

## Monotone prime numbers

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**Abstract:** After defining, monotone prime numbers will be presented from 23 to 5555555777. How many monotone prime numbers are there in the interval  $(10^{p-1}, 10^p)$  (where  $p$  is a prime number)? On the one hand, it has been counted by computer among the prime numbers with up to 31-digits. On the other hand, the function (1) gives the approximate number of monotone prime numbers in the interval  $(10^{p-1}, 10^p)$ . Near-proof reasoning has emerged from the conformity of Mills' prime numbers with monotone prime numbers. The set of monotone prime numbers is probably infinite.

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### I. INTRODUCTION

The sets of special prime numbers within the set of prime numbers are well-known. For instance, the Erdős-primes (the sum of the digits is prime) [8], Fibonacci-primes ( $F_0=0, F_1=1, F_n=F_{n-1}+F_{n-2}$ ), Gauss-primes (in the form  $4n+3$ ), Leyland-primes (in the form  $x^y+y^x$ , where  $1 \leq x \leq y$ ), Pell-primes ( $P_0=0, P_1=1, P_n=2P_{n-1}+P_{n-2}$ ), Bölcsföldi-Birkás-Ferenczi primes (all digits are prime and the number of digits is prime), etc. Question: Which further sets of special prime numbers are there within the set of prime numbers? We have found a further set of special prime numbers within the set of prime numbers. It is the set of monotone prime numbers.

### II. MONOTONE PRIME NUMBERS [3], [9], [10], [11].

Definition: a positive integer number is a monotone prime number, if

a/ the positive integer number is prime, b/ the digits of number are monotone growing primes,

c/ the number of digits is prime, d/ the sum of digits is prime.

Positive integer numbers that meet the conditions a/ and b/ have been known for a long time: these are prime numbers containing prime digits [1]. The set of prime numbers meeting the conditions a/ and c/ is also well-known: it is the set of prime-long prime numbers [3], [9]. Positive integer numbers meeting all the four conditions (a/, b/, c/, d/) at the same time are monotone prime numbers (Fig.1, Fig.2).

Bölcsföldi-Birkás prime number  $p$  has the following sum form:

$$p = \sum_{j=0}^{k(p)} e_j(p) \cdot 10^j \quad \text{where } e_j(p) \in \{2, 3, 5, 7\} \text{ and } k(p)+1 \text{ is prime and } e_0(p) \in \{3, 7\} \text{ and } \sum_{j=0}^{k(p)} e_j(p) \text{ is prime.}$$

**The monotone prime numbers are as follows (the last digit can only be 3 or 7):**

23,  
 223, 227, 337, 557, 577,  
 33377,  
 2222333, 2233337, 2235557, 3337777, 3355777, 5555777,  
 222222223, 2222222357, 2222333333, 22223335577,  
 22223335777, 22223357777, 22333355777, 22355555777,  
 22555577777, 33333333377, 33555555557, 55555555777, etc.

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$M(p)$  is the factual frequency of monotone prime numbers in the interval  $(10^{p-1}, 10^p)$ .  
 $M(2)=1, M(3)=5, M(5)=1, M(7)=6, M(11)=12, M(13)=19, M(17)=19, M(19)=32, M(23)=26, M(29)=42, M(31)=66$ , etc.  $S(p)$

function gives the number of monotone prime numbers in the interval  $(10^{p-1}, 10^p)$ . We think that

$$S(p) = p^{1.148}, \quad \text{where } p \text{ is prime.} \tag{1}$$

The function (1) is approximately linear.

The factual number of monotone primes and the number of monotone primes calculated according to function (1) are as follows:

Number of digits p	The factual number of monotone primes in the interval $(10^{p-1}, 10^p)$ $M(p)/S(p)$	The number of monotone primes $M(p)$	The number of monotone primes according to function $S(p)=p^{1,148}$
2	1	2,22	0,45
3	5	3,53	1,42
5	1	6,34	0,16
7	6	9,34	0,64
11	12	15,69	0,76
13	19	19,00	1,00
17	19	25,86	0,74
19	32	29,38	1,09
23	26	36,58	0,71
29	42	47,73	0,88
31	66	51,53	1,28

Fig.1

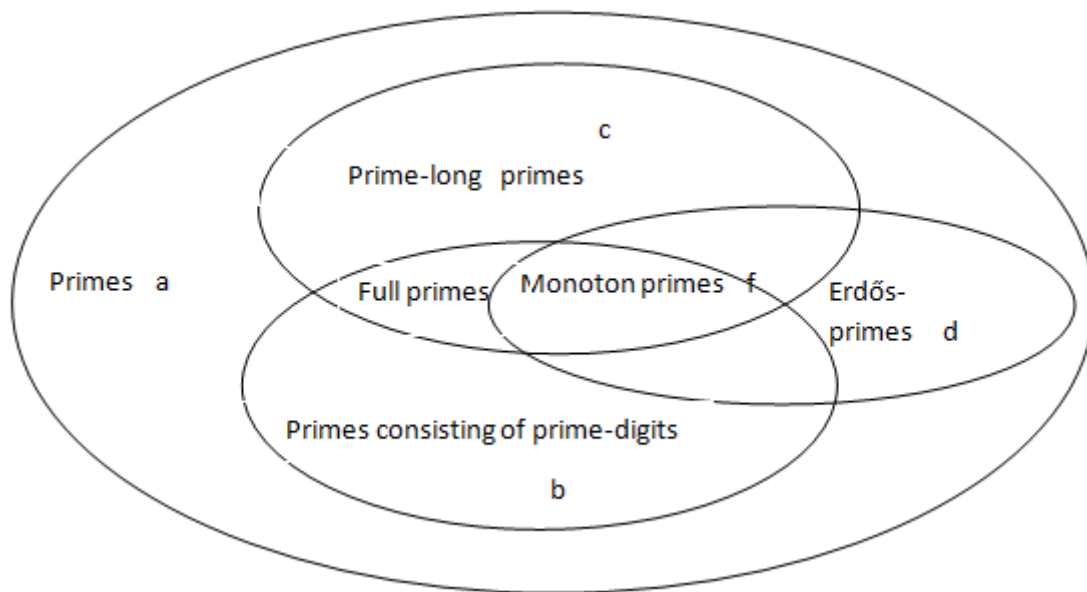
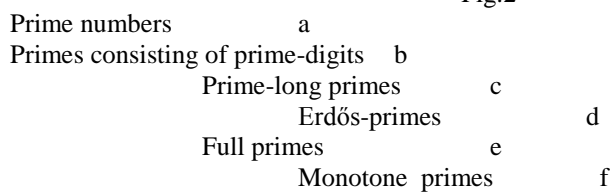


Fig.2



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**Number of the elements of the set of monotone prime numbers [3], [9], [10], [11].**

Let's take the set of Mills' prime numbers!

Definition: The number  $m=[M \text{ ad } 3^n]$  is a prime number, where

$M=1,306377883863080690468614492602$  is the Mills' constant, and  $n=1,2,3,\dots$  is an arbitrary positive integer number. It is already known that the number of the elements of the set of Mills' prime numbers is infinite. The Mills' prime numbers are the following:  $m=2, 11, 1361, 2521008887,\dots$

The connection  $n \rightarrow m$  is the following:  $1 \rightarrow 2, 2 \rightarrow 11, 3 \rightarrow 1361, 4 \rightarrow 2521008887, \dots$ . The Mills' prime number  $m = [M \text{ ad } 3^n]$  corresponds with the interval  $(10^{m-1}, 10^m)$  and vice versa. For instance:  $2 \rightarrow (10, 10^2), 11 \rightarrow (10^{10}, 10^{11}), 1361 \rightarrow (10^{1360}, 10^{1361})$ , etc. and vice versa. The number of the elements of the set of Mills' prime numbers is infinite. As a consequence, the number of the intervals  $(10^{m-1}, 10^m)$  that contain at least one Mills' prime number is infinite. The number of monotone primes in the interval  $(10^{m-1}, 10^m)$  is  $S(m) = m^{1.148}$ . The number of monotone prime numbers is probably infinite:  $\lim_{p \rightarrow \infty} T(p) = \infty$  is probably where  $p$  is prime.

$p \rightarrow \infty$

### III. CONCLUSION

Countless different sets of special prime numbers have been known. We have found the following set of special prime numbers within the set of prime numbers. There may be further sets of special prime numbers that we do not know yet. Finding them will be task of researchers of the future.

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### REFERENCES:

- [1]. <http://oeis.org/A019546>
- [2]. Freud, Robert – Gyarmati, Edit: *Number theory* (in Hungarian), Budapest, 2000
- [3]. <http://ac.inf.elte.hu> → VOLUMES → VOLUME 44 (2015) → VOLLPRIMZAHLENMENGE → FULL TEXT
- [4]. <http://primes.utm.edu/largest.html>
- [5]. <http://mathworld.wolfram.com/SmarandacheSequences.html>
- [6]. Dubner, H.: "Fw:(Prime Numbers) Rekord Primes All Prime digits" Februar 17. 2002
- [7]. <http://listserv.nodak.edu/scripts/wa.exe?A2=ind0202&L=nmbtrthy&P=1697>
- [8]. Harman, Glyn: Counting Primes whose Sum of Digits is Prime. *Journal of Integer Sequences* (2012. , Vol. 15, 12.2.2.)
- [9]. ANNALES Universitatis Scientiarum Budapestiensis de Rolando Eötvös Nominatae Sectio Computatorica, 2015, pp 221-226
- [10]. International Journal of Mathematics and Statistics Invention, February 2017: <http://www.ijmsi.org/Papers/Volume.5.Issue.2/B05020407.pdf>
- [11]. International Organisation of Scientific Research, April 2017  
Bölcsföldi Birkás prime numbers: [http://www.iosrjournals.org/iosr-jm/pages/v13\(2\)Version-4.html](http://www.iosrjournals.org/iosr-jm/pages/v13(2)Version-4.html) or <http://dx.doi.org> or [www.doi.org](http://www.doi.org) Article DOI is: 10.9790/5728-1302043841

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