

A Review on the static pile-soil interaction

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Abstract: In civil engineering, in order to increase the factor of safety for heavy loads buildings or any other structure built upon special conditions such as, the presence of highly compressible upper soil layers or comparatively weak to support the whole load of the structure, pile foundations are preferably used thus to transmit loads from superstructures built on them deep into a hard deeper layer. The fact that piles are driven deep into the ground thus to interact mostly with saturated soil layers, their interaction with surrounding soils remain one of the most crucial essential parts of the whole structure. Besides, the pile-soil system behavior is predominantly nonlinear and this makes the problem complicated. Nowadays, the analysis became easier with the debut of powerful computers and simulation tools. A detailed review on the static interaction between piles and homogenous and non-homogenous soils from different existing kinds of literature is presented in this paper. Further, this review paper uses a combination of older and most recent literature to provide a historical perspective on the topic and to illustrate advances on this topic.

Keywords: -fictitious pile method, pile-soil interaction, soil, transverse isotropy property, vertically loaded piles

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I. INTRODUCTION

In civil engineering, pile foundations are essential elements in structural design in soft soils. They permit structures to be constructed upon weak or erratic soils by transferring loads of the building to a hard deeper soil layer. Piles vary in length, diameter, and material, which are all determined based on the given site conditions and loading. They can withstand axial loads, lateral loads or a combination of both, as long as they are designed in a proper way. These are required in special situations such as: where the upper soil layers are highly compressible and comparatively weak to support the whole load of the structure; the structure is subjected to horizontal forces; there is a presence of expansive soil; the foundation is subjected to uplifting forces; there is a possibility of the soil erosion at the surface of the ground due to water flows; etc. The fact that piles are driven deep into the ground thus to interact mostly with saturated soil layers, their interaction with surrounding soils remain one of the most urgently important parts of the whole structure.

Natural soil deposits are, in many practical conditions, both non-homogeneous and cross-anisotropic with a vertical axis of symmetry. The stratification nature of soil deposits is a result of the deposition history of the soil and also reflects the behavior of the soil under sustained increasing overburden pressure. Apart from stratification, most engineering soils also present the transverse or cross-anisotropic behavior. This property generally originates from the sedimentation process and the variation of the orientation in different directions of the soil particles, resulting in different soil's Young's modulus in both horizontal and vertical directions.

The object of this current paper is to present a review of pile-soil interaction studies based on lateral and vertical soil properties. This review also discusses some of the results presented by different authors. It is noted that, this paper mainly discusses the static interaction between vertically loaded piles and soils. So far a number of methods have been presented by different authors for the investigation of the pile-soil interaction. A few and important studies on the pile response in homogeneous and non-homogenous soil medium are presented in the following sections.

II. PILE IN HOMOGENEOUS SOIL

In early studies, several investigators [1-5] outlined approximate methods based on elastic theory for analyzing different aspects of the load-displacement behavior of single axially loaded piles and piers. Poulos and Davis [6, 7] proposed accurate analyses to the problem of the axial loading of incompressible floating piles embedded in an ideal elastic homogenous half-space (Fig. 1) by employing a discretization procedure. As shown in Fig. 2, the interface between the bar and the elastic soil was discretized into N series of ring elements of

finite length, each acted upon by a uniform shear loading P_k ($k = 1, \dots, i, \dots, j, \dots, N$). Here, Q and P_b represent the vertically applied load on the pile top and the load carried by the pile base. The expressions for the axial displacement due to stresses acting vertically on these elements were derived by utilizing Mindlin's solution [8] for concentrated axial load acting at the interior of the half-space.

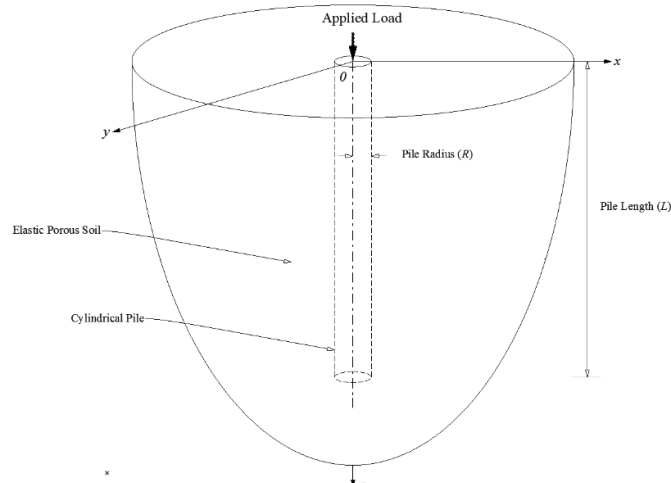


Figure 1. Schematic of a vertically loaded pile embedded in a homogeneous soil medium

Later, Mattes and Poulos[9] utilized a mirror image technique for end-bearing piles seated on a firm base. In order to eliminate the assumption of a smooth disc pile base and taking into consideration the disturbance of the half-space due to the presence of the piles that was ignored in the above mentioned studies, Butterfield and Banerjee [10] presented a rigorous analysis based on the Mindlin's equations [8] together with the analytical method to study the response of a rigid and compressible single piles and pile groups with a floating cap in a homogeneous linear elastic soil. The effect of smooth disc pile base on under reamed piles and very short plain piles was shown to be significant, besides, the pile compressibility was also illustrated to be an important aspect in a pile group behavior.

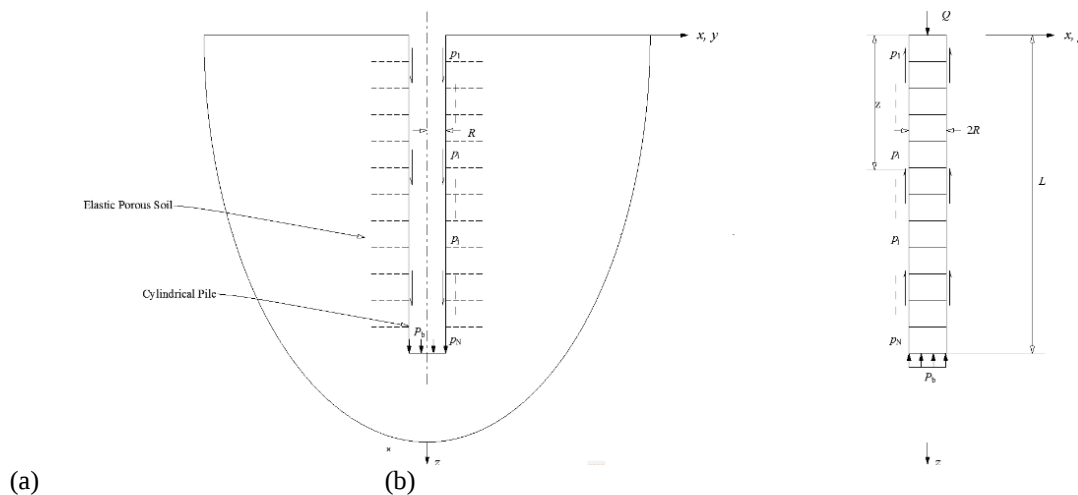


Figure 2. Stress acting on (a) adjacent soil and (b) pile

Muki and Sternberg [11, 12] proposed an important approach for partially embedded rod in an elastic half-space under axial load based on an assumption that decomposes the bar-half-space system into a one-dimensional bar and an extended half-space (Fig. 3). Such an assumption essentially reduces the problem to the following solution of a Fredholm integral equation of the second kind in which the axial force in the rod becomes the primary unknown.

$$\varepsilon_z(z) = [Q + P_*(0)] \varepsilon_z^{(G)}(0, z) - P_*(L) \varepsilon_z^{(G)}(L, z) - \int_0^L q_z(\xi) \varepsilon_z^{(G)}(\xi, z) d\xi, \quad (1)$$

here, $A = \pi R^2$ denotes the cross-section area of the pile and Q is the total external load applied at the top of the pile. As shown in Fig. 3, the top and the bottom of the fictitious pile is subjected to the forces $P_*(0)$ and $P_*(L)$ concentrated within the terminal $\Pi(0)$ and $\Pi(L)$, while $q_z(z)$ denotes the bond force per unit pile-length exerted by the extended soil on the fictitious pile a depth z . The expression $\varepsilon_z(z)$ in equation (1) represent the vertical strain of the extended half-space at the center of the cross-section $\Pi(z)$ and the influence function $\varepsilon_z^{(G)}(\xi, z)$ denote the vertical strain of the semi-infinite soil at the depth z caused by a uniform circular load over the cross-section $\Pi(\xi)$. Once the axial force $P_*(z)$ of the fictitious pile has been found, the solution for the real pile axial force, vertical displacement and soil pore pressure on the pile-side can also be obtained.

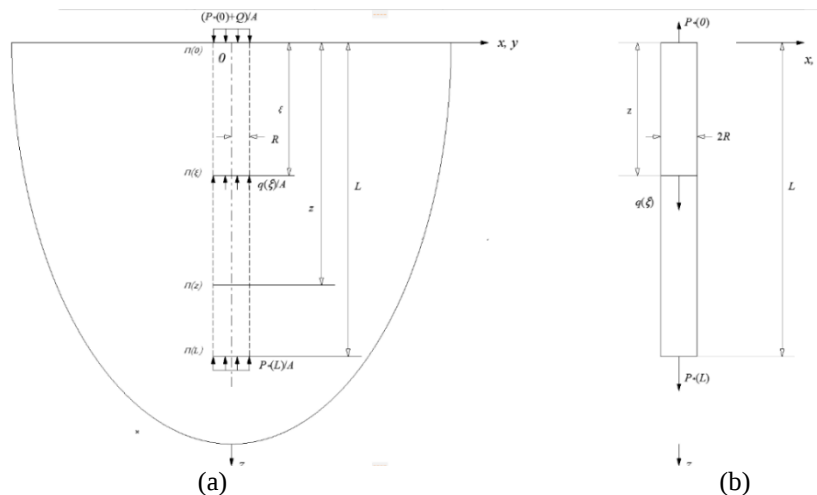


Figure 3. Schematics of (a) the extended saturated half-space and (b) the fictitious pile

Owing to the importance of this class of problems, various authors [13-17] have attempted to reformulate the problem within similar frameworks. For instance, on the basis of an energy principle and the Muki and Sternberg approach [11, 12], Rajapakse and Shah [18] proposed an approximate treatment of this problem through a discretization of the rod-medium interfacial tractions. Pak and Saphores[19] reconsidered the same problem by using Muki and Sternberg concepts, however, the displacement was taken as the basic unknown. To account for the influence of the pore pressure on the pile-soil interaction, Niumpradit&Karadushi[20] used the Biot's theory [21] of poroelasticity and the same fictitious pile method due to Muki&Sternberg to present the first time-dependent theoretical study which deals with the interaction between a pile and a saturated homogeneous half-space. However, only the final and initial state solutions were obtained. In their study, Niumpradit&Karadushi[20] displayed significant numerical results, such as the influence of the pile-soil stiffness ratio (E_p / E) on the initial and the final state force along the pile together with the initial state soil pore pressure on the pile-side along the depth as shown in Fig.4 and 5. These results were greatly used later by several other authors [22, 23] to validate their proposed models. Liang [24] analyzed the pile-to-pile interaction under different situations of consolidation for piles loaded vertically embedded in homogeneous poroelastic saturated half-space employing Muki and Sternberg approach.

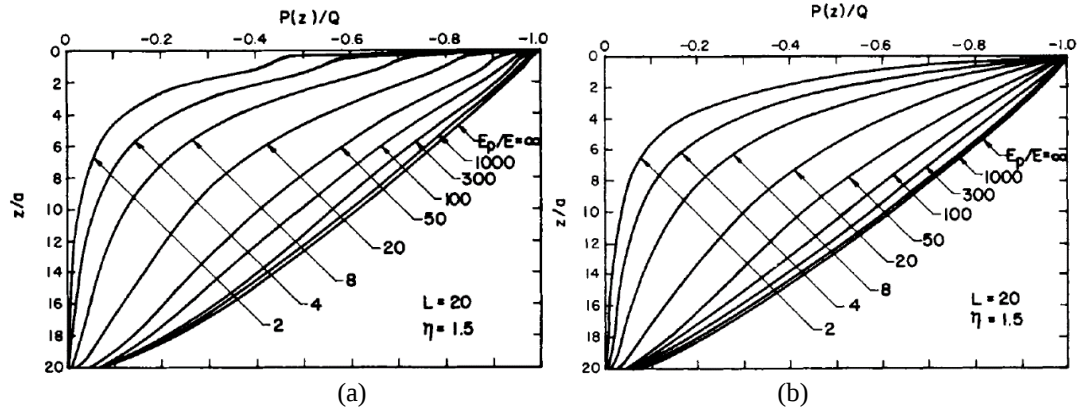


Figure 4. Variation of the pile axial force $P(z)$ along the depth (pile length $L = 20$ m): (a) initial state force profile (b) final state force profile [20]

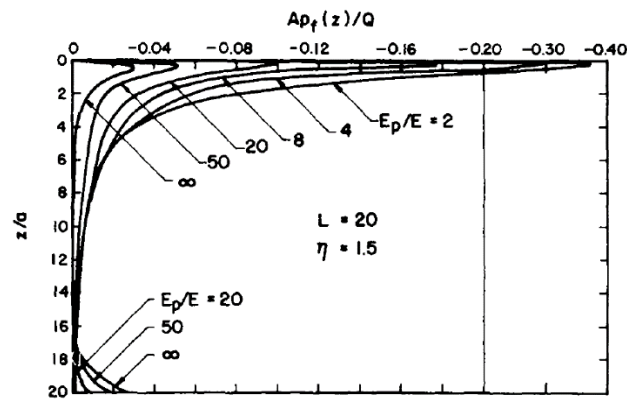


Figure 5. Variation of the initial state soil pore pressure $p_f(z)$ along the depth (pile length $L = 20$ m) [20]

III. PILE IN NON-HOMOGENEOUS SOIL

Normally, piles are rarely embedded in an ideal homogenous single soil layer. Thus, to account for the soil non-homogeneity, Banerjee and Davies [25] employed the analytical solutions proposed by Davies and Banerjee [26] for a two-layered soil and analyzed the behavior axially and laterally loaded single piles embedded in a soil layer whose modulus increases linearly with depth by the following equation:

$$E(z) = E(0) + mz, \quad (2)$$

in which m is the rate of increase of Young's modulus with depth. $E(0)$ and $E(z)$ denote the Young's modulus at the ground level and at arbitrary depth z as displayed in Fig. 6. The soil Poisson's ratio was kept constant in this study.

Randolph and Wroth [27] presented an approximate closed-form solution for the settlement of a pile in a Gibson soil [28], then, Guo and Randolph [29] extended this work with an elastic-plastic solution that considers the case of soil response with stiffness and strength varying as a power law of depth. Poulos[30] proposed an empirical technique, in which a modified soil modulus is used in the Mindlin solutions. Lee et al. [31] and Chin et al. [32] employed the fundamental solutions for an interior point load in a two-layered soil proposed by Chan et al. [33], then studied the response of axially loaded piles and pile groups in layered soils using a simple elastic continuum boundary element method, respectively.

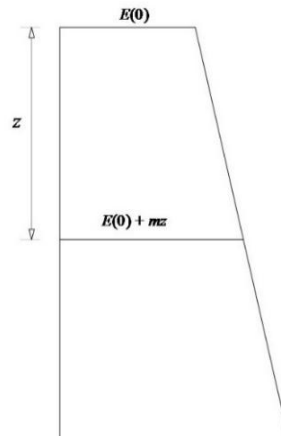


Figure 6. Atypical idealization for the soil non-homogeneity by [25]

Chow [34] used the finite element method to present a numerical procedure that models axially loaded piles and pile groups embedded in non-homogeneous soil with transverse or cross-anisotropic behavior. Normally, the modulus of elasticity of adjacent layers of the soil may differ abruptly. Thus, the above solutions are only appropriate for axially loaded piles in a two-layered soil and they might not give accurate results for piles embedded in multilayered soil type.

IV. PILE IN MULTILAYERED SOIL

To account for the stratification of the realistic soil, a number of authors presented single phase solutions for single piles embedded in multilayered soils [35-39]. However, in practical Engineering conditions, piles do interact with saturated soils, thus time-dependent solutions will make more sense. Based on that, Senjuntichai et al. [22] utilized the Muki & Sternberg pile-soil decomposition technique and studied the time-dependent response of an axially loaded bar in a multilayered poroelastic half-space by the influence function of the multilayered poroelastic half-space and the Laplace transform method. Later, Modeling the pile with the finite element method and using the soil's fundamental solution obtained via the analytical layer-element method [40], Ai et al. [23] also provided a time-dependent solution for a pile embedded in a multilayered saturated soil. In their study, Ai et al. investigated both the influence of the time dependency and the stratification of the soil medium on the response of the pile as depicted in the following Fig. 7. Two different cases, namely, Case 1 corresponding to a single layer half-space and Case 2 a stratified soil consisting of three isotropic layers with the shear moduli ratio equal to $\mu_1 : \mu_2 : \mu_3 = 1 : 2 : 3$ were considered in their research, wherein the subscripts 1, 2 and 3 represent the first layer, second layer, and third layer, respectively. Other parameters for the pile-soil system are as follows: the pile length-diameter ratio $L/d = 20$; the soil Poisson's ratio $\nu = 0.3$; the pile-soil Young's modulus ratio $E_p/E = 300$; the coefficient of permeability $k = 1.2 \times 10^{-6} \text{ ms}^{-1}$; the soil layer's thicknesses for the second case are $h_1/d = h_2/d = 10$, with d denoting the pile diameter. For convenience, the non-dimensional vertical displacement defined as $u = u(z)AE/QR$; the time factor as $\tau = 2k\mu_v t / \gamma_w R^2$ and the non-dimensional pore pressure as $\tilde{p} = Ap(z)/Q$, in which the reference E , μ , and k are taken as the vertical Young's modulus, the shear modulus and the coefficient of vertical permeability in Case 1, respectively; $\gamma_w = 9.8 \times 10^3 \text{ Nm}^{-3}$.

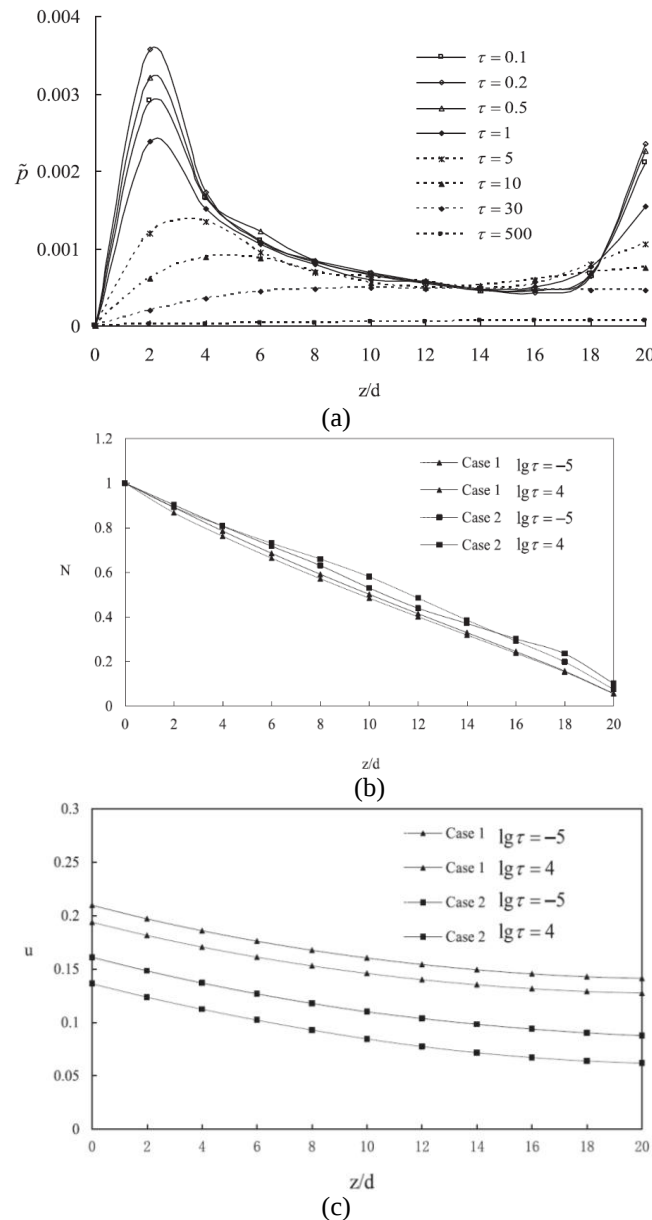


Figure 7. Variation of the non-dimensional (a) Pore pressure, (b) Pile axial force and (c) Vertical displacement along the depth [23]

As shown in Fig. 7 (a), the pore pressure value decreases with time and as the time approaches infinity, the pore pressure value reaches zero, which indicates the final stage of the consolidation when the pore pressure dissipates completely. On the other hand, Fig. 6 (b) and (c) demonstrates that the stratified behavior of the soil medium exerts a significant impact on the behavior of the embedded pile and it should be highly considered.

Fig. 8 displays a vertically loaded cylindrical pile embedded in a N layered soil medium consisting of an overlying single-phase soil layers and underlying saturated soil layers with anisotropic permeability. Accounting on this particular pile-soil interaction problem, Ai et al. [41] analyzed the effect of the overlying single phase soils and the time-dependent response of pile embedded in layered saturated soils with elastic superstrata.

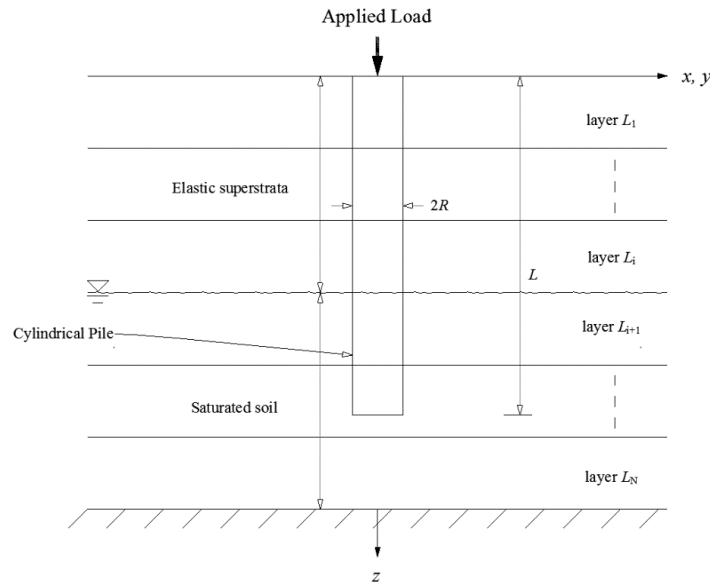


Figure 8. Schematics of a single pile in saturated soils with elastic superstrata

V. PILE IN LAYERED TRANSVERSELY ISOTROPIC SOIL

As mentioned above, apart from stratification property, in engineering practice, the soil might exhibit transverse isotropy property [42, 43], this soil property considers the stiffness differences between the horizontal and vertical direction due to some preferred orientation of the soil particles during the deposition process. Fig. 9 shows a vertically loaded pile embedded in a layered transversely isotropic soil with $N + 1$ layers, in which the vertical shear modulus G_v ; the vertical Young's modulus E_v and the horizontal Young's modulus E_h differ abruptly from layer to layer. Existing relevant researches are all concerned with the interaction between the pile and single phase transversely isotropic soil. For example, by treating the transverse isotropy property of the soil with a combination of the order reduction approach and the integral transform method, Ai & Yi [44] proposed a simple analytical layer-element solution for layered transversely isotropic media subjected to axially symmetric loads at arbitrary depth. Then, presented a boundary element method solution for a single pile embedded in a multilayered transversely isotropic soil. In order to study the influence of the soil transverse isotropy character on the pile response as shown in Fig. 10, Ai & Yi [44] provided an example of a plain cylindrical pile of length L and diameter D embedded in the soil layer of thickness $H = 3L$. Three different stiffness ratios i.e., $n = E_h / E_v = 0.5, 1$ and 3 were selected in this example to display the effect of the cross-anisotropy. Other parameters such as, $G_v = 0.4E_v$, the Poisson's ratios $\mu_{vh} / \mu_h = 0.25$ were also given, while the ratio of the pile Young's modulus to the vertical soil Young's modulus was kept constant throughout the example. In Fig. 10 (a), it can be seen that the shear stress along the pile shaft will increase with depth and with the increase of stiffness ratio. As for the pile top stiffness plotted against the pile slenderness ratio in Fig. 10 (b), the increase of the slenderness ratio will increase the pile top stiffness while keeping other conditions unchanged a bigger n reduces the pile top settlement. These results prove the overprediction of pile foundation settlement by conventional methods for piles installed in isotropic soils with n bigger than a unit, the case of overconsolidated natural deposit of clay. As mentioned earlier, pile foundations are in most cases subjected to both lateral and vertical loads, Ai et al. [45] recently, investigated the behavior of piles in a multilayered transversely isotropic soil subjected to vertical and horizontal loads simultaneously utilizing the fundamental solutions modeled via the Analytical layer element method [46].

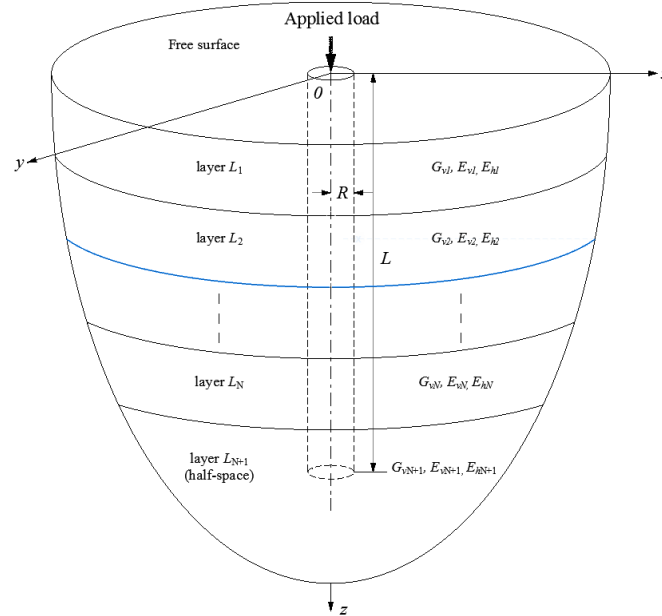


Figure 9. Schematics of an axially loaded pile embedded in a layered transversely isotropic soil half-space

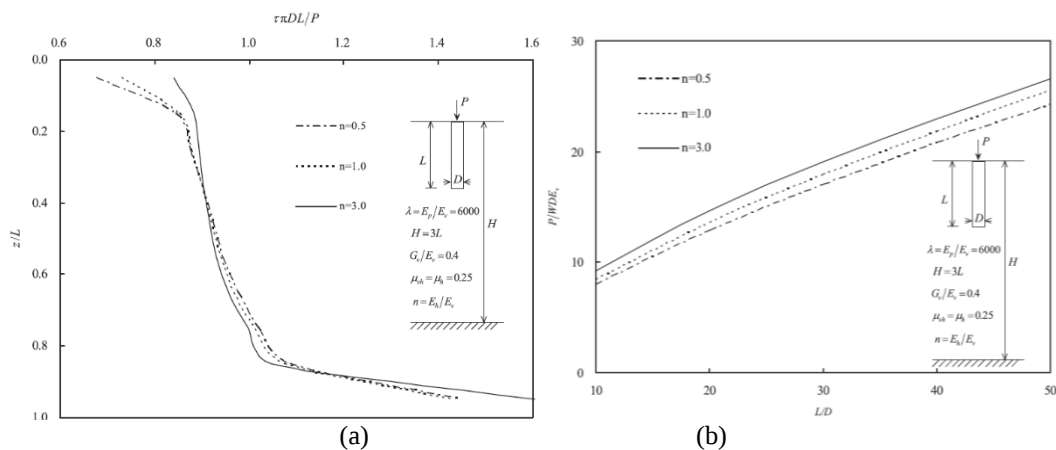


Figure 10. Variation of (a) the pile shear stresses with different n and (b) the normalized pile stiffness with different n [44]

VI. CONCLUSION

A comprehensive literature review on the static interaction between piles and soils is presented in this paper. Different properties of the pile and the soil, the time and the analysis methods are shown to play significant roles in estimating the behavior of superstructures built upon the pile-soil system. The following points can be drawn from this review:

- Pile-soil stiffness ratio mainly governed by the ratio between the pile Young's modulus E_p to that of the soil E greatly contributes toward the load carrying capacity and response of pile.
- The vertical profile of the soil medium also shows a very important impact in predicting the pile-soil system response and it should be paid more consideration. However, as shown in this study, for simplicity in conducting various studies, soil vertical profile may be considered to be either homogeneous or layered.
- Investigators proved the over prediction of pile foundation settlement by conventional methods regarding the soil deposits as isotropic soil medium since in most cases such as over consolidated natural deposit of clay, the stiffness ratio n is bigger than a unit. Hence, considering the soil medium as transversely isotropic soil medium is of a great impact in predicting the behavior of vertically loaded piles. Besides, the distributed shear stress along the pile shaft is also affected by the difference between lateral and vertical stiffness.

- Time factor has a significant influence on the pile's response as well as the consolidation and settlement of the soil. Thus, Time-dependent solutions on static pile-soil interaction problems are relatively valuable to most civil engineering problems.
- In the further studies, Numerical simulations and analysis of more actual engineering problems such as the time-dependent response of piles embedded in transversely isotropic saturated soil should be highly considered.

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