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Generating subtour elimination constraints for the Traveling Salesman Problem

Soufia Benhida[1], Ahmed Mir[2]
[1, 2](Laboratory of Industrial Engineering and Computer Science, National School of Applied Sciences ENSA, Avenue Tamesoult Agadir 80000, AGADIR, MOROCCO)
Corresponding Author: Soufia Benhida[1]

Abstract: -.The traveling salesman problem (TSP) has commanded much attention from mathematicians and computer scientists specifically because it is so easy to describe and so difficult to solve. In this work we solved the Traveling Salesman Problem, with three different formulations, the formulation DFJ (Danzig-Fulkerson-Johnson), MTZ formulation (Miller-Tucker-Zemlin) and DL formulation (Desrochers-Laporte) . The goal of this work is to solve the Traveling Salesman Problem with a big size of network, in the first we explain the resolution method and we will present some numerical result

Keywords: - generation of constraint, Linear integer programming, Traveling Salesman Problem.

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I. INTRODUCTION

When we talk about operational research, we necessarily talk about the Traveling Salesman problem (TSP). In this work we are interested to solve with exact method using constraint generation approach, and for that we will use three basic formulations for the traveling Salesman Problem.

The traveling salesman problem was formulated in two ways; the first is from Danzig et al. (1954) **[1]**, who proposed the DFJ model using n^2 binary variables xij, for the ATSP, the second is of Miller, Tucker & Zemlin (1960) **[2]**, who rewrote the problem with different way, MTZ was proposed for a Vehicle routing problem, then other formulations appeared reinforcing the MTZ as the DL formulation **[3].**

In this work we will solve the traveling salesman Problem with large instances, using the three formulations (DFJ, MTZ and DL), and see which of the formulations is stronger by applying a strategy of constraint generation. In the literature Padberg and Sung **[4]** showed that the MTZ formulation is weaker than the DFJ formulation. The weakness of the MTZ formulation has also been demonstrated by Langevin et al. **[5].** Desrochers and Laporte **[6]** reinforced the MTZ formulation, while Sherali and Driscoll **[7]** proposed a reformulation of the MTZ constraints, the aim of this work is not only to compare the formulations, but also to apply a Constraint generation strategy to solve the DFJ model that generates an exponential number of constraints, which makes the resolution a bit difficult.

Figure 1: Solution of the traveling salesman problem: the red line is the shortest path that connects all the black points (cities).

II. DESCRIPTION OF THE PROBLEM

The Traveling Salesman Problem is defined on a complete and directed graph G= (V, A), because we treat Asymmetric TSP (ATSP) (See section 3), the set V = {1 . . . n} is the vertex set (cities) with n is the number of cities, and is an arc set; a cost matrix d_{ij} is defined on A. The cost matrix satisfies the triangle inequality whenever , for all i, j, k.

III. MATHEMATICAL FORMULATIONS OF TRAVELING SALESMAN PROBLEM:

A. The Danzig-Fulkerson-Johnson formulation:

The DFJ formulation and many ATSP formulations consist of an assignment problem with integrality constraint and sub-tour elimination constraints **(SECs) [1],** they use a binary variable x_{ij} equal to 1 if and only if arc (i, j) belongs to the optimal solution and otherwise it would be equal to 0, **S** is the sub set of vertices (cities) in the sub tour. The basic model is as follows:

$$Min \sum_{i \neq j} d_{ij} x_{ij} \tag{5}$$

Subject to $$\sum_{j=1}^{n} x_{ij} = 1 \quad (\forall i \in V, i \neq j) \tag{6}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad (\forall j \in V, i \neq j) \tag{7}$$

$$\sum_{i,j \in S} x_{ij} \leq |S| - 1 \quad (S \subset V, 2 \leq |S| \leq n-2) \tag{8}$$

$$x_{ij} \in \{0;1\} \quad \forall (i,j) \in A \tag{9}$$

(5) represents the objective function, (6) and (7) represent the assignment constraints, (8) is the sub-tour elimination constraints (9) is a integrality constraints**,** This formulation becomes difficult in practice, as soon as the size of the model increases, and it is due to the exponential number of subtour elimination constraints generated, this formulation of the TSP contains *n(n-1)* variables and $2^{n-1} + n - 2$ constraints.

B. The Miller -Tucker-Zemlin formulation:

This formulation was originally proposed by Miller and all, for a traveling salesman problem, where the number of vertices of each route is limited **[2]**, they use one binary variable x_{ij} and for each arc $(i, j) \in A$, x_{ij} which takes 1 if the city j is visited immediately after i in the tour , otherwise it takes 0, they uses U_i variables to define the order in which each vertex i is visited on a tour, so the MTZ formulation of the TSP is :

$$\min \sum_{i \neq j} d_{ij} x_{ij} \tag{10}$$

Subject to $$\sum_{j=1}^{n} x_{ij} = 1 \quad (\forall i \in V, i \neq j) \tag{11}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad (\forall j \in V, i \neq j) \tag{12}$$

$$x_{ij} \in \{0;1\} \quad \forall (i,j) \in A \tag{13}$$

$$U_i - U_j + n x_{ij} \leq n - 1 \ , \quad i, j = 2..n \tag{14}$$

$$1 \leq U_i \leq n - 1 \quad i = 2..n \tag{15}$$

(10) Represents the objective function, (11) and (12) are the assignment constraints, (13) is the integrality constraint, (14) are the sub-tour elimination constraints, (15) is the integrality constraint. This TSP formulation contains $(n-1)(n+1)$ variables, and $n^2 - n + 2$ constraints.

C. The Desrochers-Laporte formulation:

Among the advantages of the MTZ formulation is that the sub-tour elimination constraints (14) and (15) can be incorporated into other types of problem formulations together with stronger constraints **[3]**. Motivated by these facts, Desrochers and Laporte have lifted sub-tour elimination constraints (14) and (15) to obtain the stronger forms:

$$Min \sum_{i \neq j} d_{ij} x_{ij} \tag{16}$$

Subject to $\sum_{j=1}^{n} x_{ij} = 1 \quad (\forall i \in V, i \neq j)$ (17)

$$\sum_{i=1}^{n} x_{ij} = 1 \quad (\forall j \in V, i \neq j) \tag{18}$$

$$x_{ij} \in \{0;1\} \quad \forall (i,j) \in A \tag{19}$$

$$U_i - U_j + (n-1)x_{ij} + (n-3)x_{ji} \leq n-2 \quad , \ \text{i,j} = 2..n \tag{20}$$

$$1 \leq U_i \leq n-1 \quad , \ \text{i} = 2..n \tag{21}$$

(16) is the objective function, the assignment constraints (17) and (18) ensure that each city is visited one and only once, (19)and (21) are the integrality constraints, (20) is the sub-tour elimination constraints .

IV. RESOLUTION METHODS FOR THE TRAVELING SALESMAN PROBLEM :

In our case, for the resolution we used exact methods **[8]**, using a linear programming solver CPLEX version 12.6.3 which by default uses the method Branch and Cut, it is an exact method for solve integer linear optimization problems, using the Branch and Bound method, and the secant plan method.

A. Resolution of MTZ and DL formulation :

For the resolution with this formulation we used an integer linear programming solver that uses the default Branch and Cut method.

B. Resolution of DFJ Formulation

For the DFJ (Danzig-Fulkerson-Johnson) formulation, taking into account the exponential number of subtour elimination of constraints (SEC) of the order of 2n, we proceed by constraint generation.

C. The generation of constraint method :

The constraint generation approach works as follows:

First of all, we solve the problem without any constraints of Elimination of sub-turns. and we solve the remaining ILP model. Then we check if the resulting whole solution contains sub-turns. Otherwise, the solution is an optimal TSP visit. And if we have sub-towers, we find all the sub-towers in the integral solution and add the corresponding SEC to the model. The resulting expanded ILP model is again resolved to optimality. Iterating this process clearly leads to an optimal TSP tour.

V. RESULTS AND DISCUSSTION

Experiments were performed on a processor Intel® Celeron® CPU 847 of 1.10 GHz, 1.10 GHz with 4 GB Memory. The solution has been provided by IBM ILOG CPLEX version 12.6.3; we have developed programs in OPL language (Language Modeler of the CPLEX Solver). To solve the TSP with the tree formulations MTZ, DL and DFJ we have generated small random instances similar to some of the TSPLIB instances to obtain 10 complete graphs from 10 to 950 nodes. These nodes have taken between 0 and 99 coordinates randomly; the distances between nodes are the Euclidean distances (d). We have calculated the Continuous Relaxation Value (R) (obtained by relaxing the integrality constraints), and the Optimal Value (Vopt), with T(s) is the calculation time in second and Relaxation quality (RQ), (Table 1 shows the results.) with QR=(Vopt-R)*100/Vopt.

INSTANCES	Vopt	RDFJ	RMTZ	RDL	RQ_{dfj} %	RQ_{mtz}%	RQ_{dl}%
TSP10	337,67	322,5	256,78	263,76	4,49	23,96	21,89
TSP20	412,79	411,13	344,37	366,97	0,40	16,58	11,10
TSP35	518,96	495,65	390,31	460,6	4,49	24,79	11,25
TSP45	537,68	489,28	371,96	402,8	9,00	30,82	25,09
TSP60	603,42	578,33	481,69	510,5	4,16	20,17	15,40
TSP75	645,76	588,10	476,77	489,8	8,93	26,17	24,15
TSP85	678,97	678,97	-	-	0,00	-	-
TSP90	691,41	691,41	-	-	0,00	-	-
TSP100	738,09	738,09	-	-	0,00	-	-
TSP150	888,64	888,64	-	-	0,00	-	-
TSP200	1015,87	1015,87	-	-	0,00	-	-
TSP250	1146,44	1146,44	-	-	0,00	-	-
TSP500	1537,20	1537,20	-	-	0,00	-	-
TSP600	1679,00	1679,00	-	-	0,00	-	-
TSP800	1926,00	1926,00	-	-	0,00	-	-
TSP900	2058,86	2058,57	-	-	0,01	-	-
TSP950	2111,27	2110,97	-	-	0,01	-	-

Table 1: Optimal Value (Vopt), Solution time T(s), Continuous Relaxation Value(R) , and Relaxation quality (RQ) for the DFJ , MTZ and DL formulations.

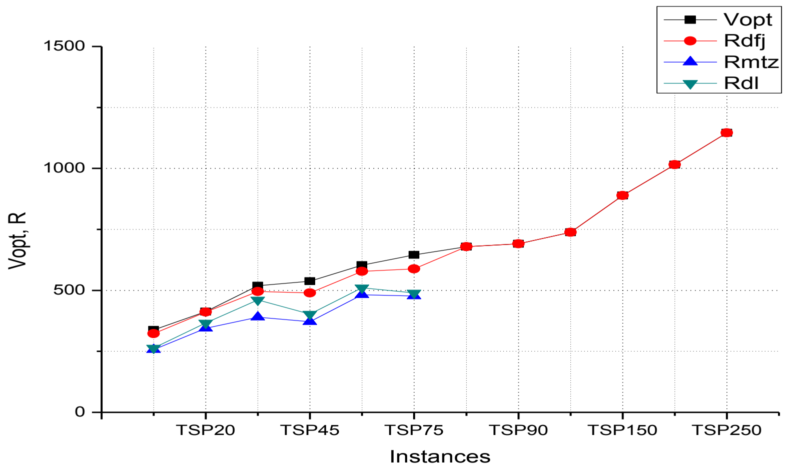

Figure 2. The Continuous Relaxation Value (R) of the DFJ, MTZ and DL formulations

It is apparent from the results that:

• The DFJ Relaxation value is better than the MTZ and DL Relaxation value.

• From 35 cities, the formulation MTZ and DL become very slow in calculation time, we took as limit time 30 min.

VI. CONCLUSION

The study leads to the following conclusions:

- The MTZ formulation is seductive because it is easy to implement but it gives a low continuous relaxation.
- In practice it is more interesting to use the DFJ formulation, even this requires to implement the generation of constraints.
- The DL formulation, although significantly strengthens the MTZ formulation, does not compete with the DFJ formulation.

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