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Peristaltic Flow Characteristics of Blood through Stenotic Artery under the influence of Magnetic Field and Slip Velocity.

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ABSTRACT: The current analysis deals with a theoretical study on combined effect of both slip velocity and magnetic field on unsteady generalized non-Newtonian peristaltic flow with permeable wall of artery. A constant transverse magnetic field is applied on pulsatile blood flow and we are treating blood as an elastico-viscous, electrically conducting and incompressible fluid. Appropriate transformation methods are adopted to solve unsteady non-Newtonian axially symmetric momentum equation in cylindrical polar co-ordinate system with suitably prescribed conditions. To validate the applicability of the proposed analysis, analytical expressions for axial velocity, fluid acceleration, wall shear stress and volumetric flow rate are computed and obtained numerical results are depicted graphically for different values of flow variables of interest, to analyse the influence in axial velocity, wall shear stress and volumetric flow rate of streaming blood.

KEYWORDS: Elastico-viscous fluid, slip velocity**,** selective permeability, transverse magnetic field, peristalsis, knudsen number.

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I. INTRODUCTION

Today rheology of streaming blood in arteries is a very important issue of research in both physiological and clinical situations since one of world- wide leading cause of death is due to malfunction of the cardiovascular system in human beings. As we know that the major contribution of the entire cardiovascular system is to supply oxygen rich blood to every tissue under a sufficient pressure gradient for exchanging materials through the blood vessels wall by peristaltic pumping of heart. It is a two- way exchange through the walls of blood vessels that the nutrients are carried to tissues and cells and then bodily fluid returns along with the waste from cellular metabolism. But this transport of materials by progressive wave of regular contraction and expansion of heart (termed as peristalsis) is not always so regular. Many cardiovascular diseases- particularly atherosclerosis (medically called stenosis) is common disease due to narrowing of arteries lumen, are very much related to the nature of blood movement and dynamic behaviour of vessel walls. The word, atherosclerosis has been taken from the Greek words athero (meaning paste or gruel) and sclerosis (hardness). The word stenosis means the narrowing of artery due to deposition of arteriosclerotic plaque or other types of abnormal tissue along the walls of the channel. This results into disorder in circulatory system by reducing or occluding the blood supply which may develop cardiovascular diseases especially heart attack, stroke etc and when blood flow to a tissue becomes restricted or reduced, necrosis will eventually occur. Therefore, in the present scenario, the study of blood flow disorders in constricted human arteries has of much interest for further scientific investigations and applications in biomedical science.

To examine the flow characteristics of blood through stenosed vessel, Beaver and Joseph [1] employed boundary conditions at a naturally permeable vessel wall and it would help in understanding that why no-slip condition at vessel wall should be replaced by slip velocity. Lee and Fung [2] studied the blood flow at low Reynolds number in the range 0-25 in locally constricted tube by using conformal mapping method. Oka and Murata [3] presented hydro- dynamical theory for flow of blood through a blood vessel with permeable wall and investigated the exchange of fluid as a steady slow motion across the permeable wall upon the motion of the fluid within rigid circular tube. Saffmann [4] further discussed boundary conditions for flow of fluid at the surface of a porous medium in suitable improved manner (given by Beaver and Joseph [1]) and justify it. Popel et al. [5] have investigated a continuum approach for blood flow with couple stresses. Steady flow of blood through modelled vascular stenosis has been investigated by McDonald [7]. Shukla et al. [8] discussed the effect of stenosis on non-Newtonian flow of blood in an artery but later on they took into account the effect of radial distribution of cells and the existence of the peripheral plasma layer near the wall and studied the flow of blood through an artery with mild stenosis by assuming blood as a power law fluid. Srivastava and Srivastava [9] had developed a two- phase mathematical model for pulsatile flow of blood with entrance effects. Blood flow through an artery with mild stenosis has been investigated by Sinha and Singh [10] and couple stresses effects on blood flow also discussed by them. Srivastava [12] studied the flow through stenotic blood vessel and he considered blood as couple stress fluid. A numerical study on flow of fluid have been developed by Lee [13] and fluid passed through tubes with double constrictions. The flow characteristics in the neighbourhood of double constrictions were studied numerically in a circular cylindrical tube. Misra et al. [16] have presented a non-Newtonian model in which they have discussed flow of blood through arteries under stenotic conditions. Haldar and Ghosh [17] have studied blood flow under the effect of magnetic field through an indented artery in the presence of erythrocytes taking blood as Newtonian fluid and discussed the expressions for blood velocity, pressure and flow rate. Murata [18] has proposed a sedimentation model on flow properties of aggregating red cell suspension in constricted horizontal tubes in which he has considered constant hematocrit level and Newtonian viscosity in the central region circular tube. Chakravarty and Mandal [19] investigated a two dimensional flow of blood under stenotic conditions through tapered artery. Srivastava [20] has investigated the flow of blood flow in a mild stenosed artery taking central layer (core region) is as couple stress fluid as it is the suspension of erythrocytes (RBC) and a peripheral layer of plasma as Newtonian fluid. Pralhad and Schultz [21] have studied the modeling of arterial stenosis and employed its application to blood diseases assuming blood as non- Newtonian couple stress fluid. Bali and Awasthi [22] have discussed the resistance to flow of blood having single mild stenosis in an artery having effect of magnetic field.

A large number of investigations have been described by several researchers to understand disorders in flow of blood due to development of stenosis by considering blood as Newtonian or non-Newtonian fluid. Mathematical modeling of flow of blood in vertebral artery with stenosis has been presented by Ali et al. [23]. Rathod and Tanveer [24] have employed the effects of periodic body acceleration on pulsatile blood flow through a porous medium under the influence of magnetic field and they have found that local exposure of a magnetic field could relax blood vessel and enhance blood flow. Varshney et al. [25] have also presented the effect of magnetic field on blood flow in a multiple stenosed artery and they have analysed that all the flow variables are affected by the intensity of magnetic field in the presence of multiple stenosis. Shit and Roy [26] have employed a theoretical study of hydro- magnetic pulsatory blood flow in a channel which is constricted and porous. A mathematical modeling of non- Newtonian blood flow in a stenosed artery has been presented by Mathur and Jain [27]. They have considered blood as Power- law fluid and provided much evidences for the vital role of hydro-dynamic factors in the development and the progression of stenosis in an artery. Rathee and Singh [28] have presented the effect of externally applied magnetic field on the two- layered model of blood flow through stenosed artery in porous medium and analysed that exact strength of magnetic field can help in controlling the bood flow in hypertensive patients and in stenosed arteries. Kumar et al. [29] have designed a model to investigate flow of non-Newtonian blood through an artery with multiple stenosis. The unsteady slip flow of blood through constricted artery has been investigated by Gaur and Gupta [30] and the obtained results have shown the variation in flow characteristics along the axial distance with passage of time. Siddiqui and Geeta [31] have analysed the unsteady flow of blood with slip effects through an artery in the presence of stenosis. Malek and Horque [32] have recently presented a theoretical study for flow of blood through a stenosed artery with permeable wall taking consideration of hematocrit level.

Motivated from above investigations and in order to simulate the constricted artery flow problem, one step close to more realism of phenomenon, we have made an attempt to present theoretical analysis to explore the significant combined influence of slip velocity and externally applied magnetic field on flow of non-Newtonian blood through stenosed artery with permeable wall. In the current study, non-Newtonian rheology of streaming blood is characterised by its elastico-viscosity. Since the arterial wall tissue has selective permeability, no-slip condition at wall is not appropriate and we have to account the effect of the slip velocity at the wall of narrow artery for a more realism of the phenomenon. Also selective permeability of the arterial wall permits for applying slip condition in place of no-slip condition, it is one more close step to the real situation of peristaltic flow of blood through wall of narrow artery as the arterial permeable walls allow the fluid particles to slip at boundary. An extensive quantitative analysis is carried out by applying Laplace and finite Hankel transformations with specific boundary conditions to estimate effects of slip velocity and applied magnetic field with appropriate scientific discussions so as to substantiate the utility of the present study.

II. FORMULATION OF THE PROBLEM

We assume one dimensional motion of blood in a straight and rigid cylindrical stenosed artery through a porous medium by considering blood as non-Newtonian elastico-viscous, incompressible and electrically conducting fluid under the influence of transversely applied magnetic field. It is well known that when magnetic field is applied on a electrically conducting fluid like blood, an electromagnetic force will produce due to the interaction of current with magnetic field. The electromotive force is given by the proportionality relation to the speed of motion and the magnetic field intensity.

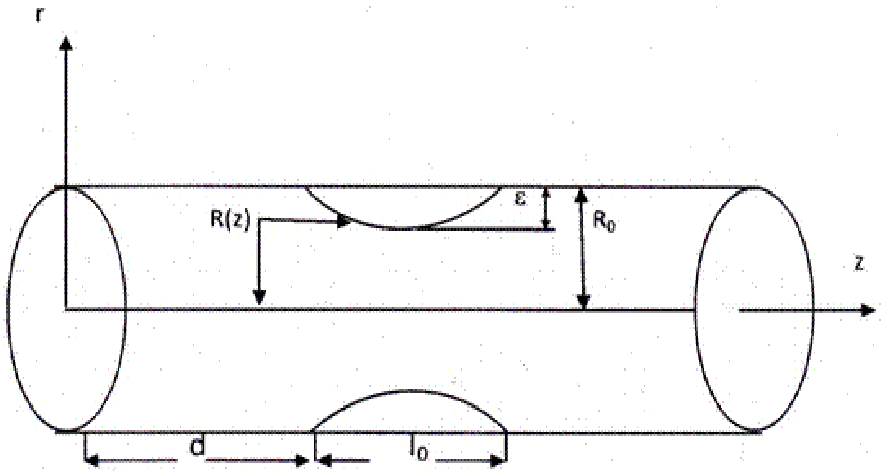
Then by Ohm's law, we have

$$\vec{J} = \sigma(\vec{E} + \vec{u} \times \vec{B}) \qquad (1)$$

where $\vec{E}$ is the electric field intensity vector, σ is the electrical conductivity, $\vec{u}$ is the velocity vector, $\vec{B} = \vec{B}_0 + \vec{B}_1$ is the total magnetic flux intensity vector in which $\vec{B}_1$ is the induced magnetic field vector which is of very small amount, hence assumed to be negligible in comparison with the external applied magnetic field vector $\vec{B}_0$ for MHD (magneto-hydrodynamic) flow. We also assume that the electric field intensity vector $\vec{E}$, due to the polarization of charge is also negligible. Therefore in the momentum equation, the electromagnetic force is included and is defined as

$$\vec{F} = \vec{J} \times \vec{B} = -\sigma B_0^2 \vec{u}, \qquad \text{where} \qquad \left|\vec{B}_0\right| = B_0 \qquad (2)$$

Further we assume that the blood flow is laminar, unsteady, axially-symmetric and fully developed. Now the geometry of artery with symmetric shape stenosis as proposed by Haldar and Ghosh (1994)



$$\frac{R(z)}{R_0} = 1 - A\left[l_0^{s-1}\left(z-d\right) - (z-d)^s\right], \qquad d \le z \le d + l_0 \qquad (3)$$

where, $s \ge 2$ is a stenosis shape determining parameter, d denotes the position of stenosis, l_0 is expressing the length of stenosis, R_0 is radius of normal artery, *R(z)* is radius of stenosed circular cylindrical artery, z denotes the axial position and A is a parameter given by

$A = \frac{\varepsilon}{R_0 l_0^s} \frac{s^{s/(s-1)}}{(s-1)}$, ε is the maximum height of developed stenosis at $z = d + \frac{l_0}{s^{1/(s-1)}}$ with $\varepsilon / R_0 << 1$.

III. GOVERNING EQUATIONS

Now the equation of motion which is Navier-Stokes equations for flow of blood, including Lorenz's force and with above assumptions, in cylindrical polar co-ordinates is

$$\rho \frac{\delta u}{\delta t} = -\frac{\delta p}{\delta z} + (\mu + \mu_1 \frac{\delta}{\delta t})\nabla^2 u - \frac{\mu}{k} u - \sigma B_0^2 u \ , \qquad (4)$$

Where

$$\nabla^2 \cong \frac{1}{r}(\frac{\delta}{\delta r}(r\frac{\delta}{\delta r})) \qquad (5)$$

And

$-\dfrac{\delta p}{\delta z} = p_o + p_1 \cos(\omega t)$, p_o is the steady-state part of the pressure gradient or constant amplitude of pressure gradient, p_1 is the amplitude of the pulsatile component that arises the systolic and diastolic pressure, $\omega = 2\pi f_p$ and f_p is the frequency of heart pulse, z is the axial distance and t is time variable.

$u(r,t)$ is velocity component in axial direction
ρ is the density of blood
μ is the viscosity of blood
μ_1 is elastic viscosity coefficient of blood
σ is the electrical conductivity
B_0 is the external magnetic field
K is the permeability of the isotropic porous medium
r is the radial coordinate.

The dimensionless quantities are defined below:

$$u^* = \frac{u}{\omega R_0},\ r^* = \frac{r}{R_0},\ t^* = t\omega,\ p_0^* = \frac{R_0}{\mu\omega} p_0,\ p_1^* = \frac{R_0}{\mu\omega} p_1,\ a_0^* = \frac{\rho R_0}{\mu\omega} a_0,\ z^* = \frac{z}{R_0},\ K^* = \frac{K}{R_0^2} \tag{6}$$

On dropping stars, we have

$$\alpha^2 \frac{\partial u}{\partial t} = p_0 + p_1 \cos t + (1+\beta \frac{\partial}{\partial t})(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}) - (H^2 + M^2)u, \tag{7}$$

where $\alpha = R_0 \sqrt{\dfrac{\rho\omega}{\mu}}$ is the Womersley parameter, $H = B_0 R_0 \sqrt{\dfrac{\sigma}{\mu}}$ is the Hartmann number, $M = \sqrt{\dfrac{1}{K}}$ is the permeability parameter and $\beta = \dfrac{\mu_1 \omega}{\mu}$.

It is supposed that for $t < 0$, only heart's pumping action is present and when $t = 0$, artery's blood flow leads to the instant pressure gradient i.e.

$$-\frac{\delta p}{\delta z} = p_0 + p_1 \tag{8}$$

IV. THE INITIAL AND BOUNDARY CONDITIONS

When intensity of permeability is low, the boundary condition proposed by Beaver and Joseph [1], which was further explained by Saffman [4] (also known as the Saffman's slip condition) as $\dfrac{du}{dr} = \dfrac{\eta}{\sqrt{K}} u$ "where η is a constant depending on the properties of the porous material and on its structure" and K is taken as the permeability parameter (or Darcy number) of porous material of the wall. This condition has effective in case of unsteady flows and even when MHD effects on fluids is taken. Now initial and boundary conditions are prescribed below as:

$\dfrac{\partial u}{\partial r} = -hu$ (Slip condition)

or

$u' = -hu$

Or

(i) $u' + hu = 0$ when $r = a$ and $t \geq 0$ (9)

where

$$h = -\frac{\eta}{R_0\sqrt{K}} \quad \text{and} \quad a = \frac{R(z)}{R_0}$$

(ii) $u(r,0)=\dfrac{2h}{a}\sum_{n=1}^{\infty}\dfrac{p_0+p_1}{(H^2+M^2+\lambda_n^2)(\lambda_n^2+h^2)}\dfrac{J_0(r\lambda_n)}{J_1(a\lambda_n)}$ when $0\le r\le a$ (10)

(iii) $u(0,t)$ is finite.

V. REQUIRED INTEGRAL TRANSFORMATIONS

If $g(t)$ is continuous function and is of exponential order for $t\ge 0$ then its Laplace transformation is defined as

$$\bar{g}(s)=\int_0^{\infty} e^{-st} g\left(t\right)dt, \qquad s>0 \tag{11}$$

If f (r) satisfies the Dirichlet conditions in finite interval $\left[\,0,a\,\right]$ then its finite Hankel transformation defined as

$$f(\lambda_n)=\int_0^{a} r\,f\left(r\right)J_0\left(r\lambda_n\right)dr \tag{12}$$

where λ_n are the roots of the equation $\lambda J_0{}'(a\lambda)+hJ_0(a\lambda)=0$, and $J_0(r)$ and $J_1(r)$ are Bessel's functions of first kind then f(r) is given by

$$f(r)=\frac{2}{a^2}\sum_{n=1}^{\infty}\frac{\lambda_n^2 J_0\left(r\lambda_n\right)}{\left(h^2+\lambda_n^2\right)J_o^2\left(a\lambda_n\right)}\,f\left(\lambda_n\right) \tag{13}$$

VI. ANALYSIS

Applying Laplace transformation on eq. (7) in light of eq. (11), we obtain;

$$\left[\alpha^2 s+\left(H^2+M^2\right)\right]\tilde{u}\left(r,\,s\right)=p_0\frac{1}{s}+p_1\frac{s}{s^2+1}$$
$$+\left\{\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r})\right\}\left(\tilde{u}(r,s)+\beta s\tilde{u}(r,s)-\beta u(r,0)\right)+\alpha^2 u\left(r,0\right) \tag{14}$$

Now applying finite Hankel transformation of first kind and zeroth order on eq. (14) in light of eq. (12), we obtain;

$$\left[\alpha^2 s+\left(H^2+M^2\right)\right]\tilde{u}\left(\lambda_n,\,s\right)=\left[p_0\frac{1}{s}+p_1\frac{s}{s^2+1}\right]\left(\frac{a}{\lambda_n}\right)J_1\left(a\lambda_n\right)-\left(1+\beta s\right)\lambda_n^2\tilde{u}\left(\lambda_n,s\right)$$
$$\lambda_n^2\beta u\left(\lambda_n,0\right)+\alpha^2 u\left(\lambda_n,0\right)$$

Or

$$\tilde{u}\left(\lambda_n,s\right)=\left[\left(p_0\frac{1}{s(s+l)}+p_1\frac{s}{\left(s+l\right)(s^2+1)}\right)\frac{1}{(\alpha^2+\beta\lambda_n^2)}+\frac{p_0+p_1}{(s+l)(H^2+M^2+\lambda_n^2)}\right]\left(\frac{a}{\lambda_n}\right)J_1\left(a\lambda_n\right) \tag{15}$$

where $\quad l=\dfrac{H^2+M^2+\lambda_n^2}{\alpha^2+\beta\lambda_n^2}$

Applying inverse Laplace transformation on eq. (15), we obtain;

$$u\left(\lambda_n,t\right)=\left[\begin{array}{l}\dfrac{p_0}{l}+e^{-lt}\left\{-\dfrac{p_0}{l}-\dfrac{l p_1}{(1+l^2)(\alpha^2+\beta\lambda_n^2)}+\dfrac{p_0+p_1}{H^2+M^2+\lambda_n^2}\right\}+\dfrac{l p_1}{(1+l^2)(\alpha^2+\beta\lambda_n^2)}\cos t\\+\dfrac{p_1}{(1+l^2)(\alpha^2+\beta\lambda_n^2)}\sin t\end{array}\right]\frac{a}{\lambda_n}J_1(a\lambda_n) \tag{16}$$

Applying inverse finite Hankel transformation on eq. (16), we obtain the expression for fluid velocity which is

$$u\left(r,t\right)=\frac{2h}{a}\sum_{n=1}^{\infty}\frac{1}{(h^2+\lambda_n^2)}\frac{J_0(r\lambda_n)}{J_0(a\lambda_n)}\left[\begin{array}{l}\dfrac{p_0}{l}+e^{-lt}\left\{-\dfrac{p_0}{l}-\dfrac{l p_1}{(1+l^2)(\alpha^2+\beta\lambda_n^2)}+\dfrac{p_0+p_1}{H^2+M^2+\lambda_n^2}\right\}\\+\dfrac{l p_1}{(1+l^2)(\alpha^2+\beta\lambda_n^2)}\cos t+\dfrac{p_1}{(1+l^2)(\alpha^2+\beta\lambda_n^2)}\sin t\end{array}\right] \tag{17}$$

The expression for fluid acceleration is given by

$$F(r,t)=\frac{\partial u}{\partial t}$$

or

$$F\left(r,t\right)=2\frac{h}{a}\sum_{n=1}^{\infty}\frac{J_0\left(r\lambda_n\right)}{\left(\lambda_n^2+h^2\right)J_0\left(a\lambda_n\right)}\left[\begin{array}{l}e^{-lt}\left(p_0+\dfrac{l^2 p_1}{(1+l^2)(\alpha^2+\beta\lambda_n^2)}-\dfrac{l(p_0+p_1)}{(H^2+M^2+\lambda_n^2)}\right)\\-\dfrac{lp_1}{(1+l^2)(\alpha^2+\beta\lambda_n^2)}\sin t+\dfrac{p_1}{(1+l^2)(\alpha^2+\beta\lambda_n^2)}\cos t\end{array}\right] \tag{18}$$

Similarly the expression for the volumetric flow rate is given by

$$Q(r,t)=2\pi\int_0^a r\,u(r,t)\,dr$$

or

$$Q\left(r,t\right)=\frac{4\pi h}{a}\sum_{n=1}^{\infty}\frac{J_1(a\lambda_n)}{\lambda_n(h^2+\lambda_n^2)J_0(a\lambda_n)}\left[\begin{array}{l}\dfrac{p_0}{l}+e^{-lt}\left\{-\dfrac{p_0}{l}-\dfrac{l p_1}{(1+l^2)(\alpha^2+\beta\lambda_n^2)}+\dfrac{p_0+p_1}{H^2+M^2+\lambda_n^2}\right\}\\+\dfrac{l p_1}{(1+l^2)(\alpha^2+\beta\lambda_n^2)}\cos t+\dfrac{p_1}{(1+l^2)(\alpha^2+\beta\lambda_n^2)}\sin t\end{array}\right] \tag{19}$$

and the expression for the shear stress is given by

$$\tau(r,t)=\mu\frac{\partial u}{\partial r}$$

or

$$\tau(r,t)=\frac{2\mu h}{a}\sum_{n=1}^{\infty}\frac{1}{(h^2+\lambda_n^2)}\frac{(-\lambda_n)}{J_0(a\lambda_n)}J_1(r\lambda_n)\left[\begin{array}{l}\dfrac{p_0}{l}+e^{-lt}\left\{-\dfrac{p_0}{l}-\dfrac{l p_1}{(1+l^2)(\alpha^2+\beta\lambda_n^2)}+\dfrac{p_0+p_1}{H^2+M^2+\lambda_n^2}\right\}\\+\dfrac{l p_1}{(1+l^2)(\alpha^2+\beta\lambda_n^2)}\cos t+\dfrac{p_1}{(1+l^2)(\alpha^2+\beta\lambda_n^2)}\sin t\end{array}\right] \tag{20}$$

VII. GRAPHICAL RESULTS AND DISCUSSIONS

In this analysis, the expression for axial velocity, volumetric flow rate, fluid acceleration and wall shear stress are obtained and plotted in respect of axial distance 'z' for different value of Hartmann number H, permeability parameter M, slip parameter h and time variable t. The flow characteristics of the blood can be discussed in two different ways; either the effect of individual factor like artery radius, pressure gradient (average steady pressure gradient which is taken as amplitude p_0 for the pulsatile pressure gradient), various parameters which are related to the problem, is studied or we can calculate the values of flow parameters at a particular site in cardiovascular system. In normal artery, radius R = 0.008 cm, p_0 =2000 dyne /cm^2, p_1 = 4000 dyne /cm^2, in coronary artery R = 0.15cm, p_0 = 693.65 dyne /cm^2 , femoral artery R= 0.5cm, p_0 = 32 dyne /cm^2 , and $\mu =$ 0.035, β = 1, d = 20cm, l_0 = 40cm . In the present analysis, we have followed the second method. The velocity profile from the eq. (17) for different values of Hartmann number H, permeability parameter M, slip parameter h and time variable t have been shown through Fig. 1, Fig. 2 and Fig. 3 for given value of Womersley parameter α and given value of β and Fig. 4 presents a three dimensional view for velocity profile. It is observed that upto some extent, as Hartmann number H increases, axial velocity increases and attains its maximum value near centerline of the artery and attains minimum value at the walls but after that it starts decreases. The effect of permeability parameter causes to strengthen the fluid velocity with increase in value of permeability parameter M and reveals its maximum value at the maximum height of the stenosis. Also slip parameter enhances the blood velocity as compare to the no-slip condition at arterial wall. It is clear from the figures Fig. 9, Fig. 10, and Fig. 11 that volumetric flow rate is having minimum value at stenosis ends, while it is having a maximum value at stenosis' throat and it is clear from the three dimensional Fig. 12 that it increases with increasing the value of the slip parameter. It is a common fact that the wall shear stress has a significant role in arterial diseases development. So it is necessary to study the effects of these variables on wall shear stress. Fig. 5, Fig. 6 and Fig. 7 depicts the variations of wall shear stress for various values of Hartmann number H, permeability parameter M, slip parameter h and time variable t and it is ascertained from Fig. 8 that the wall shear stress decreases with increase in the value of permeability parameter M and slip parameter h. Also it is investigated that the effect of pressure gradient on stenosis' site decreases with increase in value of Hartmann number H upto some extent.

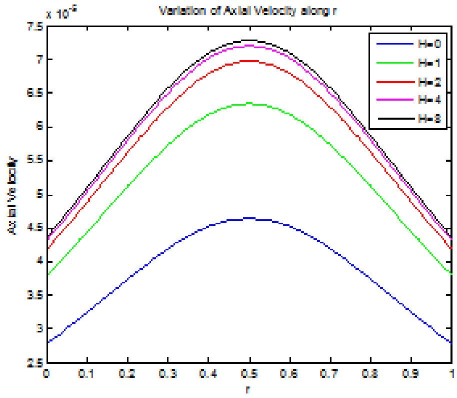


Fig. 1

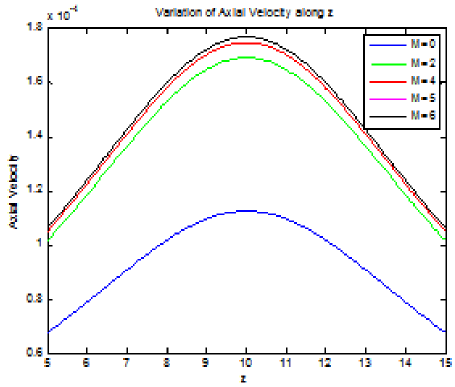


Fig. 2

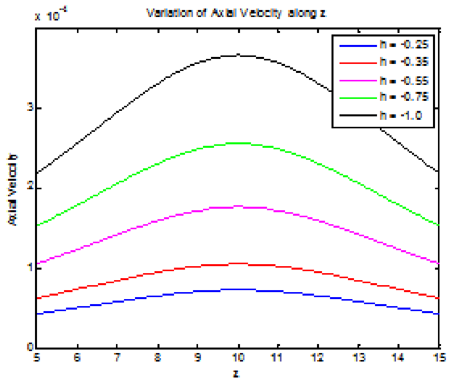


Fig. 3

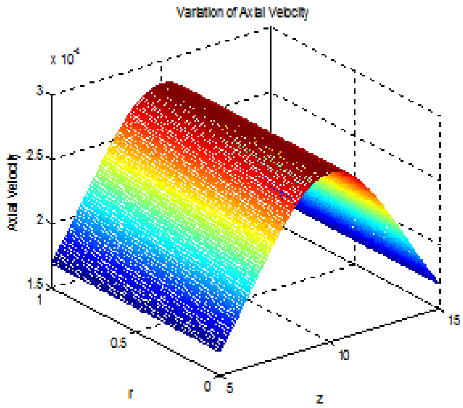


Fig. 4

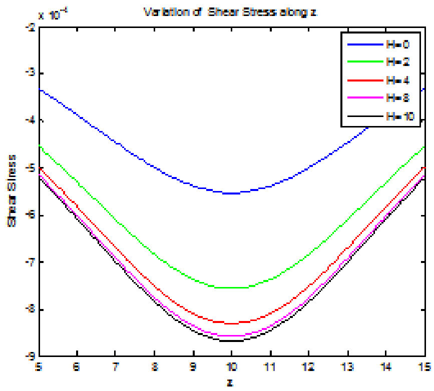
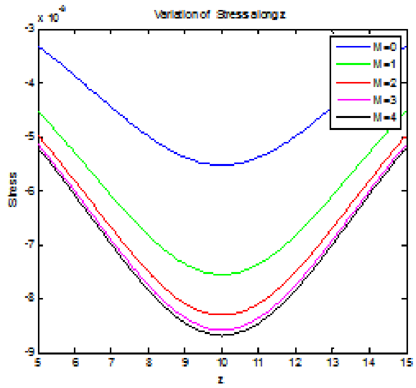


Fig. 5



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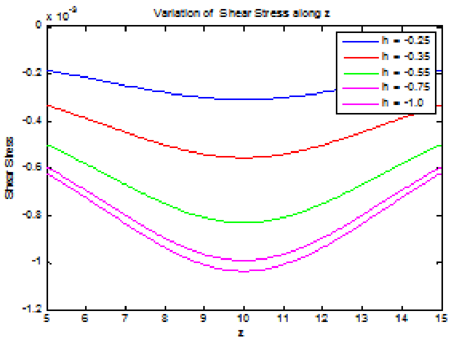

Fig. 7

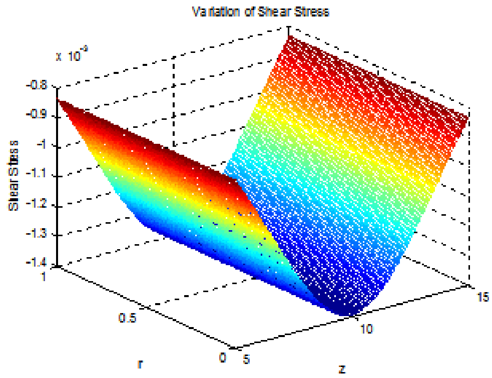

Fig. 8

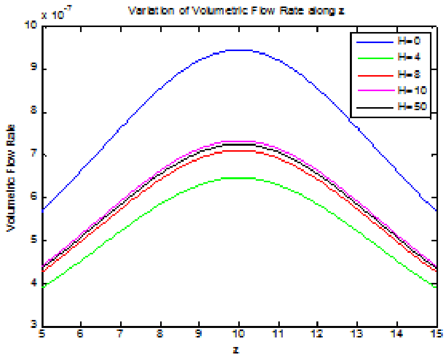

Fig. 9

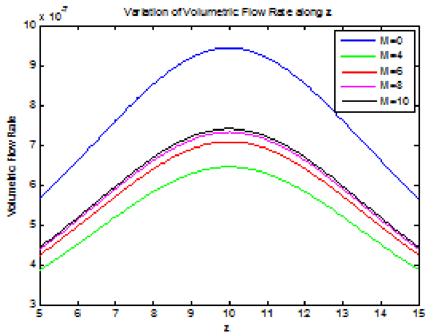

Fig. 10

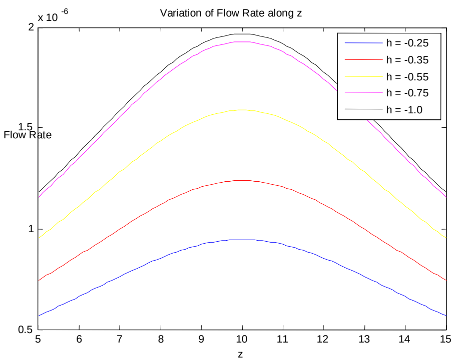

Fig. 11

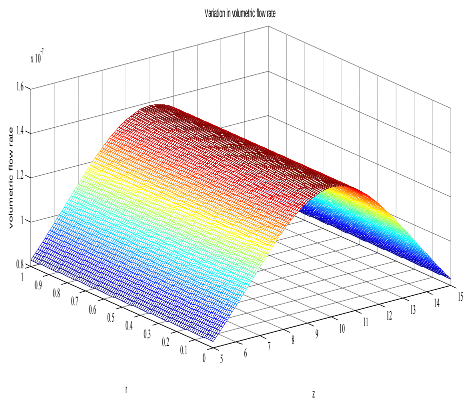

Fig. 12

VIII. CONCLUSION

In this paper, the combined effects of slip condition and external applied magnetic field on pulsatile blood flow in a constricted porous artery are analysed. It is observed that the slip velocity has an important role in decreasing wall shear stress and effective elastic viscosity of blood. We can also measure the extent of strength of magnetic field from which we can control and increase the blood flow in patients of hypertension and having blockage in their arteries. The present model can also be used as a tool to reduce the blood viscosity by using slip velocity with magnetic field at the constricted wall and it also provides scope to estimate the effect

of the various parameters provided above on various flow characteristics and to ensure which parameter is having the most significant role and can be used for better understanding of blood circulation in human body.
The main concern of this study is consideration of slip velocity at permeable wall and the conclusion of the study are given below:

(1) The pulsatile axial velocity $u(r,t)$ of the blood at the maximum height of the stenosis increases with increasing values of slip parameter h, Hartmann number H and permeability parameter M while it decreases with higher values of Hartmann number H.

(2) The volumetric flow rate $Q(r,t)$ of the blood at the maximum height of the stenosis increases with increasing values of slip parameter h, Hartmann number H and permeability parameter M while it decreases with higher values of Hartmann number H.

(3) $\tau(r,t)$ decreases at the maximum height of the stenosis when there is an increase in the values of slip parameter h, Hartmann number H and permeability parameter M while it increases with higher values of Hartmann number H.

(4) This model provides a most common form of fluid velocity, wall shear stress and volumetric flow rate and the other mathematical models related to this study can be simply possessed from this model by applying suitable values of parameters.

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