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Investment Decision Making Problem Using Interval Valued Fuzzy Soft Matrix

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Abstract: Fuzzy matrix (FM) is a very important topic in fuzzy algebra. In Fuzzy matrix, the elements belong to the unit interval [0,1]. When the elements of Fuzzy matrix are the subintervals of the unit interval [0,1] then the Fuzzy matrix is known as interval-valued fuzzy matrix. Interval valued fuzzy soft set and interval valued fuzzy soft matrix are those mathematical tools which deal with problems involving uncertain ties and imprecise (or) incomplete data. In the paper, we analyse some basic results of Interval valued fuzzy soft matrix. Investment decision making problem using the notion of Interval valued fuzzy soft matrix.

Keywords: Interval valued fuzzy set (IVFS); Interval valued soft set (IVSS),Interval valued fuzzy soft set(IVFSS), Interval valued fuzzy soft matrix (IVFSM),Interval valued fuzzy softmax min decision making method (IVFSMmDM)

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I. INTRODUCTION

Most of our real life problems in medical sciences, engineering, Management environment and social sciences often involve data which are not necessarily crisp, precise and deterministic in character due to various uncertainties associated with there problems. Such uncertainties are usually being handled with the help of the topics like probability, fuzzy sets intuitionistic fuzzy sets, interval mathematics and rough sets etc. Zadeh first of all (Zadeh) in [1] initiated the concept of fuzzy sets by the extension of classical notion of sets. Now-a-days fuzzy set theory is rapidly progressing but in particular cases we have some limitation such as how to adjust the membership functions in this theory. A new concept if soft set introduced for the solution of these limitations (Molodtsov) [2].

In recent times, researches have contributed a lot towards fuzzi fixation of soft set theory. Maji et al [3] introduced some properties regarding fuzzy soft union, intersection, complement of a fuzzy soft set, Demorgan law ets. There results were further revised and improved by Ahmad and Kharal [4].

By the extension of Interval valued fuzzy soft matrix (Sarala and Prabhavathi) [5] initiated the sanchez's approach for diagnosis of Dengue and chikangkuniya also introduced union and intersection of Interval valued fuzzy soft matrix.

In this paper, we proved commutative laws, associative laws and De-morgan laws by using And-Operation and Or-Operation of Interval valued fuzzy soft max-Min decision making Interval valued fuzzy Mm Dm method with the help of interval valued fuzzy soft max-min decision method. We construct an algorithm for Interval valued fuzzy soft Max-min Decision making method. In this paper we use this method for Investment decision making.

II. PRELIMINARIES :

Definition 2.1
A Pair (F, A) is called a soft set over U if A is any subset of E, and there exist a mapping from A to $P(U)$ is F, $P(U)$ is the parameterized family of subsets of the U but not a set.

Definition 2.2
A fuzzy set A in U is characterized by a membership function $f_A(y_i)$ which associates with each object of U in the interval [0,1], with the value of $f_A(y_i)$ where y_i representing the grade of membership of Y in A.

Definition 2.3
A Pair (F, A) is called Fuzzy soft set over U and there exist a mapping from A to $P(U)$ is F.
$P(U)$ is the collection of fuzzy subsets of U.

Definition 2.4

A Pair (F_A, E) is called a soft set over U, if A is any subset of E. Then a subset R_A of $U \times E$ is defined as $R_A = \{(U, y); y \in A, U \in F_a(y)\}$, is the relation form of (f_A, E) the characteristic function of R_A is written by $\chi_{RA}: U \times E \to \{0,1\}$

$$\chi_{RA}(U, y) = \begin{cases} 1 & (m, y) \in RA \\ 0 & (m, y) \notin RA \end{cases} \qquad (1)$$

it can be written in matrix form such as,

$$[a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

This is called an soft matrix of the soft set (f_A, E) over U of order $m \times n$.

Definition 2.5

A Pair (F, A) is called Fuzzy soft set in the fuzzy soft class (U, E). Then (F, A) is represented in a matrix form such as

$$A_{m\times n} = [a_{ij}]_{m\times n} \;\; or \; A = [a_{ij}](i = 1 \to m), (j = 1 \to n)$$

where

$$a_{ij} = \begin{cases} \mu_j(b_j), if\ y_j \in A \\ 0, \qquad if\ y_j \notin A \end{cases} \qquad (2)$$

Example : 2.1

Let $U = \{z_1, z_2, z_3, z_4\}$ be a universal set and $E = \{y_1, y_2, y_3, y_4\}$ be the set of parameters, $A = \{y_3, y_4\} \subseteq E$, then Fuzzy soft set can be written as

$$(F, A) = \begin{cases} F(y_3) = \{(z_1, 0.50), (z_2, 0.24), (z_3, 0.53), (z_1, 0.16)\} \\ F(y_4) = \{(z_1, 0.23), (z_2, 0.50), (z_3, 0.47), (z_1, 0.22)\} \end{cases}$$

This Fuzzy soft set can be represented in Fuzzy soft matrix by using equation (2) such as

$$(\mathrm{F}, \mathrm{A}) = \begin{array}{c} \mathrm{z}_1 \\ \mathrm{z}_2 \\ \mathrm{z}_3 \\ \mathrm{z}_4 \end{array} \begin{bmatrix} 0.0 & 0.0 & 0.50 & 0.23 \\ 0.0 & 0.0 & 0.24 & 0.50 \\ 0.0 & 0.0 & 0.53 & 0.47 \\ 0.0 & 0.0 & 0.26 & 0.22 \end{bmatrix}$$

Definition 2.6

A Pair (F, A) is called Interval valued fuzzy soft set over U where F is a mapping such that $F: A \to I^M$, where I^M represent the all interval valued fuzzy subsets (IVFSbs) of U.

Definition 2.7

A Pair (F, A) is called Interval valued fuzzy soft set over U where F is a mapping such that $F: A \to I^M$, where I^M represent the all interval valued fuzzy subsets (IVFSbs) of U. Then the interval valued fuzzy subsets can be in expressed in matrix form as

$$A_{m\times n} = [a_{ij}]_{m\times n} \;\; or \; A = [a_{ij}](i = 1 \to m), (j = 1 \to n)$$

where

$$a_{ij} = \begin{cases} \mu_{jL}(b_j), \mu_{jU}(b_j), if\ y_j \in A \\ \quad [0,0], \qquad if\ y_j \notin A \end{cases}$$

where $[\mu_{jL}(b_j), \mu_{jU}(b_j)]$ represent the membership of b_i in the Interval fuzzy soft set y_j

$$IVFSF = (y_j) \qquad (3)$$

Example : 2.2

Let $U = \{v_1, v_2, v_3, v_4\}$ be a universal set and $E = \{y_1, y_2, y_3, y_4\}$ be the set of parameters, Consider $A = \{y_3, y_4\} \subseteq E, F: A \to P(U)$. Then, Interval valued fuzzy soft set is

$$(F, A) = \begin{cases} F(y_3) = \begin{Bmatrix} (v_1, [0.5,0.7]), (v_2, [0.3,0.4]), \\ (v_3, [0.6,0.7]), (v_4, [0.3,0.4]) \end{Bmatrix} \\ F(y_4) = \begin{Bmatrix} (v_1, [0.3,0.4]), (v_2, [0.6,0.7]), \\ (v_3, [0.5,0.6]), (v_4, [0.3,0.4]) \end{Bmatrix} \end{cases}$$

This interval valued fuzzy soft matrix by using equation (3)

$$(\mathrm{F}, \mathrm{A}) = \begin{array}{c} \mathrm{v}_1 \\ \mathrm{v}_2 \\ \mathrm{v}_3 \\ \mathrm{v}_4 \end{array} \begin{bmatrix} (0.0,0.0) & (0.0,0.0) & (0.5,0.7) & (0.3,0.4) \\ (0.0,0.0) & (0.0,0.0) & (0.3,0.4) & (0.6,0.7) \\ (0.0,0.0) & (0.0,0.0) & (0.6,0.7) & (0.5,0.6) \\ (0.0,0.0) & (0.0,0.0) & (0.3,0.4) & (0.3,0.4) \end{bmatrix}$$

Definition 2.8
A and B are two IVFS matrix then A is said to be IVFS submatrix of B if $\mu_{AL} \leq \mu_{BL}$ and $\mu_{AU} \leq \mu_{BU}$ for all i and j, it is denoted by $A \subseteq B$.
Example 2.3
Let $A, B \subseteq E$ such that $A \subseteq B$. $A = \{y_3, y_4\}$ and $B = \{y_2, y_3, y_4\}$.
Then

$$(F,A) = \begin{cases} F(y_3) = \begin{Bmatrix} (v_1,[0.4,0.5]),(v_2,[0.3,0.4]), \\ (v_3,[0.6,0.7]),(v_4,[0.3,0.4]) \end{Bmatrix} \\ F(y_4) = \begin{Bmatrix} (v_1,[0.3,0.4]),(v_2,[0.4,0.5]), \\ (v_3,[0.5,0.6]),(v_4,[0.3,0.4]) \end{Bmatrix} \end{cases}$$

$$(F,B) = \begin{cases} F(y_2) = \begin{Bmatrix} (v_1,[0.5,0.6]),(v_2,[0.7,0.8]), \\ (v_3,[0.6,0.7]),(v_4,[0.4,0.5]) \end{Bmatrix} \\ F(y_3) = \begin{Bmatrix} (v_1,[0.4,0.5]),(v_2,[0.3,0.4]), \\ (v_3,[0.6,0.7]),(v_4,[0.2,0.3]) \end{Bmatrix} \\ F(y_4) = \begin{Bmatrix} (v_1,[0.3,0.4]),(v_2,[0.4,0.5]), \\ (v_3,[0.5,0.6]),(v_4,[0.3,0.4]) \end{Bmatrix} \end{cases}$$

There IVFSS can be represented in IVFSM such as by using equation 3

$$(\mathrm{F,A}) = \begin{array}{c} \mathrm{v}_1 \\ \mathrm{v}_2 \\ \mathrm{v}_3 \\ \mathrm{v}_4 \end{array} \begin{bmatrix} (0.0,0.0) & (0.0,0.0) & (0.4,0.5) & (0.3,0.4) \\ (0.0,0.0) & (0.0,0.0) & (0.3,0.4) & (0.4,0.5) \\ (0.0,0.0) & (0.0,0.0) & (0.6,0.7) & (0.5,0.6) \\ (0.0,0.0) & (0.0,0.0) & (0.3,0.4) & (0.3,0.4) \end{bmatrix}$$

$$(\mathrm{F,B}) = \begin{array}{c} \mathrm{v}_1 \\ \mathrm{v}_2 \\ \mathrm{v}_3 \\ \mathrm{v}_4 \end{array} \begin{bmatrix} (0.0,0.0) & (0.5,0.6) & (0.4,0.5) & (0.3,0.4) \\ (0.0,0.0) & (0.7,0.8) & (0.3,0.4) & (0.4,0.5) \\ (0.0,0.0) & (0.6,0.7) & (0.6,0.7) & (0.5,0.6) \\ (0.0,0.0) & (0.4,0.5) & (0.2,0.3) & (0.3,0.4) \end{bmatrix}$$

where $(\mathrm{F,A}) \subseteq (\mathrm{F,B})$
Definition 2.9
A $[a_{ij}]$ be an IVFSM of order $m \times n$, then transpose of IVFSM can be defined as $A^T = [a_{ji}]$ of order $n \times m$. where $[a_{ij}] = [\mu_{jL}(b_i), \mu_{jU}(b_i)](i = 1 \to m)$ and $(j = 1 \to n)$.
Example : 2.4
Consider A be Interval valued fuzzy soft matrix of order 2×2

$$\mathrm{A} = \begin{bmatrix} (0.2,0.4) & (0.3,0.4) \\ (0.4,0.2) & (0.8,0.3) \end{bmatrix}$$

$$\mathrm{A}^{\mathrm{T}} = \begin{bmatrix} (0.2,0.4) & (0.4,0.2) \\ (0.3,0.4) & (0.8,0.3) \end{bmatrix}$$

Definition : 2.10
The addition of two Interval valued fuzzy soft matrix A and B is conformable if their order is same, it is defined as $A + B = [Max(\mu_{AL}, \mu_{BL}), Max(\mu_{AU}, \mu_{BU})]$ for all i and j.
Example : 2.5
A and B are two Interval valued fuzzy soft matrices

$$\mathrm{A} = \begin{bmatrix} (0.3,0.6) & (0.5,0.7) \\ (0.3,0.5) & (0.4,0.9) \end{bmatrix}$$

$$\mathrm{B} = \begin{bmatrix} (0.5,0.8) & (0.8,0.9) \\ (0.4,0.7) & (0.8,0.9) \end{bmatrix}$$

Then

$$\mathrm{A} + \mathrm{B} = \begin{bmatrix} (0.5,0.8) & (0.8,0.9) \\ (0.4,0.7) & (0.8,0.9) \end{bmatrix}$$

Definition : 2.11
The subtraction of two Interval valued fuzzy soft matrix A and B is conformable if their order is same, it is defined as $A - B = [min(\mu_{AL}, \mu_{BL}), max(\mu_{AU}, \mu_{BU})]$ for all i and j.
Example : 2.6
A and B are two Interval valued fuzzy soft matrices

$$\mathrm{A} = \begin{bmatrix} (0.2,0.5) & (0.5,0.6) \\ (0.3,0.4) & (0.3,0.7) \end{bmatrix}$$

$$\mathrm{B} = \begin{bmatrix} (0.5,0.8) & (0.8,0.9) \\ (0.4,0.7) & (0.8,0.8) \end{bmatrix}$$

Then

$$A - B = \begin{bmatrix} (0.2,0.5) & (0.5,0.6) \\ (0.3,0.4) & (0.3,0.7) \end{bmatrix}$$

Definition 2.12

Let $A = [a_{ij}]$ and $B = [b_{ij}]$ are two Interval valued fuzzy soft matrices of order $m \times n$ and $n \times p$ respectively, then their product defined as,

$$A * B = [c_{ik}]_{m\times p} = [Max(\mu_{ALj} * \mu_{BLj}), Max(\mu_{AUj} * \mu_{BUj})] \forall\, i,j,k$$

$$A = \begin{bmatrix} (0.2,0.5) & (0.4,0.6) \\ (0.2,0.4) & (0.3,0.8) \end{bmatrix} \text{and } B = \begin{bmatrix} (0.4,0.7) & (0.7,0.8) \\ (0.3,0.6) & (0.7,0.9) \end{bmatrix}$$

Then

$$A * B = \begin{bmatrix} (0.2,0.5) & (0.4,0.6) \\ (0.2,0.4) & (0.3,0.8) \end{bmatrix} * \begin{bmatrix} (0.4,0.7) & (0.7,0.8) \\ (0.3,0.6) & (0.7,0.9) \end{bmatrix}$$

$$= \begin{bmatrix} (0.12,0.36) & (0.28,0.54) \\ (0.09,0.48) & (0.21,0.72) \end{bmatrix}$$

Definition 2.13

A $[a_{ij}]$ be an IVFSM of order $m \times n$, where $[a_{ij}] = [\mu_{jL}(b_i), \mu_{jU}(b_i)]$. Then its complement is defined as $A^c = [b_{ij}]$ where $b_{ij} = [1 - \mu_{jU}(b_i), 1 - \mu_{jL}(b_i)] \forall\, i,j$

Example : 2.8

$$A = \begin{bmatrix} (0.4,0.6) & (0.3,0.7) \\ (0.3,0.5) & (0.5,0.7) \end{bmatrix}$$

Then its complement is defined as

$$A^c = \begin{bmatrix} (0.4,0.6) & (0.3,0.7) \\ (0.5,0.7) & (0.3,0.5) \end{bmatrix}$$

Definition 2.14

An Interval valued fuzzy soft matrix of any order is called null Interval valued fuzzy soft matrix in which $[a_{ij}] = [0,0]$ for all i,j it can be represented as

$$O = \begin{bmatrix} (0,0) & (0,0) & (0,0) & (0,0) \\ (0,0) & (0,0) & (0,0) & (0,0) \\ (0,0) & (0,0) & (0,0) & (0,0) \end{bmatrix}$$

Definition 2.15

An Interval valued fuzzy soft matrix of any order is called universal Interval valued fuzzy soft matrix in which $[a_{ij}] = [1,1]$ for all i,j it can be represented as

$$U = \begin{bmatrix} (1,1) & (1,1) & (1,1) & (1,1) \\ (1,1) & (1,1) & (1,1) & (1,1) \\ (1,1) & (1,1) & (1,1) & (1,1) \end{bmatrix}$$

Definition 2.16

Let $A = [a_{ij}]$ and $B = [b_{ik}]$ are two Interval valued fuzzy soft matrices of same order $m \times n$ then And – product is defined as

$$A: A \times B \to C^2_{m\times n}, [a_{ij}]_{m\times n} \wedge [b_{ik}]_{m\times n} = [c_{ip}]^2_{m\times n}$$

where $c_{ip} = [min(\mu_{ALj}, \mu_{BLj}), min(\mu_{AUj}, \mu_{BUj})] \forall\, i,j,k$ and $p = n(j-1) + k$

Example : 2.9

Consider A and B are two Interval valued fuzzy soft matrices

$$A = \begin{bmatrix} (0.2,0.5) & (0.5,0.7) \\ (0.2,0.4) & (0.3,0.8) \end{bmatrix}_{2\times 2} \text{and } B = \begin{bmatrix} (0.4,0.7) & (0.7,0.8) \\ (0.3,0.6) & (0.8,0.9) \end{bmatrix}_{2\times 2}$$

Then

$$A \wedge B = \begin{bmatrix} (0.2,0.5) & (0.5,0.7) \\ (0.2,0.4) & (0.3,0.8) \end{bmatrix} \wedge \begin{bmatrix} (0.4,0.7) & (0.7,0.8) \\ (0.3,0.6) & (0.8,0.9) \end{bmatrix}$$

$$= \begin{bmatrix} (0.2,0.5) & (0.2,0.5) & (0.4,0.7) & (0.5,0.7) \\ (0.2,0.4) & (0.2,0.4) & (0.3,0.6) & (0.3,0.8) \end{bmatrix}$$

Definition 2.17

Let $A = [a_{ij}]$ and $B = [b_{ik}]$ are two Interval valued fuzzy soft matrices of same order $m \times n$ then Or – product is defined as

$$V: A \times B \to C^2_{m\times n}, [a_{ij}]_{m\times n} \vee [b_{ik}]_{m\times n} = [c_{ip}]^2_{m\times n}$$

where $c_{ip} = [max(\mu_{ALj}, \mu_{BLj}), max(\mu_{AUj}, \mu_{BUj})] \forall\, i,j,k$ and $p = n(j-1) + k$

$$A = \begin{bmatrix} (0.2,0.5) & (0.5,0.7) \\ (0.2,0.4) & (0.3,0.8) \end{bmatrix}_{2\times 2} \text{ and } B = \begin{bmatrix} (0.4,0.7) & (0.7,0.8) \\ (0.3,0.6) & (0.8,0.9) \end{bmatrix}_{2\times 2}$$

$$A \vee B = \begin{bmatrix} (0.2,0.5) & (0.5,0.7) \\ (0.2,0.4) & (0.3,0.8) \end{bmatrix} \vee \begin{bmatrix} (0.4,0.7) & (0.7,0.8) \\ (0.3,0.6) & (0.8,0.9) \end{bmatrix}$$

$$= \begin{bmatrix} (0.4,0.7) & (0.7,0.8) & (0.5,0.7) & (0.7,0.8) \\ (0.3,0.6) & (0.8,0.9) & (0.3,0.8) & (0.8,0.9) \end{bmatrix}$$

III. OPERATIONS ON INTERVAL VALUED FUZZY SOFT MATRICES

In This section we prove the De-morgan laws, Associative law and commutative laws by using the definitions of And-Operation and Or-Operation of Interval valued fuzzy soft matrices.

Definition : 3.1

Let $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$ are two Interval valued fuzzy soft matrices, then their OR-Operation is defined as

$A \vee B = [c_{ij}]_{m\times n}$ where $c_{ij} = [max(\mu_{AL}, \mu_{BL}), max(\mu_{AU}, \mu_{BU})] \forall\, i, j$

Commutative Law :

$$A \vee B = B \vee A$$

Proof :

Let $A = [a_{ij}] = [\mu_{AL}, \mu_{AU}]$ and $B = [b_{ij}] = [\mu_{BL}, \mu_{BU}]$ are two Interval valued fuzzy soft matrices of order $m \times n$, then

LHS $= A \vee B$

$= [\mu_{AL}, \mu_{AU}] \vee [\mu_{BL}, \mu_{BU}]$

$= [max(\mu_{AL}, \mu_{BL}), max(\mu_{AU}, \mu_{BU})]$

$= [max(\mu_{AL}, \mu_{BL}), max(\mu_{AU}, \mu_{BU})]$

$= B \vee A$

$= R.H.S.$

Associative Law :

$$(A \vee B) \vee C = A \vee (B \vee C)$$

Proof :

Let $A = [a_{ij}] = [\mu_{AL}, \mu_{AU}]$, $B = [b_{ij}] = [\mu_{BL}, \mu_{BU}]$ and $C = [c_{ij}] = [\mu_{CL}, \mu_{CU}]$ are two Interval valued fuzzy soft matrices of order $m \times n$, then

LHS $= (A \vee B) \vee C$

$= [\mu_{AL}, \mu_{AU}] \vee [\mu_{BL}, \mu_{BU}] \vee [\mu_{CL}, \mu_{CU}]$

$= [max(\mu_{AL}, \mu_{BL}), max(\mu_{AU}, \mu_{BU})] \vee [max(\mu_{CL}, \mu_{CU})]$

$= [max(\mu_{AL}, \mu_{BL}, \mu_{CL}), max(\mu_{AU}, \mu_{BU}, \mu_{CU})]$

$= A \vee (B \vee C)$

$= R.H.S.$

Commutative Law :

$$A \wedge B = B \wedge A$$

Proof :

Let $A = [a_{ij}] = [\mu_{AL}, \mu_{AU}]$ and $B = [b_{ij}] = [\mu_{BL}, \mu_{BU}]$ are two Interval valued fuzzy soft matrices of order $m \times n$, then

LHS $= A \wedge B$

$= [\mu_{AL}, \mu_{AU}] \wedge [\mu_{BL}, \mu_{BU}]$

$= [min(\mu_{AL}, \mu_{BL}), min(\mu_{AU}, \mu_{BU})]$

$= [max(\mu_{BL}, \mu_{AL}), min(\mu_{BU}, \mu_{AU})]$

$= B \wedge A$

$= R.H.S.$

Associative Law :

$$(A \wedge B) \wedge C = A \wedge (B \wedge C)$$

Proof :

Let $A = [a_{ij}] = [\mu_{AL}, \mu_{AU}]$, $B = [b_{ij}] = [\mu_{BU}, \mu_{BU}]$ and $C = [c_{ij}] = [\mu_{CL}, \mu_{CU}]$ are two Interval valued fuzzy soft matrices of order $m \times n$, then

LHS $= (A \wedge B) \wedge C$

$= ([\mu_{AL}, \mu_{AU}] \wedge [\mu_{BL}, \mu_{BU}]) \wedge [\mu_{CL}, \mu_{CU}]$

$= \left(min(\mu_{AL}, \mu_{BL}), min(\mu_{AU}, \mu_{BU})\right) \wedge (\mu_{CL}, \mu_{CU})$

$= [min(\mu_{AL}, \mu_{BL}, \mu_{CL}), min(\mu_{AU}, \mu_{BU}, \mu_{CU})]$

$= [\mu_{AL}, \mu_{AU}] \wedge \left(min(\mu_{BL}, \mu_{CL}), min(\mu_{BU}, \mu_{CU})\right)$

$= A \wedge (B \wedge C)$
$= R.H.S.$

Demorgan Laws
Let $C = [c_{ij}] = [\mu_{cL}, \mu_{cU}]$ and $D = [d_{ij}] = [\mu_{DL}, \mu_{DU}]$ are two Interval valued fuzzy soft matrices of order $m \times n$, then
(i) $(C \vee D)^c = C^c \wedge D^c$
(ii) $(C \wedge D)^c = C^c \vee D^c$
Proof : (i)
L.H.S. $(C \vee D)^c = ([\mu_{cL}, \mu_{cU}] \vee [\mu_{DL}, \mu_{DU}])^c$
$= (max[\mu_{cL}, \mu_{cU}], max[\mu_{cU}, \mu_{DU}])^c$
$= (1 - max[\mu_{cL}, \mu_{cU}], 1 - max[\mu_{cU}, \mu_{DU}])$
$= [min(1 - \mu_{cL}, 1 - \mu_{DU}), min(1 - \mu_{cL}, 1 - \mu_{DU})]$
$= C^c \wedge D^c$
$= R.H.S.$
Proof : (ii)
L.H.S. $(C \wedge D)^c = ([\mu_{cL}, \mu_{cU}] \wedge [\mu_{DL}, \mu_{DU}])^c$
$= (min[\mu_{cL}, \mu_{cU}], min[\mu_{cU}, \mu_{DU}])^c$
$= (1 - min[\mu_{cL}, \mu_{cU}], 1 - min[\mu_{cU}, \mu_{DU}])$
$= [min(1 - \mu_{cL}, 1 - \mu_{DU}), min(1 - \mu_{cL}, 1 - \mu_{DU})]$
$= [max(1 - \mu_{cL}, 1 - \mu_{DU}), max(1 - \mu_{cL}, 1 - \mu_{DU})]$
$= C^c \vee D^c$
$= R.H.S.$

IV. INTERVAL VALUED FUZZY SOFT MAX-MIN DECISION MAKING

Definition : 4.1
Let $[cip] \in IVFSM^2_{m\times n} I_{\overline{K}}\{p: \exists i, cip \neq 0, (k-1)n < p \leq kn\}$ for all $k \in I = \{1,2, \dots, n\}$. Then IVFS-max-min decision function denoted by $Mm: IVFSM^2_{m\times n} \to IVFSM_{m\times 1}$

$$Mm[cip] = [d_{i1}] = [max\{t_{ik}\}$$

where $t_{ik} = \begin{cases} \min p \in I_k \; \{cip\}, if \; I_k \neq 0 \\ [0.0,0.0], \qquad if \; I_k = 0 \end{cases}$ (5)

where $cip = [\min(\mu_{ALj}, \mu_{BLj}), \min(\mu_{AUj}, \mu_{BUj})$ for all i, j, k, such that $p = n(j-1) + k$, which is one column Interval valued fuzzy soft matrix $Mm[cip]$ is called Max-Min decision Interval valued fuzzy soft matrix.
Definition : 4.2
$M = \{m_1, m_2, \dots, m_n\}$ be a universal set and max-min $[cip] = [d_{i1}]$. Then optimum fuzzy set on U is defined as $Opt\,[d_{i1}](m) = \{d_{i1}/m_i : m_i \in M, d_{i1} \neq 0\}$ which is called optimum Interval valued fuzzy set on M.

4.1. GENERALIZED IVFSM in Decision making problem

Now, we apply the concepts of interval valued fuzzy soft matrices to investment of funds in investment decision making problem, according to these parameters such as safety of funds, high return, maximum profit in minimum period, maximum period, tax concession. Now, by using the definition of IVFSMmDn. We construct an algorithm for decision making.

ALGORITHM
1. Select the appropriate subsets from the set of parameters.
2. Construt the IVFSM be selected set of parameters.
3. Take the product of the IVFS-matrix by AND product.
4. Find Max-Min decision IVFSM.
5. Finally we find optimum IVFSM on U.

APPLICATION
Let$U = \{Bank\ Deposit, Gold, real\ Estate, Insurance\}, P =$
$\{Safety, Finds, High\ return, Maximum\ profit\ in\ Minimum\ Period,\}$. To apply Interval valued fuzzy soft matrices to this investment decision problem. Considering the various investment area as universal set $U = \{I_1, I_2, I_3, I_4\}$ and factors influence investment decision as parameters given $P = \{P_1, P_2, P_3, P_4\}$.
Example: 4.1
Two friends A and B are wants to safety their funds in any one of the universal set U. If both friends have their own set of parameters. Then we select a safety of funds on the basis of the sets of friend's parameters by using the IVFSMmDm as follows: Suppose $U = \{I_1, I_2, I_3, I_4\}$ be the universal set and $P = \{P_1, P_2, P_3, P_4\}$ be the set of parameters.

Step 1 :
First of all A and B both friends choose the set of parameters $A = \{P_1, P_2, P_4\}, B = \{P_1, P_2, P_3\}$
Step 2 :
Now, we construct the IVFS-matrices by using

$$a_{ij} = \begin{cases} \mu_{jL}(b_j), \mu_{jU}(b_j), if\ y_j \in A \\ [0,0], \qquad if\ y_j \notin A \end{cases}$$

according to selected parameters of both friends,

$$A = [a_{ij}] = \begin{bmatrix} (0.4,0.5) & (0.3,0.4) & (0.0,0.0) & (0.2,0.3) \\ (0.7,0.8) & (0.6,0.6) & (0.0,0.0) & (0.5,0.6) \\ (0.3,0.4) & (0.3,0.5) & (0.0,0.0) & (0.2,0.3) \\ (0.8,0.8) & (0.7,0.8) & (0.0,0.0) & (0.4,0.5) \end{bmatrix}$$

$$B = [b_{ij}] = \begin{bmatrix} (0.6,0.7) & (0.4,0.5) & (0.3,0.4) & (0.0,0.0) \\ (0.7,0.8) & (0.6,0.7) & (0.2,0.3) & (0.0,0.0) \\ (0.4,0.5) & (0.3,0.5) & (0.2,0.3) & (0.0,0.0) \\ (0.8,0.9) & (0.8,0.9) & (0.5,0.6) & (0.0,0.0) \end{bmatrix}$$

Step 3 :
Now, we find product of IVFS matrices by using
AND Product

$$[a_{ij}] \wedge [b_{ij}] = \begin{bmatrix} (0.4,0.5) & (0.3,0.4) & (0.0,0.0) & (0.2,0.3) \\ (0.7,0.8) & (0.6,0.6) & (0.0,0.0) & (0.5,0.6) \\ (0.3,0.4) & (0.3,0.5) & (0.0,0.0) & (0.2,0.3) \\ (0.8,0.8) & (0.7,0.8) & (0.0,0.0) & (0.4,0.5) \end{bmatrix} \wedge \begin{bmatrix} (0.6,0.7) & (0.4,0.5) & (0.3,0.4) & (0.0,0.0) \\ (0.7,0.8) & (0.6,0.7) & (0.2,0.3) & (0.0,0.0) \\ (0.4,0.5) & (0.3,0.5) & (0.2,0.3) & (0.0,0.0) \\ (0.8,0.9) & (0.8,0.9) & (0.5,0.6) & (0.0,0.0) \end{bmatrix}$$

Step 4
To calculate $\mathrm{Mm}([a_{ij}] \wedge [b_{ik}]) = [d_{i1}]$ in this step, we find d_{i1} for all $i = 1,2,3,4$. First of all we find d_{11}
$d_{11} = \max\{t_{1k}\} = \max\{t_{11}, t_{12}, t_{13}, t_{14}\}$, where $k \in \{1,2,3,4\}$ for d_{11}, we also find t_{1k} for all $k \in \{1,2,3,4\}$.
When $k = 1$ and $n = 4$, then t_{11} is $I_1 = \{p: c_{ip} \neq 0, 0 < p \leq 4\}$ and
When we find t_{2k} then $k = 2$ and $n = 4$ and $I_2 = \{p: c_{ip} \neq 0, 4 < p \leq 8\}$
Similarly, for $k = 3$, $I_3 = \{p: c_{ip} \neq 0, 8 < p \leq 12\}$ and $k = 4$, $I_4 = \{p: c_{ip} \neq 0, 12 < p \leq 16\}$
$t_{11} = \min\{c_{11}, c_{12}, c_{13}\}$, where $c_{ip} = [\min\{\mu_{ALJ}, \mu_{BLJ}\}, \min\{\mu_{AUJ}, \mu_{BUJ}\}]$
$t_{11} = \min\{[0.4,0.5], [0.4,0.5], [0.3,0.4]\}$
$t_{11} = [0.3,0.4]$
Similarly, we obtain other values,
$t_{12} = \min\{c_{15}, c_{16}, c_{17}\}$
$t_{12} = \min\{[0.3,0.4], [0.3,0.4], [0.3,0.4]\}$
$t_{12} = [0.3,0.4]$
$t_{13} = [0.0,0.0]$
$t_{14} = \min\{c_{11}, c_{14}, c_{15}\}$
$t_{14} = \min\{[0.2,0.3], [0.2,0.3], [0.2,0.3]\}$
$t_{14} = [0.2,0.3]$

So, $d_{11} = \max\{t_{11}, t_{12}, t_{13}, t_{14}\} = \max\{[0.3,0.4], [0.3,0.4][0.0,0.0], [0.2,0.3]\}$

$d_{11} = [0.3,0.4]$,

Similarly, we find d_{21}, d_{31}, d_{41}

$d_{21} = \max\{t_{2k}\} = \max\{t_{21}, t_{22}, t_{23}, t_{24}\}$ where $k \in \{1,2,3,4\}$

$t_{21} = \min\{c_{21}, c_{22}, c_{23}\}$
$t_{21} = \min\{[0.7,0.8], [0.6,0.7], [0.2,0.3]\}$
$t_{21} = [0.2,0.3]$
$t_{22} = \min\{c_{25}, c_{26}, c_{27}\}$
$t_{22} = \min\{[0.6,0.6], [0.6,0.6], [0.2,0.3]\}$
$t_{22} = [0.2,0.3]$
$t_{23} = [0.0,0.0]$

$t_{24} = \min\{c_{213}, c_{214}, c_{215}\}$
$t_{22} = \min\{[0.5,0.6], [0.5,0.6], [0.2,0.3]\}$
$t_{22} = [0.2,0.3]$

$$\text{So, } d_{21} = \max\{t_{21}, t_{22}, t_{23}, t_{24}\} = \max\{[0.2,0.3], [0.2,0.3], [0.0,0.0], [0.2,0.3]\}$$
$$d_{21} = [0.2,0.3]$$

$d_{31} = \max\{t_{3k}\} = \max\{t_{31}, t_{32}, t_{33}, t_{34}\}$
$t_{31} = \min\{c_{31}, c_{32}, c_{33}\}$
$t_{31} = \min\{[0.3,0.4], [0.3,0.4], [0.2,0.3]\}$
$t_{31} = [0.2,0.3]$
$t_{32} = \min\{c_{35}, c_{36}, c_{37}\}$
$t_{32} = \min\{[0.3,0.5], [0.3,0.5], [0.2,0.3]\}$
$t_{32} = [0.2,0.3]$
$t_{33} = [0.0,0.0]$
$t_{34} = \min\{c_{313}, c_{314}, c_{315}\}$
$t_{34} = \min\{[0.2,0.3], [0.2,0.3], [0.2,0.3]\}$
$t_{34} = [0.2,0.3]$

$$d_{31} = \max\{[0.2,0.3], [0.2,0.3], [0.0,0.0], [0.2,0.3]\}$$
$$d_{31} = [0.2,0.3]$$
$$d_{41} = \max\{t_{4k}\} = \max\{t_{41}, t_{42}, t_{43}, t_{44}\}$$

$t_{41} = \min\{c_{41}, c_{42}, c_{43}\}$
$t_{41} = \min\{[0.8,0.8], [0.8,0.8], [0.5,0.6]\}$
$t_{41} = [0.5,0.6]$
$t_{42} = \min\{c_{45}, c_{46}, c_{47}\}$
$t_{42} = \min\{[0.7,0.8], [0.7,0.8], [0.5,0.6]\}$
$t_{42} = [0.5,0.6]$
$t_{42} = \min\{c_{45}, c_{46}, c_{47}\}$
$t_{43} = [0.0,0.0]$
$t_{44} = \min\{c_{413}, c_{414}, c_{415}\}$
$t_{44} = \min\{[0.4,0.5], [0.4,0.5], [0.4,0.5]\}$
$t_{44} = [0.4,0.5]$

$$d_{41} = \max\{[0.5,0.6], [0.5,0.6], [0.0,0.0], [0.4,0.5]\}$$
$$d_{41} = [0.5,0.6]$$

Finally, we obtain the Interval valued fuzzy soft matrix by Interval valued fuzzy soft MmDm

$$\text{MmDm}[a_{ij}] \wedge [b_{ik}] = [d_{i1}] = \begin{bmatrix} d_{11} \\ d_{21} \\ d_{31} \\ d_{41} \end{bmatrix}$$

$$[d_{i1}] = \begin{bmatrix} [0.3,0.4] \\ [0.2,0.3] \\ [0.2,0.3] \\ [0.5,0.6] \end{bmatrix}$$

Finally, we obtain an optimum Interval valued fuzzy soft matrix on U according to above matrix.

$$\text{OptM}([a_{ij}] \wedge [b_{ik}])(U) = \{[0.3,0.4]/I_1, [0.2,0.3]/I_2, [0.2,0.3]/I_3, [0.5,0.6]/I_4\}$$

Where I_4 is the largest membership value $(0.5,0.6)$. Hence insurance is best suit for investor.

V. CONCLUSION :

Interval valued fuzzy matrix is one of the recent topics gaining significance in finding rational solutions in various fields of real life problems. This fuzzy matrix helps a investor to decide the right plan as well as to attain optimal solutions for his/her investment and gaining expected return on investment with the stipulated period.

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