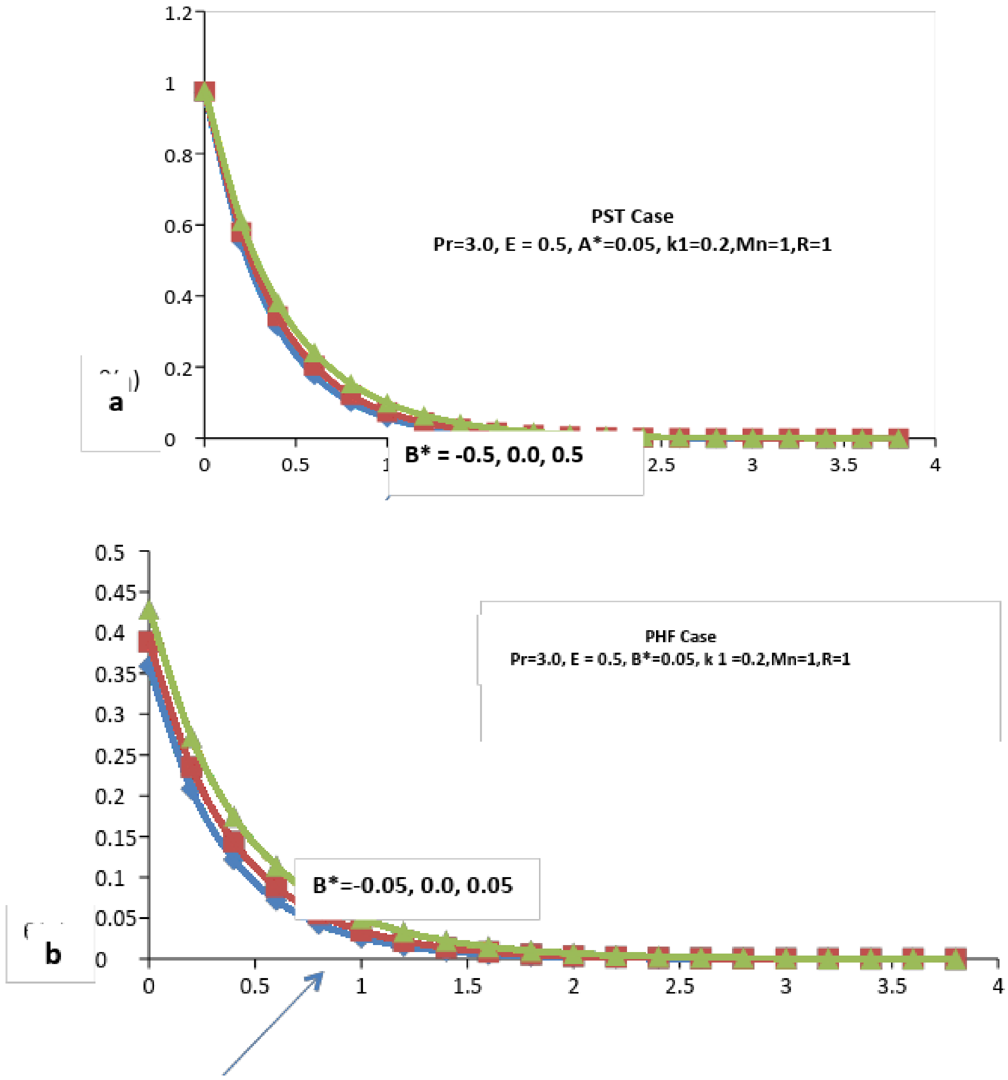


Fig. 5. Effect of non-uniform heat source/sink parameter (B*) on temperature distribution in **(a)** PST case **(b)** PHF case

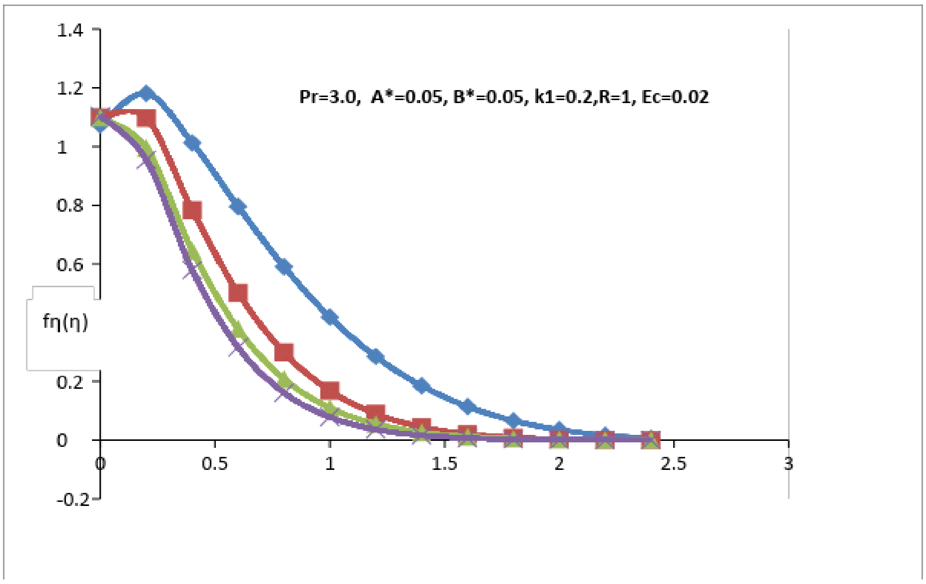


Fig6: Plot of velocity (f_η (η)) vs η for different values of Magnetic parameter (Mn)

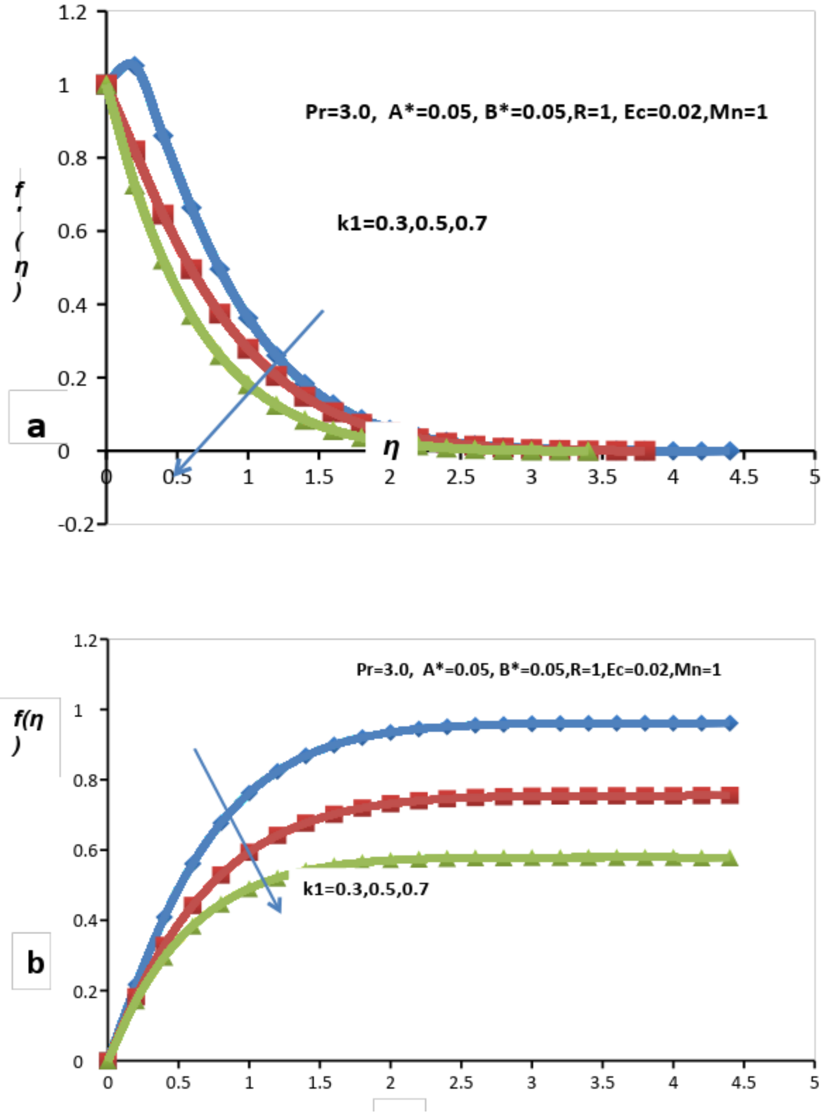


Fig7: Effect of viscoelasticity (k_1) on **(a)** transverse velocity component, **(b)** longitudinal velocity component

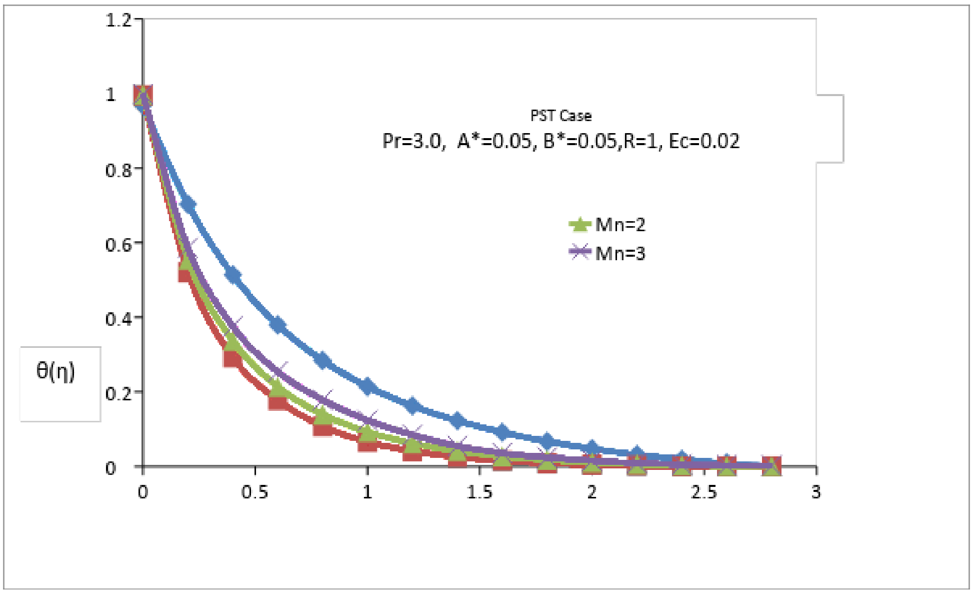


Fig 8. Effect of Magnetic field parameter (Mn) on temperature distribution θ (η)

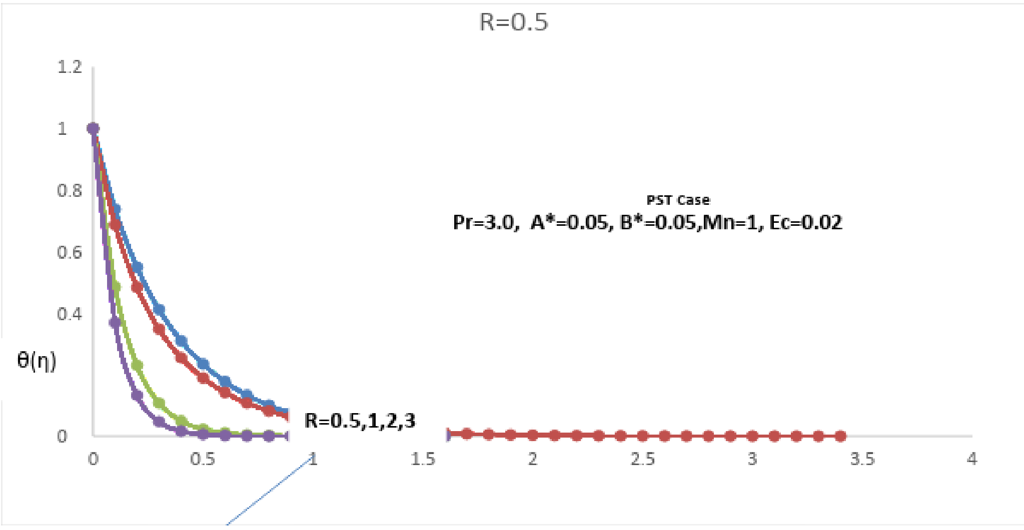


Fig9. Effect of suction parameter (R) on temperature distribution θ (η).

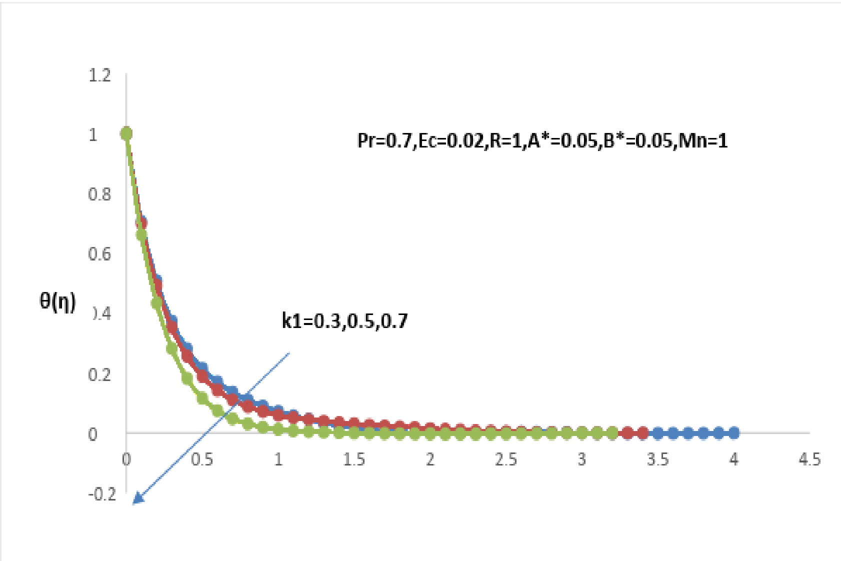


Fig10. Effect of viscoelastic parameter (k_1) on temperature distribution θ (η) in presence of Magnetic field

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