IOSR Journal of Engineering (IOSRJEN) **www.iosrjen.org**
ISSN (e): 2250-3021, ISSN (p): 2278-8719
Vol. 08, Issue 6 (June. 2018), ||V (VI) || PP 83-93

Optimal Replenishment and Shelf-Space Policy for Freshness, Promotional Effort and Stock Dependent Demand Rate Incorporating Expiration Date of Fresh Product

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Abstract: In this study, we present an EOQ model considering freshness index as demand depends on how fresh the product is. Besides, the availability of more shelf space for the displayed fresh products stimulate demand, hence it is advantageous to keep positive inventory level even at the end of inventory cycle. Generally, perishable item depletes its freshness with time and retailer is not able to sale it after its expiration date. Hence, manufacturers or retailers need to carry out promotional activities to sale the product in its lifetime. In this paper, we consider demand depending on freshness of an item, expiration date, shelf space and promotional activities carried out. Our main objective is to optimize size of shelf space, ordering cycle time and ending inventory level to maximize the total annual profit. We presented numerical example for practical implications of the theoretical model. Sensitivity analysis is also executed to examine sensitive behavior of parameters.

Keywords: EOQ model, expiration date, freshness and stock dependent stochastic demand, Inventory management, promotional efforts dependent demand, shelf space.

Date of Submission: 11-06-2018 Date of acceptance: 26-06-2018

I. INTRODUCTION

Today, consumer with good knowledge of product are no longer in receptive mode when it comes to their health needs. Hence, there is a vast opportunities for the food manufacturers and retailers to stand in this health-oriented consumer market by providing fresh and hygienic products the client wants. So, it is important to manage fresh product and shelf space.

In real life today, demand stimulates with more displayed fresh stock as displayed items inspire consumer to buy the product more. On the other hand, lower visual merchandise reduce the sales due to lacking in variety of product. Levin et al. [1] and Silver and Peterson [2] found remarkable that demand is directly proportional to the displayed inventory level. Afterwards, various inventory models have been investigated to model this phenomenon. Researchers such as Baker and Urban [3], Mandal and Phaujdar [4], Datta and Pal [5] etc. developed model with displayed stock dependent demand. Generally, demand boost with the amount of displayed stock level. Using this assumption, Urban [6] was first to investigate the EOQ model with non-zero ending inventory and demand depending on displayed stock. In this direction, the study of Urban and Baker [7], Teng and Chang [8], Dye and Ouyang [9],Yang et al. [10], Soni and Shah [11], Teng et al. [12], Soni [13],Wu et al. [14] are worth mentioning. Zhou et al. [15] and Zhong and Zhou [16] considered stock-dependent demand and limited displayed shelf space.

Freshness of an item has a key role to make sales of an item as it inspires consumer to buy product. In fact, customers' purchasing decision mainly relies on the quality of an item. Fujiwara [17] first considered the impact of the age of an item on demand for continuously deteriorating items. Sarker et al. [18] examined that the demand is negatively impacted by the age of an item and costumers lead to keep off deteriorating items which have reached near to their expiry dates. In this direction, Bai and Kendall [19] presented an EOQ model for fresh produce by considering both freshness condition and shelf space allocation dependent demand simultaneously. Albeit, past studies have contributed the inventory model with the impact of freshness and stock level of the product, major part of literature considered that perishable products deteriorate continuously and could be sold perpetually without expiration dates. In real life, perishable items do have expiry date and cannot be sold afterwards. Hence, model without taking expiration date into consideration may lead to improper policy. Chen et al. [20] presented an EOQ model in which the demand rate depends on product freshness and the amount of displayed stocks on shelf space with the assumptions that the perishable product cannot be sold after the expiration date.

Most of the item such as food product have the behavior to decline its freshness which keeps deteriorating with time and finally reaches to its expiry date. When this phenomenon is taken into consideration, it is needed to sale such an item within its maximum life time. In the fast and competitive market today it has become a challenge to sale it within certain duration. Hence, promotional activities such as price discount offers, free product offers, free coupons, advertising etc. are required to boost the demand. Researcher such as Cárdenas-Barrón and Sana [21], De and Sana [22], Pal et al. [23], Palanivel and Uthayakumar [24] considered effect of promotional effort on demand. But it is now required to give attention to contribute in the literature to examine the effect of promotional efforts on stock dependent demand. Besides, demand is not certain in today's competitive markets. Hence, considering uncertainty in the demand parameter lead to proper optimal policy. Researchers such as Maihami and Karimi [25], Sana [26], Roy et al. [27], Soni and Chauhan [28], Soni and Suthar [29] etc. examined the effect of promotional efforts on random demand. In this article we presented a model with random demand depending on freshness-shelf space, Inventory level and promotional effort. We optimize the shelf space, replenishment cycle, and ending inventory (which is non-zero) to maximize the profit.

The rest of the paper consists of the following work: Section 2 contains notations and assumptions used throughout the model. Mathematical model is formulated in Section 3. For the practical implications, numerical illustration is provided in Section 4 whereas sensitivity of the model parameters is examined in Section 5.

II. NOTATIONS AND ASSUMPTIONS

2.1. Notations

The following notation are used to formulate the problem, which are adopted from Chen et al. [20].

Decision variables	
E	Ending inventory level in units, with $E \geq 0$
T	Ordering cycle time in years
W	Number of units displayed on shelf space
Parameters	
c	Purchasing cost ($) per unit, where $0 < c < p$
h	Holding cost ($) per unit per unit time.
m	Maximum lifetime (the time to its expiration date) in years
o	Ordering cost ($) per order
p	Selling price ($) per unit, with $p \geq c$
s	Salvage price ($) per unit
u	Shelf cost ($) per unit per year
Q	Economic order quantity in units
ρ	Promotional effort, $\rho \geq 0$.
t_1	Time in years when the inventory level reaches to W
Variables	
$f(t)$	Freshness index at time t, which is a decreasing function within [0,1]
$D(t)$	Demand rate at time t, which is freshness, promotional efforts and stock dependent and close to zero at the expiration date
$I(t)$	Inventory level at time t
E^*	Optimal ending inventory level in units
Q^*	Optimal order quantity in units
T^*	Optimal ordering cycle time in years
W^*	Optimal number of units on displayed shelf space
$\Pi(E,T,W)$	Total annual profit, which is a function of E, T and W

2.2. Assumptions

1. Generally, many products such as food items deteriorate due to many factors such as atmosphere, time, humidity etc. and keep decreasing in its freshness, taste, nutrient values or its effectiveness with time. After certain period they lose its usefulness and reaches to expiration date. Such date cannot be fixed but approximated by checking its nutrient value and other ways. Hence, we assume products freshness index is 1 and gradually decreases to 0. To formulate this problem in the model, we assume the freshness index at time t is linearly decreasing function from 1 initially to 0 at the maximum lifetime and given by:

$$f(t)=\frac{m-t}{m},\ 0\le t\le m \tag{1}$$

2. It is for the most part trusted that a vast display of fresh items animates demand for an item however a large display of rancid items may create the contrary impact. Therefore, we accept in this study that the rancid things would be instantly pulled back from the displayed shelf space.
3. Inventory starts at time zero and the retailer receives *Q* units but only *W* units are displayed on the shelf and the rest of the products stored in the store room. When sales are made, stocks in the store room are moved to the rack until the time when no more stocks are in the store room at the time t_1. Hence, shelf space is full and the demand depends on the freshness index which is taken as random, during the time period $[0,\ t_1]$. Additionally, demand stimulates with promotional efforts and hence demand is taken depending on promotional efforts. This inventory system is depicted in Fig. 1. Hence demand rate at the time *t* is presented by

$$D(t)=\alpha W^{\beta}\,\frac{m-t}{m}+\frac{\tau\rho}{\rho+1}+\varepsilon,\ 0\le t\le t_1 \tag{2}$$

Where, α and β ($\alpha>0$ and $1>\beta\ge 0$) are constant parameters. It is clear from (2) that the ordering cycle time *T* is less than or equal to the lifetime of the product *m*. Otherwise $D(t)$ may become negative if $t\ge m$. Consequently, the impact of shelf space on the demand is decreasing return, and hence we assume $\beta<1$.
4. In real life, demand is influenced by displayed stock. Hence, high demand is directly proportional to high displayed stock, in this paper, we assume that ending inventory level is non-zero, i.e. $E\ge 0$.
5. During the time interval $[t_1, T]$, shelf space is partially fill up with products and demand depends on both the freshness index and displayed items. When cycle ends with inventory level *E* at ordering cycle time *T*, retailer sells those E units at a salvage price *s* per unit, cycle starts again with Q units. During this time interval, the demand rate at time *t* is given by

$$D(t)=\alpha\left[I(t)^{\beta}\right]\frac{m-t}{m},\ t_1\le t\le T \tag{3}$$

6. *Q* is assumed to be greater than or equal to the shelf space *W,* i.e. $Q\ge W$. Otherwise *W* can be reduced to *Q*. Consequently, $t_1\ge 0$.
7. The holding cost is assumed to be same for both displayed items and stored items.
8. Replenishment rate is infinite and instantaneous.
9. Shortages are not allowed.

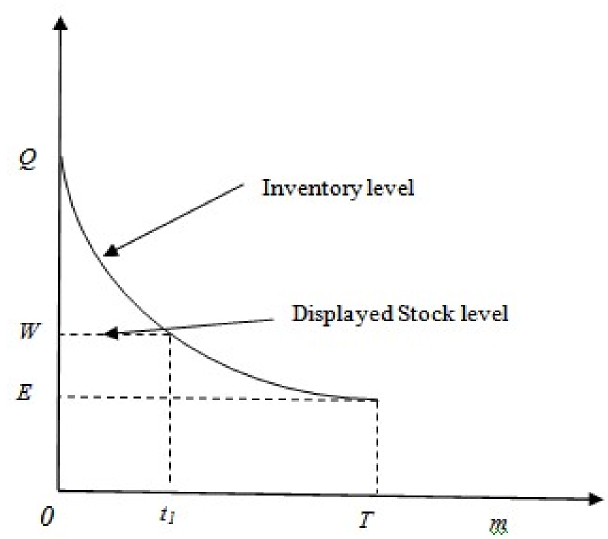

Figure 1: Graphical representation of the system
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III. MODEL FORMULATION

The inventory level $I(t)$ at time *t* during the time span is given by the differential equation

$$\frac{dI(t)}{dt} = -\alpha W^{\beta}\frac{m-t}{m} - \frac{\tau\rho}{\rho+1} - \varepsilon,\ 0 \le t \le t_1 \tag{4}$$

with the boundary condition $I(0)=Q$. By solving (1), we have

$$I(t) = \frac{1}{2m}\alpha W^{\beta} t^2 - \left(\alpha W^{\beta} + \eta\right)t + Q,\ 0 \le t \le t_1 \tag{5}$$

where, $\eta = \frac{\tau\rho}{\rho+1} + \varepsilon$

Now, by using (5) and the fact that $I(t_1)=W$ and $t_1 < T \le m$, one has,

$$t_1 = \frac{m\left(\alpha W^{\beta}+\eta\right) - \sqrt{\left(m\alpha W^{\beta}\right)^2 + 2m\alpha W^{\beta}\left(m\eta - Q + W\right) + \left(\eta m\right)^2}}{\alpha W^{\beta}} \ge 0 \tag{6}$$

Therefore, the order quantity is given by

$$Q = W + \frac{\alpha W^{\beta}}{2m}\left(2mt_1 - t_1^2\right) + \eta t_1 \tag{7}$$

Now, during the time span $[t_1, T]$, the inventory level $I(t)$ at time *t* is formulated by the differential equation as

$$\frac{dI(t)}{dt} = -\alpha\left[I(t)\right]^{\beta}\frac{m-t}{m};\quad t_1 \le t \le T \tag{8}$$

with the boundary conditions $I(t_1)=W$ and $I(T)=E$. By solving (8), one has

$$I(t) = \left\{\frac{\alpha(1-\beta)}{2m}\left[t^2 + 2m(T-t) - T^2\right] + E^{1-\beta}\right\}^{1-\beta},\ t_1 \le t \le T \tag{9}$$

Now, by using (9) and the fact that $I(t_1)=W$ and $t_1 < T \le m$, after some re-arrangements, one has,

$$t_1 = m - \sqrt{(m-T)^2 + \frac{2m\left(W^{1-\beta} - E^{1-\beta}\right)}{\alpha(1-\beta)}} \ge 0 \tag{10}$$

Substituting (10) into (7), we get the order quantity as

$$\begin{aligned} Q = W &+ \frac{\alpha W^{\beta}}{2m}\left[m^2 - (m-T)^2 - \frac{2m\left(W^{1-\beta}-E^{1-\beta}\right)}{\alpha(1-\beta)}\right] \\ &+ \eta\left(m - \sqrt{(m-T)^2 + \frac{2m\left(W^{1-\beta}-E^{1-\beta}\right)}{\alpha(1-\beta)}}\right) \end{aligned} \tag{11}$$

Next, the holding cost during $[0, t_1]$ is

$$\begin{aligned} H_1 &= h\int_0^{t_1} I(t)\,dt = h\int_0^{t_1}\left(\frac{\alpha W^{\beta}}{2m}t^2 - \left(\alpha W^{\beta}+\eta\right)t + Q\right)dt \\ &= h\left(\frac{\alpha W^{\beta}}{6m}t_1^3 - \frac{\left(\alpha W^{\beta}+\eta\right)t_1^2}{2} + Qt_1\right) \end{aligned} \tag{12}$$

The holding cost during the time period $[t_1, T]$ is

$$H_2 = \int_{t_1}^{T}\left\{\frac{\alpha(1-\beta)}{2m}\left[t^2 + 2m(T-t) - T^2\right] + E^{1-\beta}\right\}^{1-\beta},\ t_1 \le t \le T \tag{13}$$

As an expression in (13) is too complex to derive explicit analytical solution. Also H_2 contribute in less amount into overall profit. Hence, for simplicity we may consider simpler form of H_2 as follows. During the time interval $[t_1, T]$, the average inventory level can be approximated as $(W+E)/2$. Hence, the average holding cost during $[t_1, T]$ is given by

$$H_2 \approx \frac{h}{2}(W+E)(T-t_1) \tag{14}$$

Now, in this study, as ending inventory level is assumed to be positive, excerpt level of it may cause in higher holding cost. Hence, the challenge is to find optimal ending inventory level *E*, ordering cycle time *T*, and displayed shelf space *W* all together to maximize the profit.

The total profit = revenue received + salvage value-purchasing cost

- shelf space cost - ordering cost- holding cost (15)

- cost for promotional efforts

Above problem is formulated as

$$Max\ TP(E,T,W) = E\left[\frac{1}{T}\left[p(Q-E)+sE-cQ-o-h\left(\frac{\alpha W^{\beta}}{6m}t_1^3-\frac{(\alpha W^{\beta}+\gamma)t_1^2}{2}+Qt_1\right)\right.\right.$$
$$\left.\left.-\frac{h}{2}(W+E)(T-t_1)-K\rho^{\gamma}\right]-uW\right] \tag{16}$$

where, $E(\varepsilon)=\mu$

Subject to

$$Q = W+\frac{\alpha W^{\beta}}{2m}\left[m^2-(m-T)^2-\frac{2m(W^{1-\beta}-E^{1-\beta})}{\alpha(1-\beta)}\right]+\gamma\left(m-\sqrt{(m-T)^2+\frac{2m(W^{1-\beta}-E^{1-\beta})}{\alpha(1-\beta)}}\right)\geq W$$

$$t_1 = m-\sqrt{(m-T)^2+\frac{2m(W^{1-\beta}-E^{1-\beta})}{\alpha(1-\beta)}}\geq 0 \quad \text{and} \quad 0\leq E\leq W$$

It can easily be showed that total annual profit $TP(E,T,W)$ is strictly pseudo-concave in *T*. Then for any given *T*, $TP(E,T,W)$ is a strictly concave function in both *E* and *W*. Hence, there exist a unique global optimal solution (T^*, E^*, W^*) which maximizes the profit.

IV. NUMERICAL EXAMPLES

We considered same parameters as in Chen et al. [20]. The estimations of the parameters are taken as, $\alpha = 50$, $\beta = 0.7$, $c = \$20$ / unit, $h = \$4$ / unit/year, $m = 0.4$ years, $o = \$10$ / order, $s = \$10$ / unit, $u = \$5$ / unit, $p = 40$, $\tau = 200$, $K = 2$, $\rho = 3$, $\mu = 10$. We obtain a local optimal solution to maximize $TP(T,E,W)$. The results are as follows: the retailer's optimal cycle time is T^*=0.2785 years with t_1^*= 0.0351years, the optimal level of ending-inventory is E^*=2853.51 units, the optimal size of shelf-space is W^*=5299.13 units, the optimal order quantity is Q^*= 5983.36 units, and the maximum total annual profit is $TP(T^*, E^*, W^*)$ = 78610.47. When there is no effect of promotional efforts, i.e. $\rho = 0$, then optimal results are derived as follow: the retailer's optimal cycle time is T^*=0.2783 years with t_1^*= 0.0319 years, the optimal ending-inventory level is E^*= 2847.28 units, the optimal size of shelf-space is W^*= 5347.21 units, the optimal order quantity is Q^*= 5983.36 units, and the maximum total annual profit is $TP(T^*, E^*, W^*)$ = 78308.67

Now, to examine the behavior of the model parameter, we carry out the sensitivity of the parameters by changing its value at a time and keeping other parameters fixed.

V. SENSITIVITY ANALYSIS

Based on the results, we examine the sensitivity of the parameters for managerial implications. We change the value of the parameter and keep other parameter fixed at a time. We examine results calculated in Table 1 for -40% error to +40% error.

Table 1: Sensitivity analysis of model parameters (%)

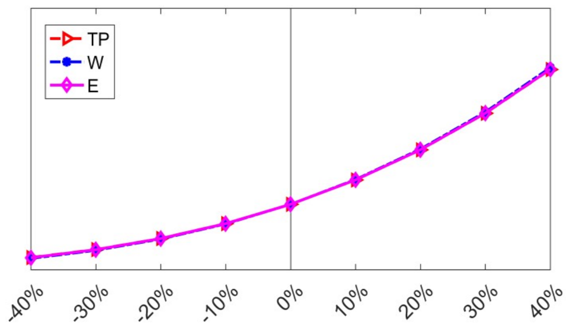
Para-meters		Percentage of under or over estimation (%)							
		-40%	-30%	-20%	-10%	10%	20%	30%	40%
α	$\Delta E / E\%$	-81.68	-69.42	-52.37	-29.55	37.31	83.44	139.46	206.49
	$\Delta T / T\%$	0.14	0.12	0.07	0.03	-0.02	-0.03	-0.05	-0.05
	$\Delta W / W\%$	-82.69	-70.27	-53	-29.91	37.76	84.43	141.11	208.93
	$\Delta TP / TP\%$	-81.37	-69.23	-52.25	-29.5	37.25	83.31	139.25	206.19
	$\Delta Q / Q\%$	-81.67	-69.42	-52.37	-29.56	37.32	83.45	139.48	206.52
β	$\Delta E / E\%$	-99.36	-98.63	-96.43	-85.58	1968.42	235926.1	78312807	-
	$\Delta T / T\%$	-24.37	-18.12	-14.96	-5.98	5.67	-4.05	-11.37	-
	$\Delta W / W\%$	-99.66	-99.26	-97.32	-85.63	1879.11	207404.3	51819627	-
	$\Delta TP / TP\%$	-94.42	-93.48	-91.09	-78.18	1235.59	168983.7	185562659	-
	$\Delta Q / Q\%$	-98.7	-97.92	-95.77	-84.52	1850.76	219217.2	91921326	-
c	$\Delta E / E\%$	6823.9	1525.45	451.03	123.17	-52.47	-76.7	-88.48	-94.39
	$\Delta T / T\%$	-25.17	-12.43	-6.14	-2.41	1.6	2.65	3.27	3.51
	$\Delta W / W\%$	4166.4	1018.54	325.67	95.13	-44.93	-68.23	-81.15	-88.68
	$\Delta TP / TP\%$	2087.62	647.73	240.18	77.43	-41.01	-64.33	-78.25	-86.83
	$\Delta Q / Q\%$	3900.48	967.22	312.15	91.82	-43.8	-66.75	-79.63	-87.25
h	$\Delta E / E\%$	16.85	12.3	7.98	3.89	-3.69	-7.21	-10.55	-13.73
	$\Delta T / T\%$	2.25	1.67	1.11	0.55	-0.54	-1.07	-1.59	-2.1
	$\Delta W / W\%$	14.63	10.7	6.96	3.4	-3.24	-6.34	-9.29	-12.12
	$\Delta TP / TP\%$	9.33	6.87	4.5	2.21	-2.14	-4.2	-6.2	-8.14
	$\Delta Q / Q\%$	14.1	10.32	6.72	3.28	-3.13	-6.13	-8.99	-11.73
m	$\Delta E / E\%$	-72.79	-59.09	-42.34	-22.61	25.35	53.3	83.66	116.27
	$\Delta T / T\%$	-36.28	-26.83	-17.65	-8.71	8.5	16.79	24.89	32.8
	$\Delta W / W\%$	-73.81	-60.12	-43.23	-23.17	26.19	55.26	87.1	121.54
	$\Delta TP / TP\%$	-61.37	-48.05	-33.26	-17.19	18.18	37.21	56.97	77.38
	$\Delta Q / Q\%$	-74.32	-60.8	-43.94	-23.67	27.05	57.42	91.01	127.74
o	$\Delta E / E\%$	0	0	0	0	0	0	0	0
	$\Delta T / T\%$	-0.01	0	0	0	0	0	0	0.01
	$\Delta W / W\%$	0	0	0	0	0	0	0	0
	$\Delta TP / TP\%$	0.02	0.01	0.01	0	0	-0.01	-0.01	-0.02
	$\Delta Q / Q\%$	-0.01	0	0	0	0	0	0	0.01
p	$\Delta E / E\%$	-99.37	-94.4	-80.07	-50.69	78.71	192.61	349.22	556.42
	$\Delta T / T\%$	-7.72	-3.65	-1.72	-0.66	0.45	0.77	1.02	1.21
	$\Delta W / W\%$	-97.95	-89.96	-73.27	-44.5	64.12	151.94	267.73	415.95
	$\Delta TP / TP\%$	-98.53	-91.69	-75.81	-46.75	69.25	166.07	295.78	463.93
	$\Delta Q / Q\%$	-96.64	-87.9	-70.95	-42.74	60.74	143.14	251	388.29
s	$\Delta E / E\%$	-53.66	-45.01	-33.91	-19.39	26.57	64.09	119.06	203.29
	$\Delta T / T\%$	3.37	2.69	1.91	1.03	-1.2	-2.64	-4.37	-6.52
	$\Delta W / W\%$	-45.09	-37.42	-27.85	-15.72	20.86	49.35	89.66	149.27
	$\Delta TP / TP\%$	-35.09	-28.71	-21.02	-11.65	14.76	33.92	59.55	95.12
	$\Delta Q / Q\%$	-44.49	-36.89	-27.44	-15.47	20.48	48.38	87.74	145.73
u	$\Delta E / E\%$	25.86	18.58	11.88	5.7	-5.28	-10.17	-14.71	-18.93
	$\Delta T / T\%$	3.57	2.63	1.73	0.85	-0.82	-1.62	-2.4	-3.15
	$\Delta W / W\%$	26.14	18.78	12	5.76	-5.33	-10.27	-14.86	-19.13
	$\Delta TP / TP\%$	15.15	11.02	7.14	3.47	-3.28	-6.39	-9.33	-12.13
	$\Delta Q / Q\%$	21.83	15.75	10.11	4.88	-4.55	-8.79	-12.77	-16.49
ρ	$\Delta E / E\%$	-0.04	-0.03	-0.02	-0.01	0.01	0.01	0.02	0.03

	$\Delta T / T\%$	-0.02	-0.01	-0.01	0	0	0.01	0.02	0.02
	$\Delta W / W\%$	0.13	0.09	0.05	0.02	-0.02	-0.04	-0.05	-0.06
	$\Delta TP / TP\%$	-0.03	-0.02	-0.01	0	0	-0.01	-0.01	-0.02
	$\Delta Q / Q\%$	-0.04	-0.03	-0.02	-0.01	0.01	0.02	0.02	0.03
	$\Delta E / E\%$	-0.08	-0.06	-0.04	-0.02	0.02	0.04	0.05	0.07
	$\Delta T / T\%$	-0.02	-0.01	-0.01	0	0	0.01	0.01	0.01
τ	$\Delta W / W\%$	0.37	0.28	0.19	0.09	-0.09	-0.19	-0.28	-0.38
	$\Delta TP / TP\%$	-0.18	-0.14	-0.09	-0.05	0.05	0.09	0.14	0.19
	$\Delta Q / Q\%$	-0.07	-0.05	-0.03	-0.02	0.02	0.03	0.05	0.06
	$\Delta E / E\%$	-0.01	0	0	0	0	0	0	0.01
	$\Delta T / T\%$	-0.01	-0.01	0	0	0	0	0.01	0.01
K	$\Delta W / W\%$	-0.01	0	0	0	0	0	0	0.01
	$\Delta TP / TP\%$	0.03	0.02	0.01	0.01	-0.01	-0.01	-0.02	-0.03
	$\Delta Q / Q\%$	-0.01	-0.01	0	0	0	0	0.01	0.01
	$\Delta E / E\%$	-0.01	-0.01	0	0	0	0.01	0.01	0.02
	$\Delta T / T\%$	-0.01	-0.01	-0.01	0	0	0.01	0.02	0.02
m_1	$\Delta W / W\%$	-0.01	-0.01	-0.01	0	0	0.01	0.01	0.02
	$\Delta TP / TP\%$	0.04	0.03	0.02	0.01	-0.01	-0.03	-0.05	-0.08
	$\Delta Q / Q\%$	-0.01	-0.01	-0.01	0	0	0.01	0.01	0.02
	$\Delta E / E\%$	0	0	0	0	0	0	0	0
	$\Delta T / T\%$	0	0	0	0	0	0	0	0
μ	$\Delta W / W\%$	0.03	0.02	0.01	0.01	-0.01	-0.01	-0.02	-0.03
	$\Delta TP / TP\%$	-0.01	-0.01	-0.01	0	0	0.01	0.01	0.01
	$\Delta Q / Q\%$	0	0	0	0	0	0	0	0

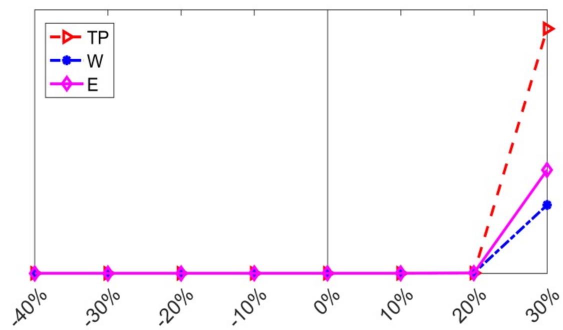
In the fast dynamic market today, some error may occurs in the estimation of the parameters. Therefore it is practicable to check the impact of errors on the optimal policy. This impact is examined by estimating $\Delta T/T$, $\Delta E/E$, $\Delta W/W$, $\Delta TP/TP$ and $\Delta Q/Q$, where $\Delta T = T' - T$, $\Delta E = E' - E$, $\Delta W = W' - W$, $\Delta TP = TP' - TP$ and $\Delta Q = Q' - Q$. T', E', W', TP' and Q' are the evaluated estimation of the parameters whereas T, E, W, TP and Q are the true value of the parameters. Hence, by changing one parameter at a time and keeping other parameters fixed, calculated outcomes are appeared in the Table 1 as well as in figures (a) to (n).

The following analysis are obtained from Table 1:

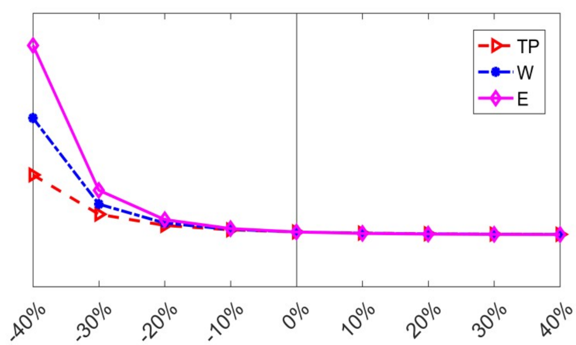
1. Inventory planner has to be most careful in estimating the demand as overestimation of β results in highly deviated profit and other decision variables.
2. All the decision variable are very less sensitive to parameters ρ, τ, K, γ,o and μ as they impact on decision variable less than 1% if their estimation errors deviate -40% to 40% from true value. Hence, model is powerful with respect to these parameters.
3. Inventory planner should be most careful in estimating purchasing cost as it impact the optimal policy more. Especially, the underestimation of it deviates optimal policy dramatically. Hence, value for *c* should be chosen leaning to the right skew.
4. Model parameters such as α, m, p and s impact more (lesser compare to *c*) to the optimal policy and it is right skewed hence decision maker estimates values of these parameters conservatively.
5. h is moderately sensitive to the optimal policy.
6. Total annual profit $TP(T^*, E^*, W^*)$ increases as $\alpha, \beta, m, p, s, \rho, \tau$ or μ increases, and decreases as c, h, o, u, K or γ increases
7. The optimal size of shelf-space W^* increases as $\alpha, \beta, m, o, p, s, K$ or γ increases, but decreases as c, h, u, ρ, τ or μ increases.
8. The optimal level of ending inventory E^* increases as $\alpha, \beta, m, o, p, s, \rho, \tau, K, \gamma$ or μ increases, but decreases as *c, h* or *u* increases.



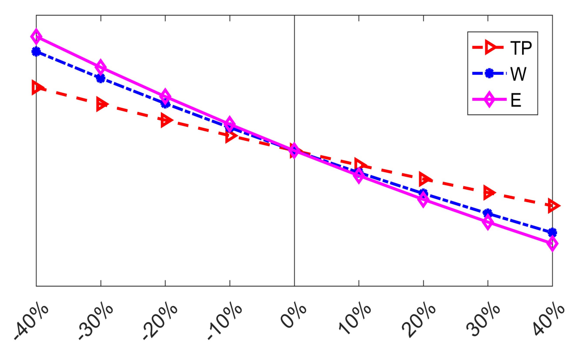
(a) sensitivity of α



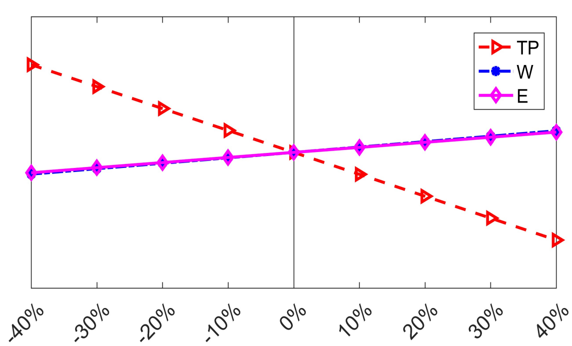
(b) sensitivity of β



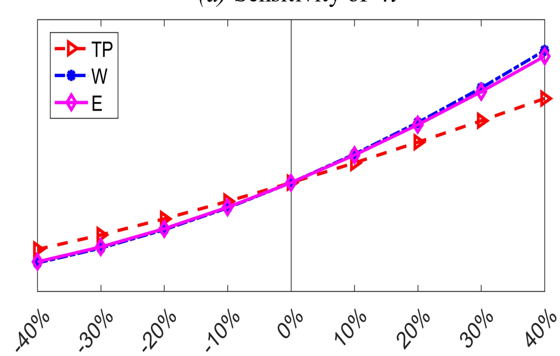
(c) *Sensitivity of* c



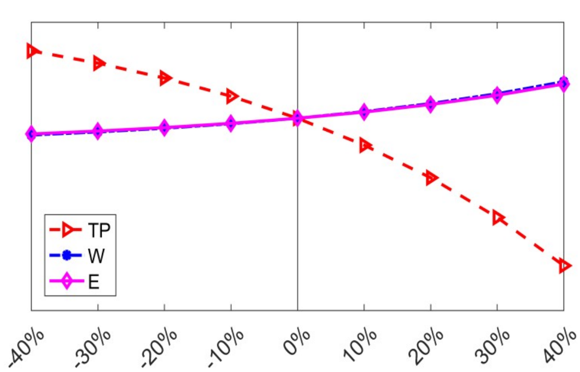
(d) Sensitivity of h



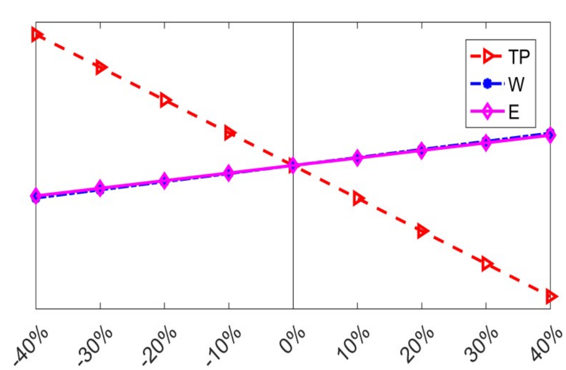
(e) Sensitivity of K



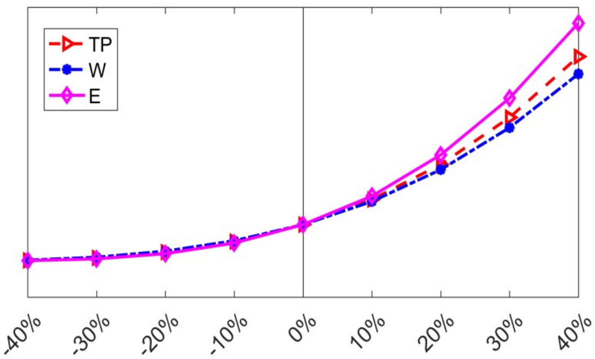
(f) Sensitivity of m



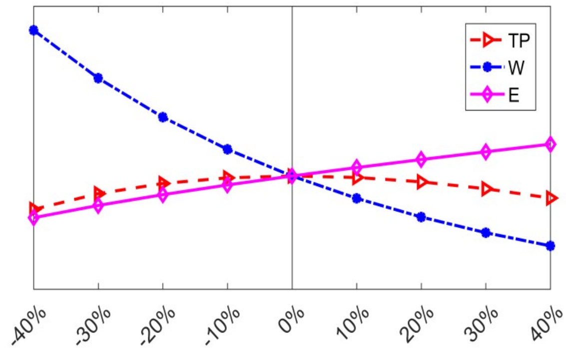
(g) Sensitivity of γ



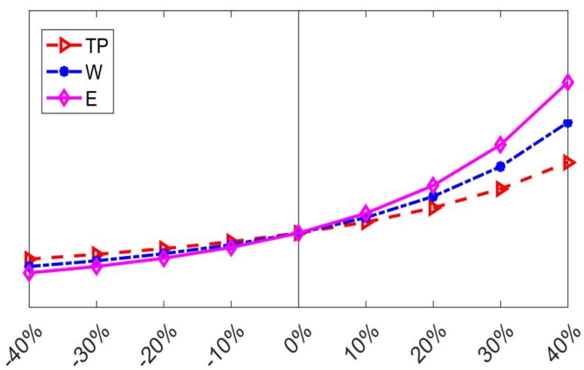
(h) Sensitivity of o



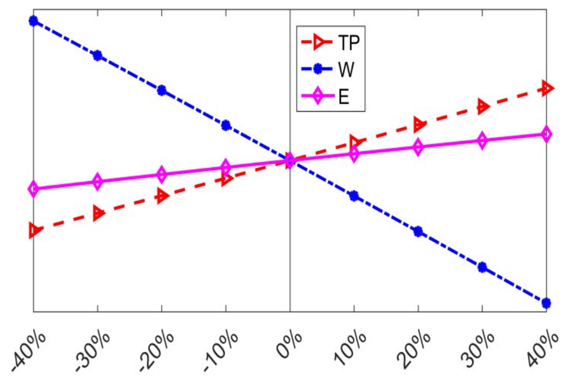
(i) Sensitivity of p



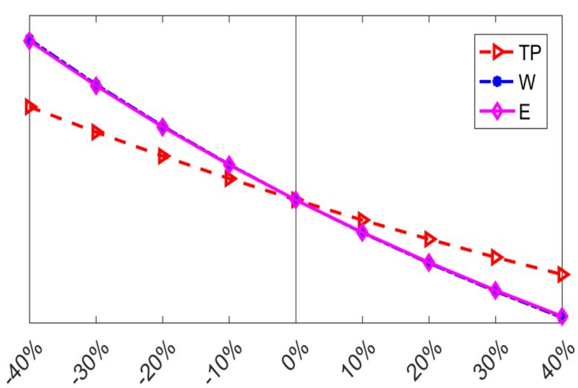
(j) Sensitivity of ρ



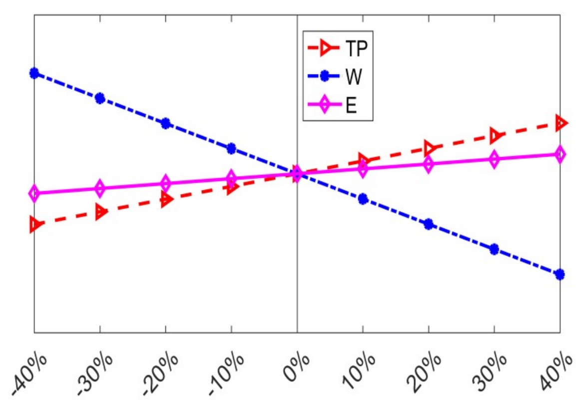
(k) Sensitivity of *s*



(l) Sensitivity of τ



(m) Sensitivity of u



(n) Sensitivity of μ

VI. CONCLUSION

This model is extension of Chen et al. [20] incorporating promotional efforts and randomness to the demand. Chen et al. [20] filled the gap by considering freshness-expiry date sensitive demand which is very practical in the real life situation. Such product having expiry date are of short shelf life, generally. Hence, it is needed to be sold in its life time. Therefore we considered promotional efforts to boost the demand. Results show that total annual profit is more and optimal size of shelf-space is less while considering promotional efforts than those without considering promotional efforts. Besides, inventory planner must have attention on the maximum lifetime of the product and its salvage value while determining replenishment, shelf space and ending inventory level policy. We have also considered the demand as uncertain parameter as in real life it cannot be estimated exactly. This paper can be extended by incorporating trade credit or pricing policy.

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IOSR Journal of Engineering (IOSRJEN) is UGC approved Journal with Sl. No. 3240, Journal no. 48995.

Dipali N. Suthar "Optimal Replenishment and Shelf-Space Policy for Freshness, Promotional Effort and Stock Dependent Demand Rate Incorporating Expiration Date of Fresh Product." IOSR Journal of Engineering (IOSRJEN), vol. 08, no. 6, 2018, pp. 83-91