Construction of a Structure from 4-Regular Planar Graph and to investigate its implications on Odd Region and Even Region

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Abstract: In this present work we have proposed the construction of a structure of the 4-regular planar graphs for G(2m+2,4m+4) where m ≥ 2 . Based on our proposed structure we have stated two theorems on odd regions and total regions of 4-regular planar graphs. The experimental results and proof of the stated theorems have also been provided.

Keywords: 4-Regular planar graphs, sub-graphs, structure, odd regions, total regions.

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I. INTRODUCTION:

Various researchers have been working on Regular planar graphs from different perspective .Therese Bied[1] have discussed drawings of planar graphs such that all edge lengths are integers. Jin Jing et al [2] have proposed that a graph G is k- choosable if it has an L-coloring whenever L is a list assignment such that $|L(y)| \ge k$ for all $y \in V(G)$. They have provided two sufficient conditions for 3-choosability of plane graphs. Dutta Anupam et al. [3] have discussed regular planar sub graphs of complete graphs and their various application. The minimum vertex cover of a class of regular planar sub-graphs H(2m+2,3m+3), K(2m+2,4m+4)for m ≥ 2 and J(2m+2,5m+5) for m ≥ 5 obtained from the complete graph K2m+2 had already been discussed by Kalita et.al [4]. They have developed an algorithm to find the minimum vertex cover of these types of regular planar sub-graphs. They have also provided the application of the same on minimum vertex cover to reduce the power consumption of sensor network. Kalita et al[5] have discussed the construction of the structures of three regular planner graphs from the graph G(2m+2,3m+3) for m ≥ 2 . They have studied its numbers of even and odd regions. They have also developed an algorithm and provided an application of it in region based. Ackerman Eyal et al [6] have proved that if an n-vertex graph G can be drawn in the plane such that each pair of crossing edges is independent and there is a crossing-free edge that connects their endpoints, then G has O(n) edges. Graphs that admit such drawings are related to quasi-planar graphs and to maximal 1-planar and fan-planar graphs. Yu Yong et al [7] have proved that planar graph G with maximum degree $\Delta \ge 12$ that the (2,1)-total labeling number $\lambda_2(G)$ is at most $\Delta + 2$. Lu Hongliang et al [8] have obtained a sharp result that for any even n \geq 34, every { D_n , $D_n + 1$ } regular graph of order n contains (n/4) disjoint perfect matchings, where Dn = 2[n/4]-1. As a consequence, for any integer $D \ge D_n$, every { D, D+1} regular graph of order n contains(D-[n/4]+1) disjoint perfect matchings. Couch PJ et al [9] have discussed The construction of a homing tour is known to be NP-complete. They have focused on split Euler tours (SETs) in 3-connected, 4-regular, planar graphs (tfps). The various results rely heavily on the structure of such graphs as determined by the Euler formula and on the construction of tfps from the octahedron. They have also construct a 2-connected 4-regular planar graph that does not have a SET.

In most of the literature, it is seen that almost all the works have been done on planar graph and regular planar graphs. Very few works are seen to be done on 3-regular planar graph and 4-regular planar graphs. But in the literature hardly any work is seen on odd region and even region of 4-regular planar graph. But the regions are used in different map coloring, image processing and biological diversity. Graph theory is used in biology and conservation efforts where a vertex represents regions where certain species exist and the edges represent migration path or movement between the regions. This information is important when looking at breeding patterns or tracking the spread of disease, parasites and to study the impact of migration that affects other species. Therefore in the present work we propose two new theorems on the odd region and even region of 4 – regular planar graphs.

Section 1 includes the introduction which contains the works of other researcher. Section 2 includes the definition. Section 3 contains two theorems which are stated and proved. Section 4 includes discussion of odd region which covered by only three edges and Section 5 includes the conclusion.

II. DEFINITION

Region: An area covered by a number of vertices with edges is known as a region. A region is two types –inner region and outer region. The inner region are two types – odd region and even region.

Odd Region: If a region is covered by odd number of vertices and odd number of edges than it is called odd region .Example



Figure 1: (bounded by odd number of edges)

Even Region: If a region covered by even number vertices and even number edges, called even region. Example-



Figure 2: (bounded by Even number of edges)

Construction of Four Regular Planar Graphs: There are so many structures of 4 regular planar graphs. In this paper we focus only one type of structure for discussing the odd region and even region of the graph.

III. Our Work:

For the graph G(2m+2,4m+4) where m ≥ 2 , we construct a structure the 4-regular planar graphs in the following ways-

A structure of 4 Regular planar graph: Let G be a graph having 2m + 2 vertices and 4m+4 edges for $m \ge 2$. For m=2, G contains six vertices $\{v_1, v_2, v_3, v_4, v_5, v_6\}$ and 12 edges. Let us join these six vertices by twelve edges as follows.

$$\begin{array}{lll} \partial(v_i) &= \begin{cases} v_{i+1} & for \ 1 \le i \le 5 \\ v_1 & for \ i = 6 \end{cases} \\ \partial(v_k) &= \begin{cases} v_1 & for \ k = 5 \\ v_2 & for \ k = 4,5 \end{cases} \\ \partial(v_p) &= \begin{cases} v_1 & for \ p = 3 \\ v_6 & for \ p = 4 \\ \end{pmatrix} \\ \partial(v_{j+2}) &= v_{7-j} & for \ 1 \le j \le 1 \end{array} \end{array}$$

Then the edge set $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_1, v_1v_5, v_2v_4, v_1v_3, v_4v_6, v_3v_6\}$ and we have the graph as shown in Figure-3, which is planar and regular of degree four.





For m=3, vertex set is $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ i.e eight vertices and sixteen edges. Let us join eight vertices with twelve edges by the following rule.

$$\partial(v_i) = \begin{cases} v_{i+1} & \text{for } 1 \le i \le 7 \\ v_1 & \text{for } i = 8 \end{cases}$$

$$\partial(v_k) = \begin{cases} v_1 & \text{for } k = 6 \\ v_2 & \text{for } k = 5,6 \end{cases}$$

$$\partial(v_p) = \begin{cases} v_1 & \text{for } p = 3 \\ v_4 & \text{for } p = 8 \\ v_7 & \text{for } p = 5 \end{cases}$$

$$\partial(v_{j+2}) = v_{9-j} & \text{for } 1 \le j \le 2$$

Then the edge set $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_7, v_7v_8, v_1v_8, v_2v_7, v_3v_6, v_4v_5, v_1v_7, v_2v_6, v_3v_5, v_1v_4, v_5v_8, v_4v_8\}$ and we have the graph as shown in Figure-4, which is planar and regular of degree four.



Figure-4: Four regular planar graph for m=3

For m=4, vertex set is $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}$ i.e ten vertices and sixteen edges. Let us join eight vertices with twelve edges by the following rule.

$$\partial(v_i) = \begin{cases} v_{i+1} & \text{for } 1 \le i \le 10 \\ v_1 & \text{for } i = 10 \end{cases}$$
$$\partial(v_k) = \begin{cases} v_1 & \text{for } k = 7 \\ v_2 & \text{for } k = 6,7 \end{cases}$$

$$\partial(v_p) = \begin{cases} v_1 & for \ p = 3 \\ v_4 & for \ p = 10 \\ v_5 & for \ p = 9 \\ v_5 & for \ p = 8 \end{cases}$$

 $\partial(v_{j+2}) = v_{11-j} \quad for \ 1 \le j \le 3$

Then the edge set $\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_7, v_7v_8, v_8v_9, v_9v_{10} \ v_1v_{10}, v_2v_9, v_3v_8, v_4v_7, v_5v_6, \ v_1v_9, v_2v_8, v_3v_7, v_4v_6, v_1v_5, v_6v_{10}, v_5v_{10}, v_5v_{10}\}$ and we have the graph as shown in Figure-5, which is planar and regular of degree four.



Figure-5 Four regular planar graph for m=4

Similarly we can construct regular planar graph of degree three having odd and even regions for $m=6,7,8,9-\cdots$ and hence we can generalize the above cases by the following rule for constructing the graph.

For the graph G having 2m+2 number of vertices and 3(m+1) edges for $m \ge 2$, we define.

$$\infty(v_i) = \begin{cases} v_{i+1} & \text{for } 1 \le i \le 2m+1 \\ v_1 & \text{for } i = 2m+2 \end{cases}$$

$$\infty(v_k) = \begin{cases} v_1 & \text{for } k = m+3 \\ v_2 & \text{for } k = m+3-1 \\ v_2 & \text{for } k = m+3 \end{cases}$$

$$(v_{i+2}) = v_{2m+3,i} & \text{for } 1 \le i \le (m-1)$$

$$(v_{j+2}) = v_{2m+3-j} \quad for \ 1 \le j \le (m-1)$$
$$(v_{j+2}) = \begin{cases} v_1 & j=1 \\ v_{2m+4-j} & for \ 2 \le j \le m \end{cases}$$

The experimental results of structure of four regular planar graph G(2m+2,4m+4) for different values of m ≥ 2 are shown in Table-1.

Value of	Graph(V,E)	No of	No of	Total	No of edges	No of edges even
m		Odd	Even	Region	odd region	region covered
		region	region		covered	
2	6,12	7	NIL	7	3	NIL
3	8,16	7	2	9	3	4
4	10,20	11	0	11	3,5	NIL
5	12,24	11	2	13	3	6
6	14,28	15	0	15	3,7	NIL
7	16,32	15	2	17	3	8
8	18,36	19	0	19	3,9	NIL
9	20,40	19	2	21	3	10

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10	22,44	23	0	23	3,11	NIL
11	24,48	23	2	25	3	12
12	26,52	27	0	27	3,13	NIL
13	28,56	27	2	29	3	14

Table -1: (Odd and Even region of three regular planar graph)

From the following results we find a theorem.

Theorem 1: The odd region of 4-Regular planar graph G(2m+2,4m+4) is 2m+3 when m=2k for $k\geq 1$ and 2m+1when m=2k+1 for k \geq 1.

Proof of the theorem by using mathematical induction:

Proof: It has been proved [3] that the sub-graph H(2m+2,4m+4) for of the complete graph K2m+2 is planar and regular of the degree 4.We now proceed to prove that the regular planar graph H(2m+2,4m+4) has odd region 2m+3 for m =2k and 2m+1 for m=2k+1 for k ≥ 1 . It is found that the result is true when m=2 and 3. That is, when m=2 for k=1, the graph H(6,12) has odd region 7 and the graph H(8,16) has also odd region 7 when m=3 for k=1 which is shown from figure-6 and figure-7.



Fig-6 For m=2 odd region is 7

Fig -7 For m=3 odd region is 7

(For both Fig 6 and 7 OR means Odd Region) The 1st part of the theorem claims for the even value of m is that if m=2k and k>=1 then Odd Region OR(m) =2m+3 and the difference between the odd regions is always 4 for two consecutive values of k. Now, OR(m) = 2m + 3

Therefore, when k = 1, 2, we have m = 2k, So $OR_{k=1} (2 \times 1 = 2) = OR_{k=1} (2) = 2 \times 2 + 3 = 7$ $(2 \times 2 - 4) - OP$ $(4) = 2 \times 4 + 3 = 11$

$$OR_{k=2} (2 \times 2 - 4) = OR_{k=2} (4) - 2 \times 4 + 3 = 11$$

 $OR_{k=3} (2 \times 3 = 6) = OR_{k=3} (6) = 2 \times 6 + 3 = 15$

Proceeding in the same manner, we get,

$$OR_{k-1} (2(k-1)) = 2(2k-2)+3=4k-1$$

$$OR_k(2k) = 2 \times 2k + 3 = 4k + 3$$

From above results, the differences of the successive odd-regions is found as,

$$OR_{k=2} - OR_{k=1} = 11 - 7 = 4$$

 $OR_{k=3} - OR_{k=2} = 15 - 11 = 4$

Proceeding in the same manner, we get,

$$OR_{k=k} - OR_{k=k-1} = (4k+3) - (4k-1)$$

= 4k+3 - 4k+1
= 4

Hence the theorem is true for all $k \ge 1$ but had to prove for k+1 getting the generalization, Now

$$OR_{k=k+1}(2 \times (k+1) = 2(2k+2)+3$$

= 4k+4+3
= 4k+7

Now

$$OR_{k=k+1} - OR_{k=k} = (4k+7) - (4k+3)$$

= 4k+7-4k-3
= 4

Since the theorem in true for k and and for k+1 and it evidently proved true for k=1,2,3. Hence the 1st part of theorem is true for any value of $k \ge 1$.

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The 2^{nd} part of the theorem claims for the even value of m is that if m=2k+1 and k>=1 then Odd Region OR(m) = 2m+1 and the difference between the odd regions is always 4 for two consecutive values of k.

Now, OR(m) = 2m + 1Therefore, when k = 1,2, we have m = 2k + 1, So $OR_{k=1} (2 \times 1 + 1 = 3) = OR_{k=1} (3) = 2 \times 3 + 1 = 7$ $OR_{k=2} (2 \times 2 + 1 = 5) = OR_{k=2} (5) = 2 \times 5 + 1 = 11$ $OR_{k=3} (2 \times 3 + 1 = 7) = OR_{k=3} (7) = 2 \times 7 + 1 = 15$ Proceeding in the same manner, we get, $OR_{k=k-1} (2(k-1)+1) = OR_{k=k-1} (2k-1)=2(2k-1)+3$ =4k-2+3=4k+1 $OR_k (2k+1) = OR_{k=k} (2k+1) = 2(2k+1) + 3 = 4k+2 + 3 = 4k+5$ From above results, the differences of the successive odd-regions is found as, $OR_{k=2} \ -OR_{k=1} = 11-7 = 4$ $OR_{k=3} - OR_{k=2} = 15 - 11 = 4$ Proceeding in the same manner, we get, $OR_{k=k} - OR_{k=k-1} = (4k+5) - (4k+1)$ = 4k+5 - 4k-1= 4 Hence the theorem is true for all $k \ge 1$ but had to prove for k+1 getting the generalization,

Now

$$OR_{k=k+1}(2 \times (k+1)+1) = OR_{k=k+1}(2k+3) = 2(2k+3)+3$$

= 4k+6+3
= 4k+9

Now

$$OR_{k=k+1} - OR_{k=k} = (4k+9) - (4k+5)$$

= 4k+9 - 4k-5
= 4

Since the theorem in true for k and and for k+1 and it evidently proved true for k=1,2,3. Hence the 2^{nd} part of theorem is true for any value of k≥1.

Theorem 2: The Total region of 4-Regular planar graph G(2m+2,4m+4) is 2m+3 when m=k+1 for $k\geq 1$. *Proof*: Proof of the theorem by using mathematical induction:

Proof: It has been proved [3] that the sub-graph H(2m+2,4m+4) for of the complete graph K2m+2 is planar and regular of the degree 4.We now proceed to prove that the regular planar graph H(2m+2,4m+4) has Total region 2m+3 for m =k+1 for k \geq 1. It is found that the result is true when m=2 and 3.That is, when m=2 for k=1, the graph H(6,12) has Total region 7 and the graph H(8,16) has a Total region 9 when m=3 for k=1.

The theorem claims for the value of m is that if m=k+1 and k>=1 then total region TR(m) = 2m+3 and the difference between the regions is always 2 for two consecutive values of k.

Now,
$$TR(m) = 2m + 3$$

Therefore, when k = 1, 2, we have m=k+1, So

$$\begin{aligned} TR_{k=1} & (1+1=2) = TR_{k=1} & (2) = 2 \times 2 + 3 = 7 \\ TR_{k=2} & (2+1=3) = TR_{k=2} & (3) = 2 \times 3 + 3 = 9 \\ TR_{k=3} & (3+1=4) = TR_{k=3} & (4) = 2 \times 4 + 3 = 1 \end{aligned}$$

Proceeding in the same manner, we get,

$$TR_{k-1}((k-1)+1) = 2k+3$$

$$TR_k(k+1) = 2(k+1)+3=2k+5$$

From above results, the differences of the successive Total regions is found as,

$$TR_{k=2} - TR_{k=1} = 9 - 7 = 2$$

 $TR_{k=3} - TR_{k=2} = 11 - 9 = 2$

Proceeding in the same manner, we get,

$$TR_{k=k} - TR_{k=k-1} = (2k+5) - (2k+3)$$

= 2k+5 - 2k-3 = 2Hence the theorem is true for all k≥1but had to prove for k+1 getting the generalization,

Now

$$TR_{k=k+1}((k+1)+1)=2((k+1)+1)+3$$

= 2k+4+3
= 2k+7

Now

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$$TR_{k=k+1} - TR_{k=k} = (2k+7) - (2k+5)$$

= 2k+7-2k-5
= 2

Since the theorem in true for k and for k+1 and it evidently proved and true for k=1,2,3. Hence the theorem is true for any value of $k \ge 1$.

IV. **Discussion:** Maximum Regions covered by three edges: The experimental results of structure of four regular planar graph G(2m+2,4m+4) for different values of m>2 are shown in Table-2.

Value	Graph(V,E)	No of	No of	Total	No of	No of
of m		Odd	Even	Region	Regions	Regions(odd
		regions	regions		covered by	or even)
					three edges	covered by
						other than 3
						edges
2	6,12	7	0	7	7	0
3	8,16	7	2	9	7	2
4	10,20	11	0	11	10	1
5	12,24	11	2	13	11	2
6	14,28	15	0	15	14	1
7	16,32	15	2	17	15	2
8	18,36	19	0	19	18	1
9	20,40	19	2	21	19	2
10	22,44	23	0	23	22	1
11	24,48	23	2	25	23	2
12	26,52	27	0	27	26	1
13	28,56	27	2	29	27	2

Table -2: Maximum edges covered by three edges.

From the above table it is seen that the maximum region of the structure of the graph are covered by three edges. For m=2, sub-graph covered by 3 edges =7 and other region covered by more then 3 edges =0. Again for m=3 sub-graph covered by 3 edges=7 and 7 and other region covered by more then 3 edges =2. Again for m=4 sub-graph covered by 3 edges =10 and other regions are covered by more then 3 edges =1.

From the above results, it is clear that the number of regions covered by the three edges change with the change of the values of 'm', let it be f(x), and it is seen that number of other regions covered by other edges remain oscillated between 1 and 2 irrespective the values of m. Hence the total regions obtained by the edges will be f(x) + k, where k is either 1 or 2. But if the graph is very large and many edges are involved at that situation, the regions covered by the three edges will be distinctly high but k remains at 1 or 2 which can be neglected as it becomes so small as compared to the large value of m and f(x). so if we ignored the region which are covered by more than three edges then we can state that the structure of the graph G(2m+2,4m+4) region are covered by 3 edges.

IV. CONCLUSION:

The above discussion is useful for the researchers of different fields like: mathematical science, region-based segmentation, biological diversity, region-based map coloring etc. , as the theorems discussed here justified our claims that the odd region of 4-Regular planar graph G(2m+2,4m+4) is 2m+3 when m=2k for $k\ge 1$ and 2m+1 when m=2k+1 for $k\ge 1$ and also the total region of 4-Regular planar graph G(2m+2,4m+4) is 2m+3 when m=k+1 for $k\ge 1$.

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