

Effect of Micro-inertia on Reflection of Plane Waves in a Transversely Isotropic Micropolar Elastic Solid

Rahul Hooda¹, Asha Sangwan², Shankar Gulia³

¹Assistant Professor, Department of Mathematics, A.I.J.H.M. College, Rohtak-124001, Haryana, India

²Assistant Professor, Department of Mathematics, Govt. College, Sampla, Rohtak-124001, Haryana, India

³Research Scholar, Department of Mathematics, I.G. University, Meerpur(Riwari)-123401, Haryana, India

Abstract: The paper deals with the problem of reflection of plane waves at the free surface of transversely isotropic micropolar elastic solid. The governing equations of transversely isotropic micropolar material are specialized in x-z plane. The plane wave solutions of these equations indicate the existence of three reflected waves in a transversely isotropic micropolar half-space. The relations in amplitude ratios and the expressions for energy ratios corresponding to reflected waves are obtained numerically with a suitable Snell's law. The speeds, amplitude ratios and the square root of energy ratios are plotted against the angle of incidence. The effect of micro-inertia on speeds, amplitude ratios and energy ratios are studied graphically for a particular model.

Keywords: Plane wave, reflection, micro-inertia, amplitude ratio, energy ratio.

Date of Submission: 26-08-2018

Date of acceptance: 07-09-2018

I. INTRODUCTION

The linear theory of elasticity has numerous applications in engineering structural materials. The linear elasticity describes the mechanical behavior of concrete, wood and coal. However, the linear theory of elasticity does not describe the behavior of some new synthetic materials, for example, polymethyl-methacrylate, polyethylene, polyvinyl chloride. The behavior of such materials is described by the theory of micropolar elasticity. Micropolar theory is an extension of elasticity with extra independent degrees of freedom for local rotation. The theory explains certain static and dynamic effects, i.e. new types of waves and coupled stress of the materials. In this theory, the motions of the particles are expressed in terms of displacement and micro-rotation vector.

Eringen [3] introduced the linear theory of micropolar elasticity and explained the micro-rotational motion and spin inertia that can support coupled stress and body couples in the materials. Many problems of waves and vibrations of micropolar elasticity have been investigated by several researches e.g. Nowacki and Nowacki [4-5], Nowacki [6], Parameshwaran and Koh [7], Smith [8], Achenbach [12] etc. Parameshwaran and Koh [7] discussed the problem of wave propagation in micro-isotropic and microelastic solids. Smith [8] discussed the problem of waves in micropolar elastic solid and obtained the velocity of surface wave. Ariman [9] studied the wave propagation in micropolar elastic solids. Parfitt and Eringen [10] investigated the plane wave propagation in an infinite isotropic homogeneous micropolar elastic solid half-space. Iesan [11] derived the uniqueness and existence theorems in the orthotropic micropolar elastic solids. Tomar and Gogna [13] discussed the problem of reflection and refraction of a longitudinal microrotational wave at an interface between two micropolar elastic media. Kumar and Choudhary [14] discussed the plane strain problem in homogeneous micropolar orthotropic elastic solids. Singh [15] studied the problem of wave propagation in an orthotropic micropolar elastic solid and obtained reflection coefficients of the reflected waves.

In this paper, we investigate the problem of reflection in a transversely isotropic micropolar medium. We obtain speeds, amplitude ratios and the square root of energy ratios of reflected waves for incident longitudinal wave. The effect of micro-inertia on the speed, amplitude ratios and energy ratios of reflected waves are discussed for a particular model.

II. BASIC EQUATIONS

Following Iesan [11], the equations of motion for a homogeneous transversely isotropic micropolar solid in the absence of body forces and body couples consists are $\sigma_{ji,j} = \rho \ddot{u}_i$ (1)

$$m_{ik,i} + \epsilon_{ijk} \sigma_{ij} = \rho j \ddot{\phi}_k \quad (i, j, k = 1, 2, 3) \quad (2)$$

The constitutive equations are written as

$$\sigma_{ij} = A_{ijkl}e_{kl} + G_{ijkl}\psi_{kl} \tag{3}$$

$$m_{ij} = G_{kl ij}e_{kl} + B_{ijkl}\psi_{kl} \tag{4}$$

The geometrical equations are written as

$$e_{ij} = u_{j,i} + \varepsilon_{ijk}\phi_k, \quad \psi_{ij} = \phi_{j,i} \tag{5}$$

where σ_{ij} is the stress tensor, ρ is the mass density, \bar{u} is the displacement vector, $\bar{\phi}$ is the microrotation vector, j is the micro-inertia, m_{ij} is the couple stress tensor, ε_{ijk} is the alternating symbol, e_{ij} and ψ_{ij} are kinematic strain measures and A_{ijkl} , B_{ijkl} and G_{ijkl} are constitutive coefficients. Superposed dot denote partial differentiation with respect to the time t . Subscripts preceded by a comma denote partial differentiation with respect to the corresponding cartesian coordinate and the repeated index in the subscript implies summation.

III. FORMULATION OF THE PROBLEM

We consider a homogeneous transversely isotropic micropolar solid half-space (medium M). The origin of the Cartesian coordinate system (x,y,z) is taken at any point on the plane interface and z -axis pointing vertically downwards into M is taken which is designated as $z \geq 0$. The present study is restricted to the plane strain parallel to x - z plane. For two-dimensional problem, the displacement vector \bar{u} and microrotation vector $\bar{\phi}$ in medium M are taken as

$$\bar{u} = (u_1, 0, u_3) \quad \text{and} \quad \bar{\phi} = (0, \phi_2, 0) \tag{6}$$

Using eqs. (1) to (6), the equations of motion for transversely isotropic micropolar medium M are expressed as

$$A_{11} \frac{\partial^2 u_1}{\partial x^2} + (A_{13} + A_{56}) \frac{\partial^2 u_3}{\partial x \partial z} + A_{55} \frac{\partial^2 u_1}{\partial z^2} + K_1 \frac{\partial \phi_2}{\partial z} = \rho \frac{\partial^2 u_1}{\partial t^2} \tag{7}$$

$$A_{66} \frac{\partial^2 u_3}{\partial x^2} + (A_{13} + A_{56}) \frac{\partial^2 u_1}{\partial x \partial z} + A_{33} \frac{\partial^2 u_3}{\partial z^2} + K_2 \frac{\partial \phi_2}{\partial x} = \rho \frac{\partial^2 u_3}{\partial t^2} \tag{8}$$

$$B_{77} \frac{\partial^2 \phi_2}{\partial x^2} + B_{66} \frac{\partial^2 \phi_2}{\partial z^2} - \chi \phi_2 - K_1 \frac{\partial u_1}{\partial z} - K_2 \frac{\partial u_3}{\partial x} = \rho j \frac{\partial^2 \phi_2}{\partial t^2} \tag{9}$$

$$\text{where } K_1 = A_{56} - A_{55}, K_2 = A_{66} - A_{56}, \chi = K_2 - K_1 \tag{10}$$

IV. SOLUTION OF THE PROBLEM

We seek the plane wave solution of the eqs. (7) to (9) in x - z plane in the following form

$$\{u_1, u_3, \phi_2\} = (A, B, C) \exp\{ik(x \sin \theta + z \cos \theta - vt)\} \tag{11}$$

where k is the wave number and v is the speed of wave propagating in x - z plane along a direction making an angle θ with z -axis.

Making use of eq. (11) in eqs. (7-9), we obtain three homogeneous equations in A , B and C and which have non-trivial solution if

$$\Gamma^3 - S_1 \Gamma^2 + S_2 \Gamma - S_3 = 0 \tag{12}$$

where

$$\Gamma = \rho v^2$$

$$S_1 = D_1 + D_2 + D_3^*,$$

$$S_2 = D_1 D_2 + D_2 D_3^* + D_3^* D_1 - L^2 - K_1 K_1^* \cos^2 \theta - K_2 K_2^* \sin^2 \theta,$$

$$S_3 = D_1 D_2 D_3^* - D_3^* L^2 - K_2 K_2^* D_1 \sin^2 \theta - K_1 K_1^* D_2 \cos^2 \theta + 2LK_1 K_2^* \sin \theta \cos \theta,$$

$$D_1 = A_{11} \sin^2 \theta + A_{55} \cos^2 \theta, \quad D_2 = A_{66} \sin^2 \theta + A_{33} \cos^2 \theta, \quad D_3 = B_{77} \sin^2 \theta + B_{66} \cos^2 \theta,$$

$$L = (A_{13} + A_{56}) \sin \theta \cos \theta, \quad D_3^* = \frac{D_3}{j} + \frac{\chi}{jk^2}, \quad K_1^* = \frac{K_1}{jk^2}, \quad K_2^* = \frac{K_2}{jk^2}.$$

The three real roots of cubic eq. (12) in v^2 correspond to the speeds of three quasi plane waves in a transversely isotropic micropolar medium. The three roots v_1, v_2 and v_3 ($v_1 > v_2 > v_3$) correspond to the speeds of Coupled Longitudinal Displacement (CLD), Coupled Transverse Displacement (CTD) and Coupled Transverse Microrotational (CTM) waves, respectively.

V. REFLECTION FROM FREE SURFACE

A train of longitudinal wave with amplitude A_0 is incident at the plane interface making an angle θ_0 with the normal. This wave gives three reflected coupled waves in the half-space M. The complete geometry of the problem is shown in Fig.1.

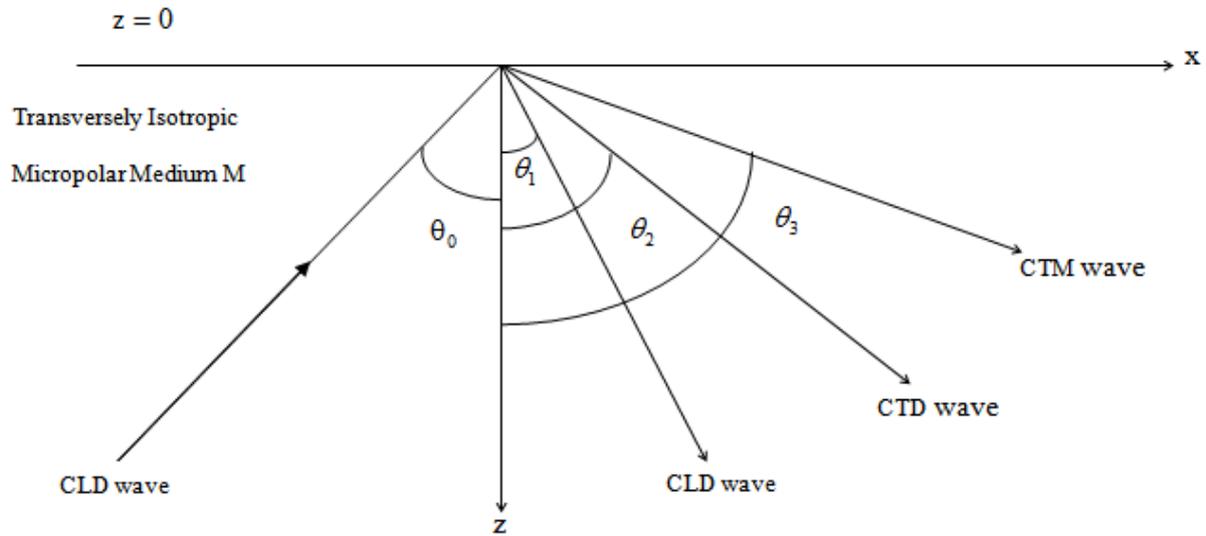


Fig.1 Geometry of the Problem

The formal solution for the displacement components u_1, u_3 and microrotation component ϕ_2 are as follow:

$$u_1 = A_0 \exp\{ik_1(x \sin \theta_0 + z \cos \theta_0 - v_1 t)\} + \sum_{j=1}^3 A_j \exp\{ik_j(x \sin \theta_j - z \cos \theta_j - v_j t)\}, \tag{13}$$

$$u_3 = \varepsilon_1^* A_0 \exp\{ik_1(x \sin \theta_0 + z \cos \theta_0 - v_1 t)\} + \sum_{j=1}^3 \varepsilon_j A_j \exp\{ik_j(x \sin \theta_j - z \cos \theta_j - v_j t)\}, \tag{14}$$

$$\phi_2 = \eta_1^* A_0 \exp\{ik_1(x \sin \theta_0 + z \cos \theta_0 - v_1 t)\} + \sum_{j=1}^3 \eta_j A_j \exp\{ik_j(x \sin \theta_j - z \cos \theta_j - v_j t)\}, \tag{15}$$

where v_i ($i = 1, 2, 3$) are real speeds of CLD, CTD and CTM waves, respectively.

The coupling constants $\varepsilon_1^*, \eta_1^*, \varepsilon_j$ and η_j ($j = 1, 2, 3$) are defined as

$$\varepsilon_1^* = -\varepsilon_1, \eta_1^* = -\eta_1, \varepsilon_j = \frac{\Delta_{1j}}{\Delta_j}, \eta_j = \frac{\Delta_{2j}}{\Delta_j},$$

where

$$\Delta_j = C_j K_{2j} - Q_j K_{1j}, \Delta_{1j} = P_j K_{2j} - C_j K_{1j}, \Delta_{2j} = C_j^2 - P_j Q_j,$$

$$P_j = \rho v_j^2 - A_{11} \sin^2 \theta_0 \left(\frac{v_j}{v_1}\right)^2 - A_{55} \left[1 - \sin^2 \theta_0 \left(\frac{v_j}{v_1}\right)^2\right], \quad Q_j = \rho v_j^2 - A_{66} \sin^2 \theta_0 \left(\frac{v_j}{v_1}\right)^2 - A_{33} \left[1 - \sin^2 \theta_0 \left(\frac{v_j}{v_1}\right)^2\right],$$

$$C_j = -(A_{13} + A_{56}) \sin \theta_0 \left(\frac{v_j}{v_1}\right) \sqrt{1 - \sin^2 \theta_0 \left(\frac{v_j}{v_1}\right)^2}, \quad K_{1j} = iK_1 \sqrt{1 - \sin^2 \theta_0 \left(\frac{v_j}{v_1}\right)^2}, \quad K_{2j} = iK_2 \sin \theta_0 \left(\frac{v_j}{v_1}\right),$$

VI. BOUNDARY CONDITIONS

The appropriate boundary conditions at an interface $z = 0$ are the continuity of normal and tangential force stress components and continuity of tangential couple stress component i.e.,

$$\sigma_{33} = 0, \sigma_{31} = 0, m_{32} = 0 \tag{16}$$

where

$$\sigma_{33} = A_{13} u_{1,1} + A_{33} u_{3,3}$$

$$\sigma_{31} = A_{56} u_{3,1} + A_{55} u_{1,3} + (A_{56} - A_{55}) \phi_2$$

$$m_{32} = B_{66}\phi_{2,3}$$

The displacement components and microrotation component are given by eqs. (13) to (15) satisfy boundary conditions (16) if following relations (Snell's law) hold

$$k_1 \sin \theta_0 = k_1 \sin \theta_1 = k_2 \sin \theta_2 = k_3 \sin \theta_3 \tag{17}$$

$$k_1 v_1 = k_2 v_2 = k_3 v_3 \tag{18}$$

and we obtain following non-homogeneous system of three equations in amplitude ratios of reflected waves

$$\sum_{j=1}^3 a_{ij} Z_j = b_i, \quad (i=1,2,3) \tag{19}$$

where $Z_j = \frac{A_j}{A_0}$, ($j=1,2,3$) are amplitude ratios of reflected CLD, CTD and CTM waves, respectively and

$$a_{1j} = \frac{A_{13} \sin \theta_0 - A_{33} \epsilon_j \sqrt{\left(\frac{v_1}{v_j}\right)^2 - \sin^2 \theta_0}}{A_{13} \sin \theta_0 + A_{33} \epsilon_1^* \cos \theta_0}, \quad (j=1,2,3)$$

$$a_{2j} = \frac{A_{56} \epsilon_j \sin \theta_0 - A_{55} \sqrt{\left(\frac{v_1}{v_j}\right)^2 - \sin^2 \theta_0} - i(A_{56} - A_{55}) \left(\frac{v_1}{v_j}\right) \left(\frac{\eta_j}{k_j}\right)}{A_{56} \epsilon_1^* \sin \theta_0 + A_{55} \cos \theta_0 - i(A_{56} - A_{55}) \left(\frac{\eta_1}{k_1}\right)}, \quad (j=1,2,3)$$

$$a_{3j} = \frac{\eta_j \sqrt{\left(\frac{v_1}{v_j}\right)^2 - \sin^2 \theta_0}}{\eta_1^* \cos \theta_0}, \quad (j=1,2,3)$$

$$b_1 = -1, b_2 = -1, b_3 = 1$$

VII. ENERGY RATIOS

We shall now consider the partitioning of energy between different reflected waves at a surface element of unit area. Following Achenbach [12], the instantaneous rate of work of surface traction is the scalar product of the surface traction and the particle velocity. This scalar product is called the power per unit area, denoted by P^* , and represents the rate at which the energy is communicated per unit area of the surface, i.e., the energy flux across the surface element. The time average of P^* over a period, denoted by $\langle P^* \rangle$, represents the average energy transmission per unit surface area per unit time. For the micropolar medium, the rate of energy transmission at the free surface $z = 0$ is given by

$$P^* = \sigma_{33} \dot{u}_3 + \sigma_{31} \dot{u}_1 + m_{32} \dot{\phi}_2 \tag{20}$$

The time rate of average energy transmission for the respective wave to that of the incident wave, denoted by E_j ($j=1, 2, 3$) for reflected CLD, reflected CTD, reflected CTM waves respectively, are given as

$$E_j = \frac{\langle P_j^* \rangle}{\langle P_0^* \rangle}, \quad (j=1,2,3) \tag{21}$$

where $\langle P_0^* \rangle$ denotes the average energy transmission per unit surface area per unit time for incident CLD wave in micropolar medium M .

The expressions for energy ratios at an interface $z = 0$ are given as

$$E_j = \left(\frac{p_j - q_j - r_j}{w_0} \right) Z_j^2, \quad (j=1,2,3) \tag{22}$$

where

$$p_j = (A_{13} \epsilon_j + A_{56} \epsilon_j) \sin \theta_0, \quad q_j = (A_{55} + A_{33} \epsilon_j^2 + B_{66} \eta_j^2) \sqrt{\left(\frac{v_1}{v_j}\right)^2 - \sin^2 \theta_0}, \quad r_j = i(A_{56} - A_{55}) \left(\frac{v_1}{v_j}\right) \left(\frac{\eta_j}{k_j}\right)$$

$$w_0 = (A_{13} + A_{56})\epsilon_1^* \sin \theta_0 + (A_{55} + A_{33}\epsilon_1^{*2} + B_{66}\eta_1^{*2}) \cos \theta_0 - i(A_{56} - A_{55}) \begin{pmatrix} \eta_1^* \\ k_1 \end{pmatrix}$$

VIII. NUMERICAL RESULTS AND DISCUSSION

We are interested in the computation of speeds, amplitude ratios and the square root of energy ratios of reflected waves for incident longitudinal wave. We have developed programs on MATLAB for the computation of speed, amplitude ratios and the square root of energy ratios and will discuss the effects of micro-inertia. To illustrate the numerical results graphically, the value for relevant parameters for transversely isotropic micropolar solid are taken as (modified value of Singh [16])

$$A_{11}=17.8 \times 10^{11} \text{Nm}^{-2}, A_{33}=18.43 \times 10^{11} \text{Nm}^{-2}, A_{13}=7.59 \times 10^{11} \text{Nm}^{-2}, A_{56}=1.89 \times 10^{11} \text{Nm}^{-2},$$

$$A_{55}=4.357 \times 10^{11} \text{Nm}^{-2}, A_{66}=4.42 \times 10^{11} \text{Nm}^{-2}, B_{77}=0.278 \times 10^{10} \text{N}, B_{66}=0.268 \times 10^{10} \text{N}, \rho=1.74 \times 10^3 \text{Kg m}^{-3}$$

The variations of speeds of reflected CLD, CTD and CTM waves with angle of incidence at different values of micro-inertia are shown in Figs. 2-4. The variations of absolute values of amplitude ratios of reflected waves with angle of incidence at different values of micro-inertia are shown in Figs. 5-7 and those of square root of energy ratios are depicted in Figs 8-10. In all figures, we take values of micro-inertia as follow:

$$j = 0.019, j = 0.02, j = 0.021.$$

Effect of Micro-inertia on Speeds of Reflected waves

In Figs. 2-4, it is observed that there is a little effect of micro-inertia on the speed of reflected CLD wave and the speeds of reflected CTD and reflected CTM waves decrease with increase of micro-inertia.

Effect of Micro-inertia on Amplitude Ratios of Reflected waves

From Fig. 5, it may be noted that there is a little effect of micro-inertia on the amplitude ratios of reflected CLD wave. From Figs. 6 and 7, it is observe that value of amplitude ratios increase for reflected CTD wave and decrease for reflected CTM wave with increase of micro-inertia. The minimum effect of micro-inertia on amplitude ratios of reflected CTD and CTM waves is observed near grazing angle of incidence.

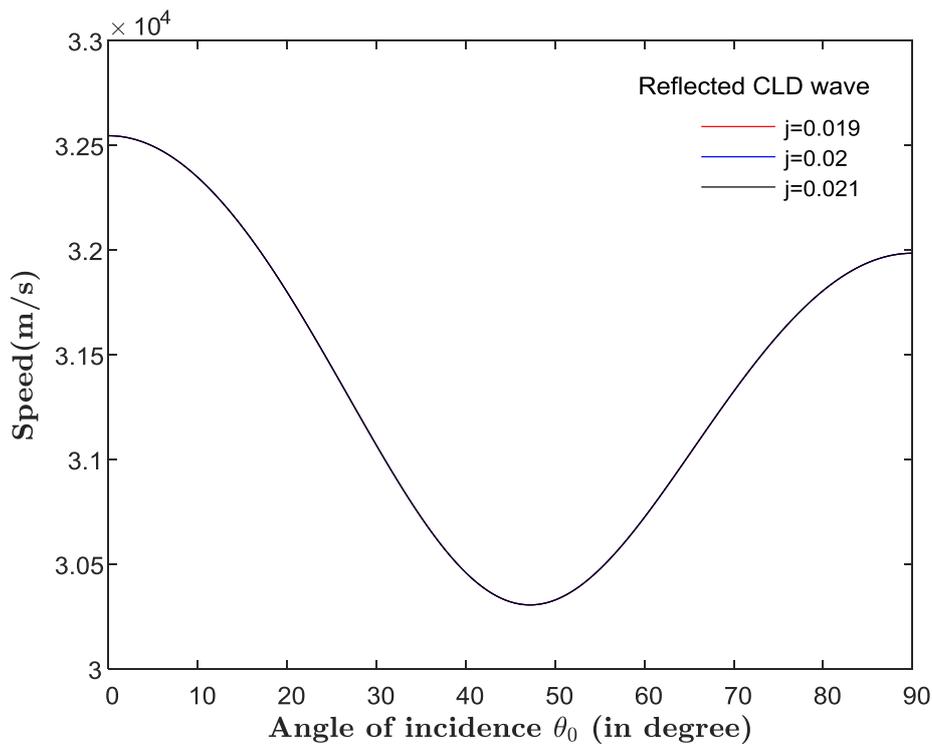


Fig.2 Variations of speeds of reflected coupled longitudinal displacement (CLD) wave against the angle of incidence of incident CLD wave at different values of micro-inertia

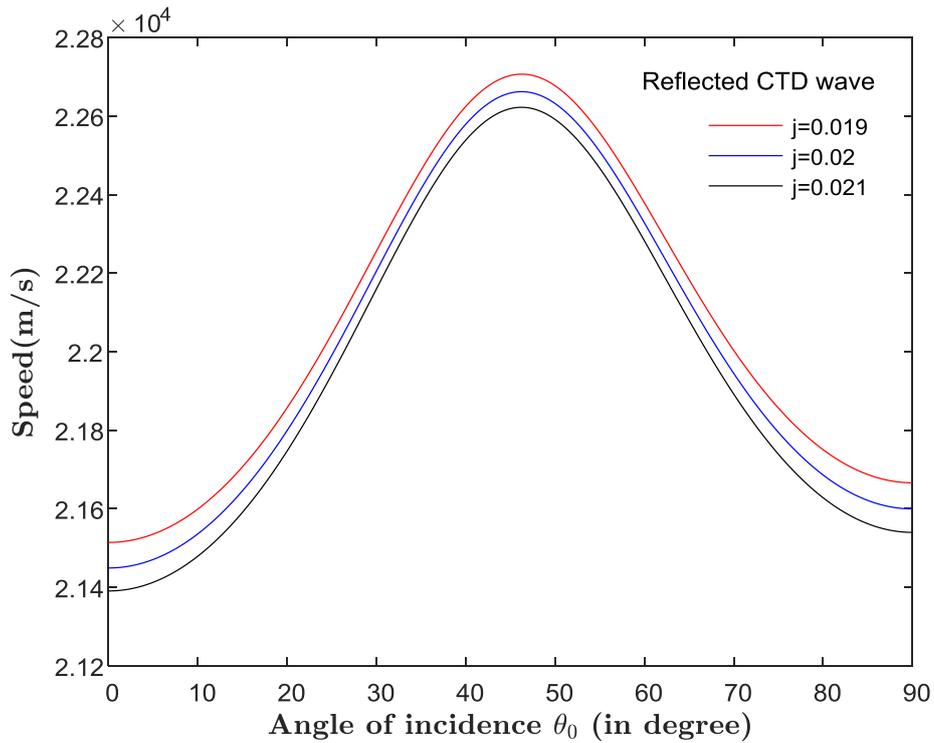


Fig.3 Variations of speeds of reflected coupled transverse displacement (CTD) wave against the angle of incidence of incident CLD wave at different values of micro-inertia

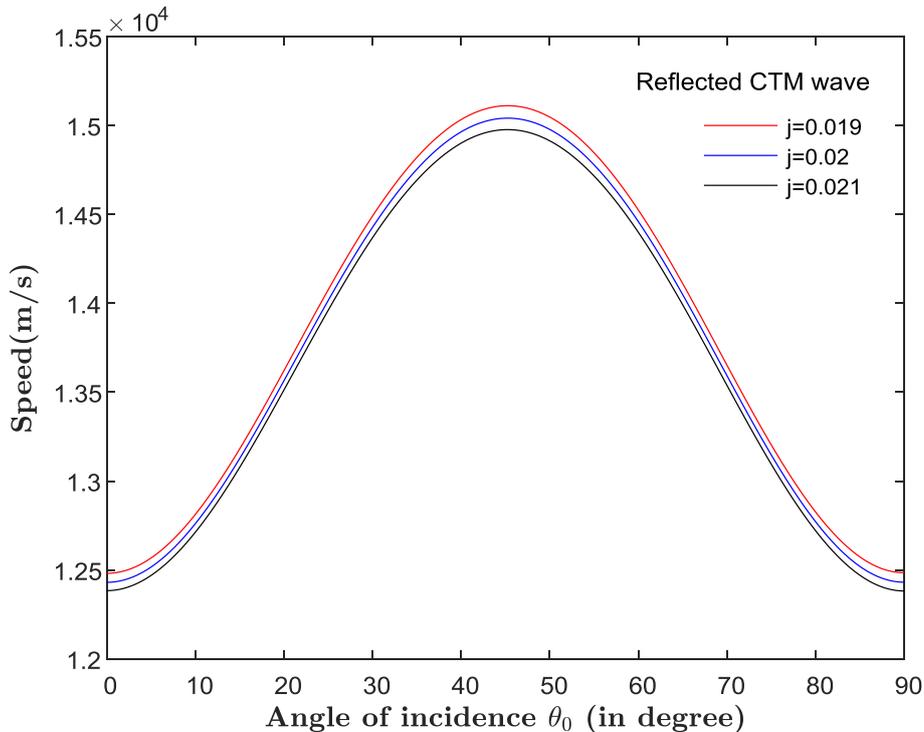


Fig.4 Variations of speeds of reflected coupled transverse microrotational (CTM) wave against the angle of incidence of incident CLD wave at different values of micro-inertia

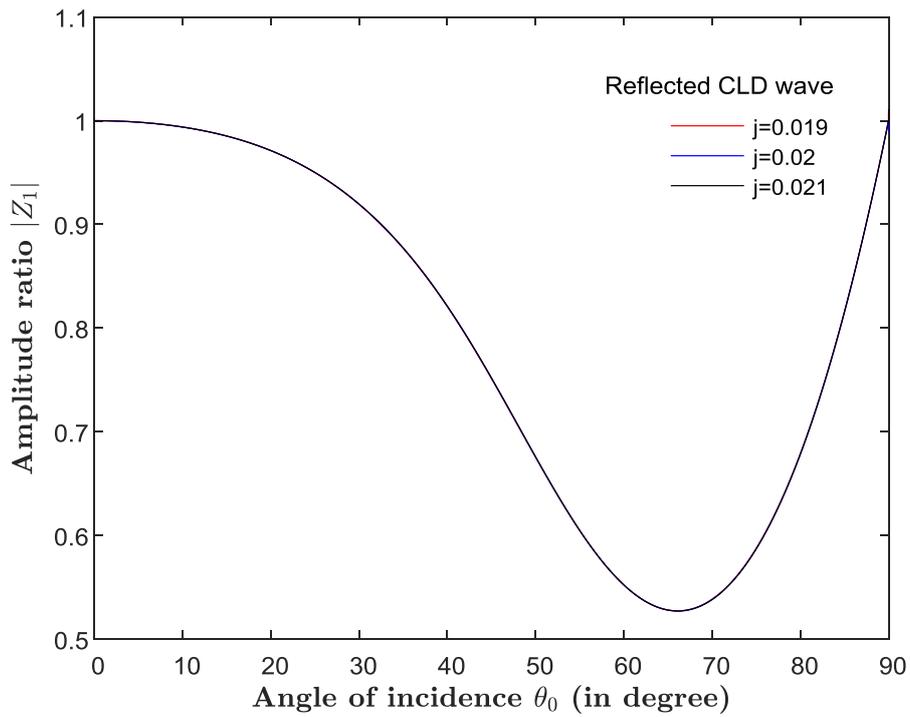


Fig.5 Variations of amplitude ratios of reflected coupled longitudinal displacement (CLD) wave against the angle of incidence of incident CLD wave at different values of micro-inertia

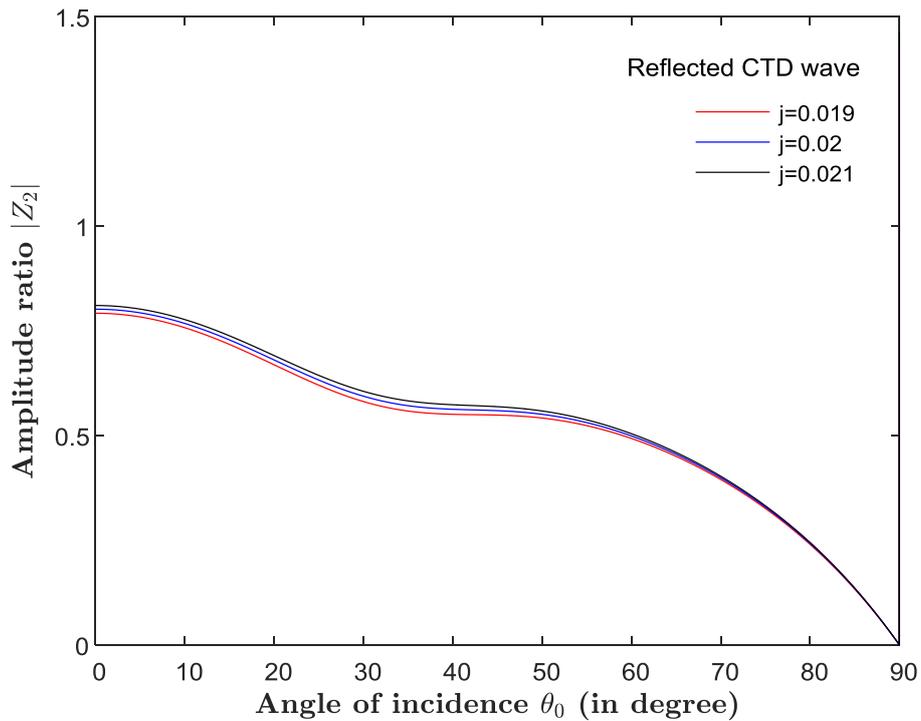


Fig.6 Variations of amplitude ratios of reflected coupled transverse displacement (CTD) wave against the angle of incidence of incident CLD wave at different values of micro-inertia

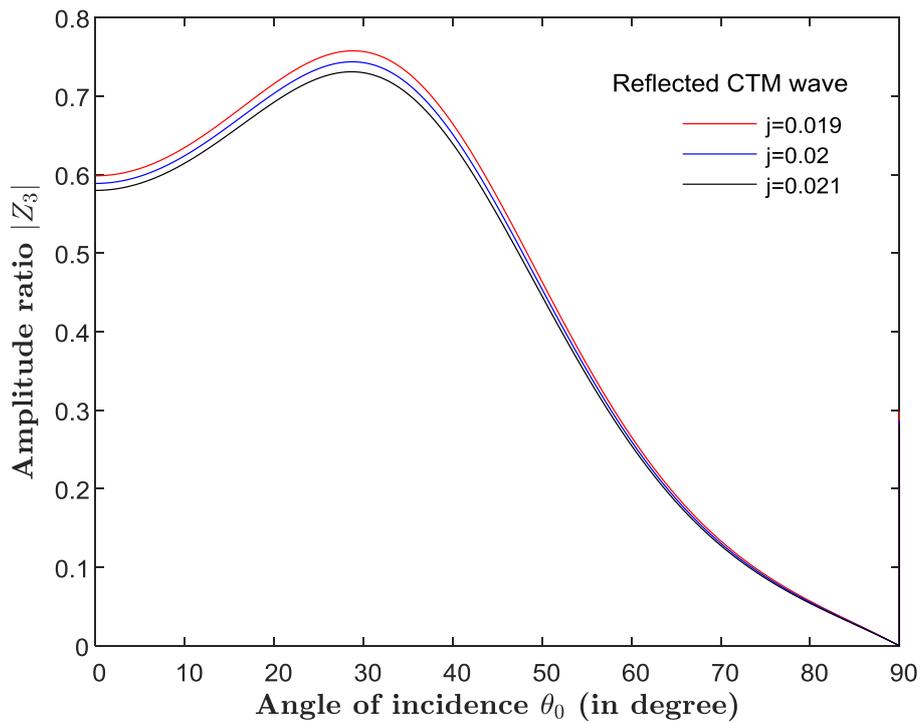


Fig.7 Variations of amplitude ratios of reflected coupled transverse microrotational (CTM) wave against the angle of incidence of incident CLD wave at different values of micro-inertia

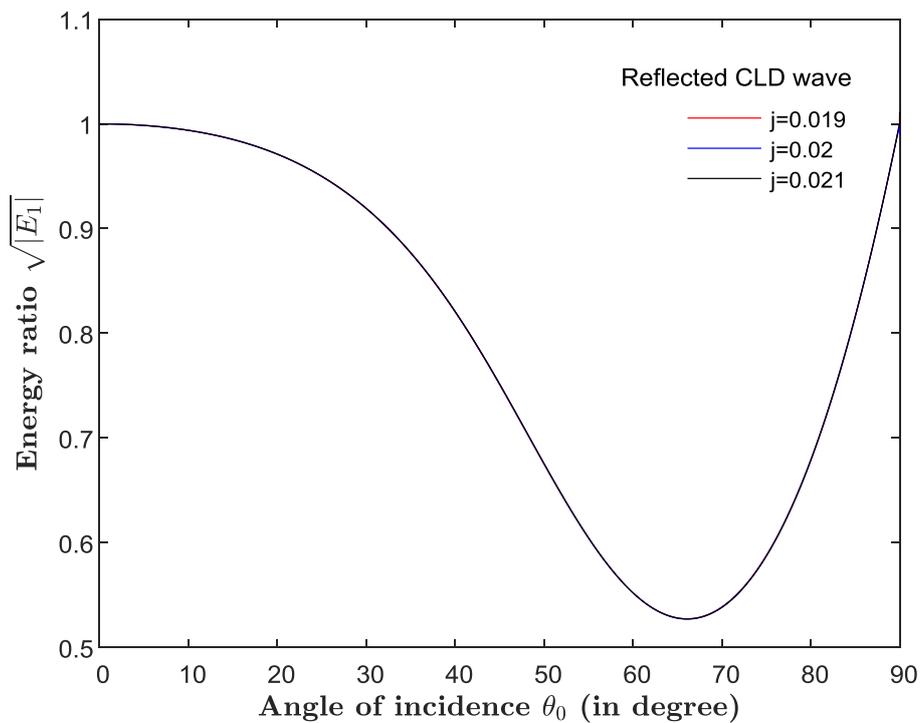


Fig.8 Variations of the square root of energy ratios of reflected coupled longitudinal displacement (CLD) wave against the angle of incidence of incident CLD wave at different values of micro-inertia

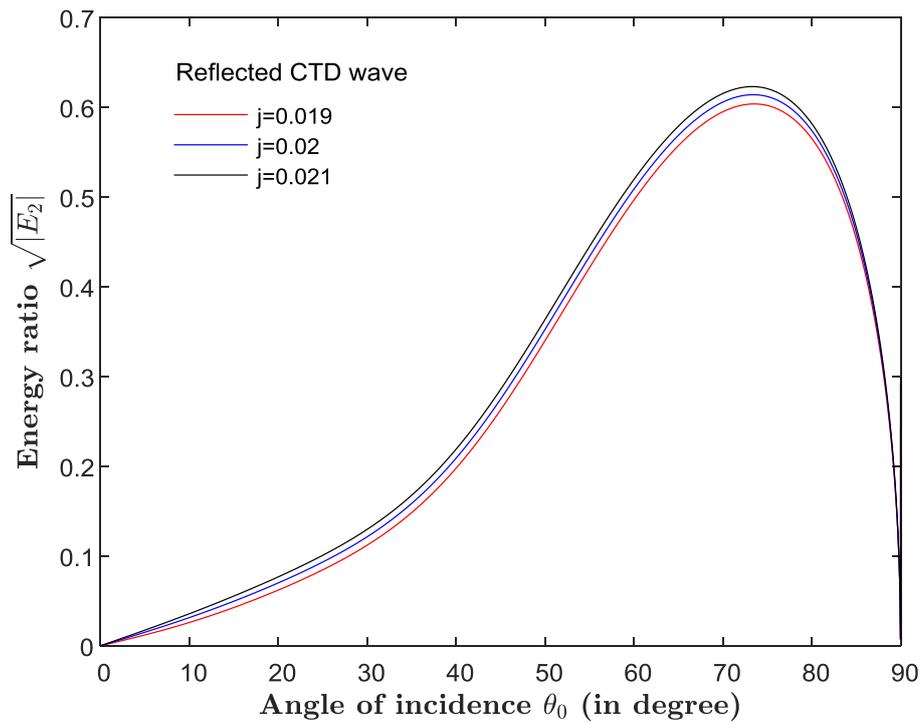


Fig.9 Variations of the square root of ratios of reflected coupled transverse displacement (CTD) wave against the angle of incidence of incident CLD wave at different values of micro-inertia

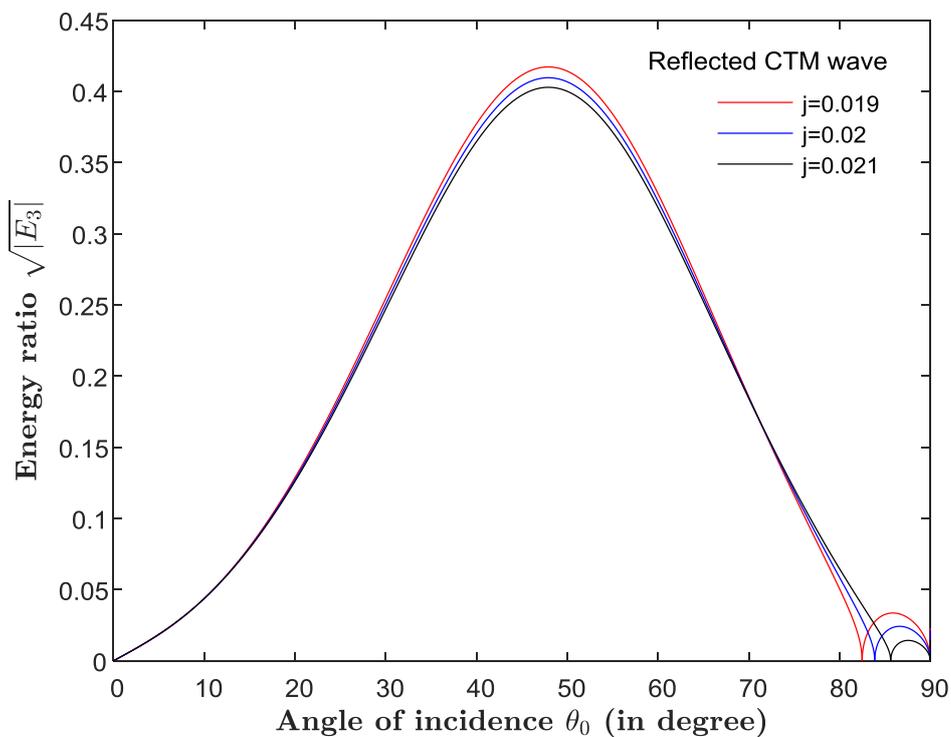


Fig.10 Variations of the square root of ratios of reflected coupled transverse microrotational (CTM) wave against the angle of incidence of incident CLD wave at different values of micro-inertia

From Fig. 8, it is observed that there is a little effect of micro-inertia on the value of square root of energy ratio of reflected CLD wave. From Fig. 9, we conclude that the value of square root of energy ratios of reflected CTD wave increase with increase of micro-inertia and the minimum effect of micro-inertia has observed near grazing angle of incidence. From Fig. 10, it may be noted that minimum effect of micro-inertia on value of square root of energy ratio of reflected CTM wave is observed in the range $\theta_0 = 0^\circ - 16^\circ$. Thereafter, value of square root of energy ratios of reflected CTM wave decrease with increase of micro-inertia upto $\theta_0 = 70^\circ$ and then there is a little change from $\theta_0 = 70^\circ$ to 72° . Thereafter, it increases with increase of micro-inertia upto $\theta_0 = 83^\circ$ and decrease with increase of micro-inertia from $\theta_0 = 85.7^\circ$ to $\theta_0 = 90^\circ$.

IX. CONCLUSION

The problem of the effect of micro-inertia on the reflection of plane waves at the free surface of transversely isotropic micropolar half-space has been investigated. The speeds, amplitude ratios and the square root of energy ratios of the reflected CLD, CTD and CTM waves due to incident CLD wave are obtained. These speeds, amplitude ratios and energy ratios are computed numerically for different values of micro-inertia and study the effects. We may summarize with the following concluding remarks:

- i. The speeds, amplitude ratios and energy ratios of reflected waves are functions of angle of incidence, frequency and micro-inertia.
- ii. There is a little effect of micro-inertia on the speeds, amplitude ratios and square root of energy ratios of reflected CLD wave.
- iii. The speeds of reflected CTD wave decrease and amplitude ratios and the square root of energy ratios of reflected CTD wave increase with increase of micro-inertia.
- iv. The speeds and amplitude ratios of reflected CTM wave decrease with increase of micro-inertia and have mixed effect on the square root of energy ratios of CTM wave.

REFERENCES

- [1]. Eringen A.C and Suhubi E.S. Non-linear theory of simple microelastic solids I. International Journal of Engineering Science. 1964; 2: 189-203.
- [2]. Eringen A.C and Suhubi E.S. Non-linear theory of simple microelastic solids II. International Journal of Engineering Science. 1964; 2: 389-404.
- [3]. Eringen A.C. Linear theory of micropolar elasticity. Journal of Mathematics and Mechanics. 1966; 15: 909-924.
- [4]. Nowacki W. and Nowacki W.K. Generation of waves in infinite micropolar elastic solid body I. Bulletin of the Polish Academy of Sciences Technical Sciences. 1969; 17: 39-47.
- [5]. Nowacki W. and Nowacki W.K. Generation of waves in infinite micropolar elastic solid body II. Bulletin of the Polish Academy of Sciences Technical Sciences. 1969; 17: 49-56.
- [6]. Nowacki W. Theory of micropolar elasticity. International Centre for Mechanical Sciences, Courses and Lectures, No. 25. 1970; Springer-Verlag, Berlin.
- [7]. Parameshwaran S. and Koh S.L. Wave propagation in micro-isotropic, microelastic solids. International Journal of Engineering Science. 1973; 11: 95-107.
- [8]. Smith A.C. Waves in micropolar elastic solids. International Journal of Engineering Science. 1967; 5(10): 741-746.
- [9]. Ariman T. Wave propagation in a micropolar elastic half-space. Acta Mechanica. 1972; 13(1-2): 11-20.
- [10]. Parfitt V.R. and Eringen A.C. Reflection of plane waves from a flat boundary of a micropolar elastic half-space. Journal of the Acoustical Society of America. 1971; 45(5): 1258-1272.
- [11]. Iesan D. The plane micropolar strain in orthotropic elastic solids. Archives of Mechanics. 1973; 25(3): 547-561.
- [12]. Achenbach J. D. Wave propagation in elastic solids .Vol. 16, North-Holland Publishing Company. 1973; Amsterdam, Elsevier.
- [13]. Tomar S.K. and Gogna M.L. Reflection and refraction of longitudinal wave at an interface between two micropolar elastic solids in welded contact. Journal of the Acoustical Society of America. 1995; 97: 822-830.
- [14]. Kumar R. and Choudhary S. Response of orthotropic micropolar elastic medium due to time harmonic source. Sadhana. 2004; 29(1): 83-92.
- [15]. Singh B. Wave propagation in an orthotropic micropolar elastic solid. International Journal of Solids and Structure. 2007; 44: 3638-3645.
- [16]. Singh B. and Goyal M. Wave propagation in a transversely isotropic microstretch elastic solid. Mechanics of Advanced Materials and Modern Processes. 2017; 3(8): 1-10.