# Squeeze Film Lubrication BetweenHydromagneticPorousParallel Stepped Plates

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Abstract: Theoretical study of squeeze film lubrication between magnetohydrodynamic porous parallel stepped plates is presented in this paper. The modified Reynolds equation which account for magnetic field and porous medium is derived. Closed form solution is obtained for the pressure, load carrying capacity and squeezing time as a function of various physical parameters. Comparing with the non-magnetic case (NMC), the effect of applied magnetic field on the squeeze film lubrication between porous parallel stepped plates is to increase in load carrying capacity significantly and to delay the time of approach. The pressure, load carrying capacity and time of approach are predominant for smaller values of permeability parameters as compared to the solid case.

**Key words:**Magnetic field, Couple stress, Squeeze film, Porous Parallel stepped plates

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## I. INTRODUCTION

During last few decades the study of self-lubricating porous bearings have received great importancebecause of their industrial applications and machine manufacturing. The advantage of porous bearings is that the pores can be filled with lubricating oil so that the bearing requires no further lubrication during the whole life of the machine in which it is used. When the normal load is applied, the fluid is supplied through the interconnected pores to the bearing surface and when load is removed from the loaded zone of the bearing, fluid is reabsorbed by capillary action. Thus, porous metal bearings have been using in the manufacturing of the vehicles, home appliances, machines etc. Hence, by these importance of porous bearings, Wu[1] presented the squeeze film effects between two rectangular plates in which the both the plates have a porous layered material. Sanni and Ayomidele<sup>[2]</sup> have investigated the lubrication characteristics of a plane porous slider without the use of the simplifying the assumption of a small porous facing thickness. Numerous paper are available in the history, for the study of the different types of porous bearings.

With the development of modern machine elements, different types of lubricants are selected to meet the specific requirements for bearings operating under various severe conditions. The property of high thermal conductivity reveals that the heat from the source of generation is conducted away. In addition, the property of high electrical conductivity implies that hydrodynamic flow behaviour can be adjusted by the application of an external magnetic field. Typical theoretical researches are found in the magneto-hydrodynamic(MHD) squeeze film bearings by Shukla[3], Lin et.al.[4] and Lu et.al.[5] and MHD journal bearings by Kamiyama[6] and Malik and Singh[7]. Experimental and theoretical study is analysed by in the MHD hydrostatic bearings by Maki and Kuzma<sup>[8]</sup>.

Many researchers have been presented in MHD slider bearings, Huges[9] studied the MHD characteristics of step slider bearing using an electrically conducting liquid metal lubricant. It shows that significant increases in load carrying capacity can be achieved for both transverse and tangential magnetic fields. Ramanaiah[10] predicted the optimal load capacity of a parallel plate slider bearing with non-uniform magnetic field. The magnetic field profile for the maximum load capacity of the bearing is determined by using the calculus of variations. Rodkiewitcz and Anwar[11] studied the MHD characteristics of inclined slider bearings with arbitrary magnetic field.

Hence, in the present paper an attempt has been made to study the effect of MHD squeeze film lubrication between porous parallel stepped plates.

## **II. MATHEMATICAL FORMULATION OF THE PROBLEM**

In the present paper, we have considered a fluid film of thickness *h* between parallel stepped plate and a porous flat plate approaching each other with a normal velocity V (= -dh/dt) as shown in Figure 1. The uniform magnetic field  $B_0$  is applied perpendicular to the plates. The basic equations of motion for the steady flow in the film region in the presence of applied magnetic field are given by

$$\frac{\partial^2 u}{\partial z^2} - \frac{M_0^2}{h_2^2} u = \frac{1}{\mu} \frac{\partial p}{\partial x}$$
(1)  
$$\frac{\partial p}{\partial z} = 0 \quad (2)$$
  
$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \quad (3)$$

Where  $M_0 = B_0 h_2 (\sigma/\mu)^{1/2}$  is the Hartmann number.  $B_0$  is the applied magnetic field and  $\sigma$  being the conductivity of the lubricant and u and w are the velocity components in the xand z directions respectively and p is the fluid film pressure.

The relevant boundary conditions are: At the upper surface z=h

$$u=0 \qquad \qquad w=\frac{\partial h}{\partial t} (4)$$
  
At the lower surface z=0  
$$u=0 \qquad w=w^{*}(5)$$

The modified Darcy's equations are:

$$u^{*} = -\frac{k}{\mu \left(1 - \beta + \frac{kM_{0}^{2}}{mh_{2}^{2}}\right)} \frac{\partial p^{*}}{\partial x} (6) \quad w^{*} = -\frac{k}{\mu \left(1 - \beta\right)} \frac{\partial p^{*}}{\partial z} (7)$$
$$\frac{\partial u^{*}}{\partial x} + \frac{\partial w^{*}}{\partial z} = 0 \tag{8}$$

where  $\beta = (\eta/\mu)/k$ ,  $\mu$  is the isotropic viscosity of the fluid, *k* is the permeability of the porous matrix. On substituting (6) and (7) in (8) and integrating once with respect to z from  $-\delta$  to 0 and using the Margon-

Cameron approximation with the condition  $\partial p^* / \partial z = 0$  when  $z = -\delta$  gives

$$\frac{\partial p^*}{\partial z}\Big|_{z=0} = -\frac{\delta(1-\beta)}{c^2}\frac{\partial^2 p}{\partial x^2}$$
$$c^2 = \left(1-\beta + \frac{kM_0^2}{mh_2^2}\right)$$

The solution of equation (1) for u by using boundary conditions (4) and (5)

$$u = \frac{h_2^2}{\mu M_0^2} \frac{\partial p}{\partial x} \left\{ Cosh(M_0 z/h_2) - 1 - \frac{Cosh(M_0 h/h_2) - 1}{Sinh(M_0 h/h_2)} Sinh(M_0 z/h_2) \right\}$$
(9)

On integrating the continuity equation (3) using the boundary conditions(4) and (5) we obtain

$$\frac{\partial}{\partial x} \left\{ \frac{h_2^3}{\mu M_0^3} \frac{\partial p}{\partial x} \left( \frac{M_0 h}{h_2} - 2tanh \frac{M_0 h}{2h_2} \right) + \frac{k\delta}{\mu c^2} \frac{\partial p}{\partial x} \right\} = -V \quad (10)$$

Integrating the modified Reynolds equation (10) with respect to x we get

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$$\frac{\partial p_i}{\partial x} = -\frac{\mu V x}{h_2^3 \left\{ f_i(h_i, M_0) + \frac{k\delta}{h_2^3 c^2} \right\}}$$
(11)

Where

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$$f_i(h_i, M_0) = \frac{1}{M_0^3} \left( \frac{M_0 h_i}{h_2} - 2tanh \frac{M_0 h_i}{2h_2} \right)$$

and  $i=1, h=h_1$  in the region  $0\leq x\leq KL$ 

$$i=2, h=h_2$$
 in the region  $\mathit{KL}\,{\leq}\,x\,{\leq}\,L$ 

The relevant boundary conditions for pressure are:

$$p_1 = 0 \ at \ x = 0$$
(12a)  
 $p_1 = p_2 \ at \ x = KL$ (12b)  
 $p_2 = 0 \ at \ x = L$ (12c)

The solution of equation (11) subject to the boundary conditions (12a) (12b) and (12c) we get

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$$p_{1} = \frac{\mu V}{2h_{2}^{3}} \left\{ \frac{K^{2}L^{2} - x^{2}}{f_{1}(h_{1}, M_{0}) + \frac{k\delta}{h_{2}^{3}c^{2}}} + \frac{L^{2}(1 - K^{2})}{f_{2}(h_{2}, M_{0}) + \frac{k\delta}{h_{2}^{3}c^{2}}} \right\} \text{ in region } 0 \le x \le KL (13)$$

$$p_{2} = \frac{\mu V}{2h_{2}^{3}} \frac{(L^{2} - x^{2})}{f_{2}(h_{2}, M_{0}) + \frac{k\delta}{h_{2}^{3}c^{2}}} \text{ in region } KL \le x \le L (14)$$

Introducing non-dimensional quantities

$$x^* = \frac{x}{L}, h_1^* = \frac{h_1}{h_2}, \ \delta^* = \frac{\delta}{h_2}, \ \psi = \frac{k\delta}{h_2^3}$$

in equations (13) and (14), the non-dimensional pressure is obtained in the form  $\begin{pmatrix} & & \\ & & \end{pmatrix}$ 

$$P_{1} = \frac{2p_{1}h_{2}^{3}}{\mu VL^{2}} = \left\{ \frac{K^{2} - x^{*2}}{f_{1}^{*}(h_{1}^{*}, M_{0}) + \frac{\psi}{C^{2}}} + \frac{(1 - K^{2})}{f_{2}^{*}(1, M_{0}) + \frac{\psi}{C^{2}}} \right\} \text{ in region } 0 \le x^{*} \le K$$
(15)  

$$P_{2} = \frac{2p_{2}h_{2}^{3}}{\mu VL^{2}} = \frac{(1 - x^{*2})}{f_{2}^{*}(1, M_{0}) + \frac{\psi}{C^{2}}} \text{ in region } K \le x^{*} \le 1$$
(16)  
where  $f_{1}^{*}(h_{1}^{*}, M_{0}) = \frac{1}{M_{0}^{3}} \left( M_{0}h_{1}^{*} - 2tanh\frac{M_{0}h_{1}^{*}}{2} \right)$   
 $f_{2}^{*}(1, M_{0}) = \frac{1}{M_{0}^{3}} \left( M_{0} - 2tanh\frac{M_{0}}{2} \right)$   
 $C^{2} = \left( 1 - \beta + \frac{\psi M_{0}^{2}}{m\delta^{*}} \right)$ 

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(17)

The load carrying capacity W is given by

$$W = 2b\int_{0}^{K} p_{1}dx + 2b\int_{KL}^{L} p_{2}dx$$
$$W = \frac{2\mu bL^{3}V}{3h_{2}^{3}} \left\{ \frac{K^{3}}{f_{1}(h_{1}, M_{0}) + \frac{k\delta}{h_{2}^{3}c^{2}}} + \frac{\left(1 - K^{3}\right)}{f_{2}(h_{2}, M_{0}) + \frac{k\delta}{h_{2}^{3}c^{2}}} \right\} (18)$$

The non-dimensional load carrying capacity is given by

$$W^* = \frac{3Wh_2^3}{2b\mu L^3 V} = \frac{K^3}{f_1^*(h_1^*, M_0) + \frac{\psi}{C^2}} + \frac{\left(1 - K^3\right)}{f_2^*(1, M_0) + \frac{\psi}{C^2}}$$
(19)

Writing  $V = -\frac{dh_2}{dT}$  in equation (18) ,the time of approach the squeezing time for reducing the film thickness

from an initial value  $h_0$  of  $h_2$  to a final value  $h_f$  is given by,

$$T = -\frac{2b\mu L^3}{3Wh_0^3} \int_{h_0}^{h_f} \left( \frac{K^3}{f_1(h_1, M_0) + \frac{\psi}{c^2}} + \frac{(1 - K^3)}{f_2(h_2, M_0) + \frac{\psi}{c^2}} \right) dh_2 (20)$$

Taking non-dimensional quantities

$$h_f^* = \frac{h_f}{h_0}, \quad h_2^* = \frac{h_2}{h_0}, \quad h_s^* = \frac{h_s}{h_0}$$

The non-dimensional squeeze film time is obtained in the form

$$T^{*} = -\frac{3Wh_{0}^{2}T}{2b\mu L^{3}} = \int_{1}^{h_{f}^{*}} \left( \frac{K^{3}}{f_{1}^{*}(h_{2}^{*}, h_{s}^{*}, M_{0}) + \frac{\psi}{C^{2}}} + \frac{(1-K^{3})}{f_{2}^{*}(h_{2}^{*}, M_{0}) + \frac{\psi}{C^{2}}} \right) dh_{2}^{*}$$
(21)

where

$$f_1^*(h_2^*, h_s^*, M_0) = \frac{1}{M_0^3} \left( M_0(h_2^* + h_s^*) - 2tanh \frac{M_0(h_2^* + h_s)}{2} \right)$$
$$f_2^*(h_2^*, M_0) = \frac{1}{M_0^3} \left( M_0 h_2^* - 2tanh \frac{M_0 h_2^*}{2} \right)$$

### **III. RESULTS AND DISCUSSION**

In the present analysis the effect of MHD on the squeeze film lubrication between porous parallel stepped plates has been studied on the basis the magneto-hydrodynamic thin film lubrication theory. The squeeze film characteristics are analysed with respect to the dimensionless parameters Hartmann number  $M_0$  and couple stress parameter  $l^*$  and porous parameter  $\psi$ . The following range of values for these parameters are used in the numerical computations of the results.  $M_0 = 0, 2, 3, 4.$   $\psi = 0, 0.01, 0.001.$ 

#### 4.1 Pressure

Figures 2 shows the variation of non-dimensional pressure P with  $x^*$ . It is observed that, the effect of magnetic field is to increase the pressure as compared to non magnetic case ( $M_0=0$ ). It is also observed that the

magnetic field effects are predominant only in the first region  $(0 \le x^* \le K)$  and for smaller values of permeability parameter.

## 4.2 Load carrying capacity

The variation of non-dimensional load carrying capacity  $W^*$  with non-dimensional film height  $h_1^*$  is presented in Figures 3 for different values of Hartmann number  $M_0$ . It is observed that, the effect of magnetic field is to increase the load carrying capacity as compared to non magnetic case. It is also observed that the load carrying capacity decreases with increasing values of film height. In figure 4 the variation of non-dimensional load carrying capacity  $W^*$  with non-dimensional film height  $h_1^*$  for different values of  $\psi$  and K is presented. Figure 5 depicts the variation of non-dimensional load carrying capacity  $W^*$  with K, from figure 4 and figure 5 it is observed that the load carrying capacity decreases with increasing values of K.

## 4.3 Squeeze film time

Figures6 shows the variation of non-dimensional squeeze film time  $T^*$  with  $h_f^*$  for different values of Hartmann number  $M_0$  and it is observed that the effect of magnetic field increases the squeeze film time as compared to the non-magnetic case. Also it is observed that squeeze film time  $T^*$  decreases with increasing values of permeability parameter  $\psi$ . Figure 7 depicts that the variation of squeeze film time  $T^*$  with  $h_f^*$  for different values of  $h_s^*$ . It is observed that the squeeze film time decreases with increasing values of  $h_s^*$ . The variation of non-dimensional squeeze film time  $T^*$  with K is presented in figure 8. It is observed that the squeeze film time decreases with increasing values of K.

## **IV. CONCLUSIONS**

The effect of Magneto hydrodynamic lubrication on the porous parallel stepped plates is studied according to the magneto-hydrodynamic thin film lubrication theory, from the results obtained and discussed, the following conclusions can be drawn.

1. The effect of MHD increases the pressure, the load carrying capacity and the squeeze film time.

2. The load carrying capacity and squeeze film time decreases with increasing values of *K*.

3. The pressure, load carrying capacity and squeezing time are found to be decreasing for increasing permeability parameter compared to its corresponding classical case.

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Figure 1 Porous Parallel stepped plates



Figure 2 Variation of non-dimensional pressure P with  $x^*$  for different values of  $\psi$  and  $M_0$  with  $h_1^* = 1.2$ ,



 $K = 0.7, m = 0.6, \beta = 0.2, \delta^* = 0.01$ 



Figure 4 Variation of non-dimensional Load carrying capacity W with  $h_1^*$  for different values of  $\psi$  and K



with  $h_1^* = 1.2, K = 0.7, m = 0.6, \beta = 0.2, \delta^* = 0.01$ 



Figure 6 Variation of non-dimensional squeeze film time  $T^*$  with  $h_f^*$  for different values of  $\psi$  and  $M_{\theta}$  with K = 0.7, m = 0.6,  $\beta = 0.2$ ,  $\delta^* = 0.01$ ,  $h_s^* = 0.15$ 



Figure 7 Variation of non-dimensional squeeze film time  $T^*$  with  $h_f^*$  for different values of  $\psi$  and  $h_s^*$  with  $M_o = 2$ ,  $K = 0.7, m = 0.6, \beta = 0.2, \delta^* = 0.01$ 

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