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Bending Analysis of Rectangular Thick Plate Using Polynomial Shear Deformation Theory

[1]Ibearugbulem, Owus M., [2]Ezeh, John C., [3]Ettu, Lawrence O., [4]Gwarah, Ledum S.
[1,2,3,4]Civil Engineering Department, Federal University of Technology, Owerri, Nigeria
Corresponding Author: 1Ibearugbulem

Abstract: This paper presents bending analysis of all clamped (CCCC) rectangular thick plate using polynomial shape function in shear deformation theory. The theory presented herein is based on Ritz energy method and the displacement function based on polynomial function. The theory derived transverse shear stress from constitutive relation that satisfied zero shear stress condition on the top and bottom surfaces of the plate, hence like other higher order theories no shear correction factor is required. The total potential energy equation of a thick plate was formulated from the principle elasticity. The governing equations for determination of displacement coefficients were derived by subjecting the total potential energy equation to direct variation. A rectangular thick plate with all edges clamped was considered for numerical studies. The results obtained herein for displacements and stresses were compares with those from previous works to show the sufficiency of this theory. It was observed that the present results agreed with those of previous works. Also the obtained non-dimensional values of vertical shear stress ($\overline{\tau_{xz}}$) were used to delineate the boundary between thick and thin plate based on span to depth ratio. The values of non-dimensional vertical shear stress ($\overline{\tau_{xz}}$) between span to depth ratios (α) of 60 and 100 were equal to values obtained from classical plate theory (CPT), therefore they can be idealized to be thin plate. The values of the vertical shear stress ($\overline{\tau_{xz}}$) of plate, whose span to depth ratio (α) falls between 20 and 50 varied minimally and differed from those of classical plate theory, so the plate can be taken to be moderately thick. Furthermore, values of the vertical shear stress ($\overline{\tau_{xz}}$) of plate whose span to depth ratio (α) falls between 4 to 15 varied significantly with span to depth ratio. Therefore, the plate can be taken to be thick..

Keywords: shear deformation, shear correction factor, vertical shear stress, deflection, displacement, potential energy

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I. INTRODUCTION

The wide spread application of shear flexible materials has stimulated interest in predicting accurately the structural behavior of thick plates. Thick beams and plates, either isotropic or anisotropic, basically form two and three dimensional problems of elasticity theory. The main objective of researchers has been the reduction of these problems to the corresponding one and two dimensional approximate problems for their analysis. The shear deformation effects are more pronounced in thick plates subjected to transverse loads than in the thin plates under similar loading (Sayyad & Ghugal, 2012b; Touratier, 1991). The analyses of thick plates by previous authors have been predominantly characterized by the use of trigonometric and exponential displacement function. It has been witnessed that most scholars have obtained the closed form solutions and others have obtained approximate solution by use of energy method. However, one thing is common in them all - the use of trigonometric displacement functions to approximate the deformed shapes of the plates. (Chikalthankar et al., 2013; Sayyad, 2011; Akavci, 2007; Sayyad and Ghugal, 2012; Sadrnejad et al., 2009; Daouadji et al., 2013;Hashemi and Arsanjani, 2005; Reddy, 2014; Shimpi and Patel, 2006; Murthy, 1981; Daouadji, Tounsi, Hadji, Henni and El Abbes, 2012; Zhen-qiang, Xiu:xi and Mao-guang, 1994). Others have applied the polynomial displacement functions in numerical methods like finite element method and differential quadrature element methods (Matikainen, Schwab and Mikkola, 2009; Goswami and Becker, 2013, Liu, 2001). In the course of development of refined plate theory, the assumption that the shear deformation line is not varying linear with depth of the plate was introduced. This according to many scholars helps to ensure that the vertical shear stress across the plate section does not remain constant, but varies parabolically with zero values at both the top and bottom surfaces (Kruszewski, 1949; Ambartsumian, 1958 Krishna, 1984; Touratier, 1991; Karama and Mistou, 2003; Sayyad, 2011). They came up with different shear deformation line functions, here-in-after called F(z). However, their F(z) are not strictly based on the vertical shear stress mathematical formulation. As discussed earlier, scholars had been assuming displacement functions in thick plate bending analysis. The correctness of the analysis through variational or energy approaches depend more on the exactness of the assume displacement function. This seems to be the major factor discouring engineers in petronizing thick plate analysis, and instead resort to idealizing thick plate as thin plates. In this paper, the authors tried to integrate the thick plate governing equation to obtain general polynomial displacement function, which was easy to satisfy the boundary condintion for various plates. They also, tried to propagate an easy and straightforwad approach to bending analysis of thick rectangular plates..

II. TEORITICAL FORMULATION

The displacement field include two in-plane displacements (u and v) and out-of-plane displacement (w). While the inplane displacements are differentiable with respect to the three cardinal coordinates, the out-of-plane displacement is only differentiable with respect to x and y coordinates. The in-plane domains are in the following range: $0 \le x \le a; 0 \le y \le b$. Where a and b are the in-plane lengths of the plate as shown on Figure 1. The out-of-plane domain is within the following range $-t/2 \le z \le t/2$.

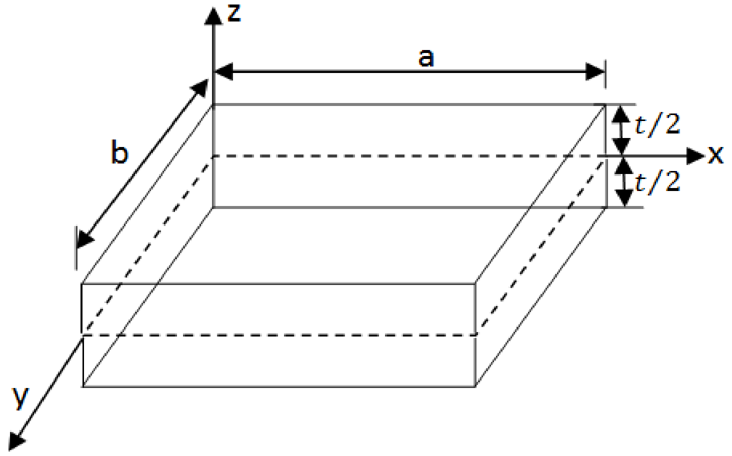

Figure 1: Three dimensions and cordinates of rectangular plate

The in-plane displacements comprized of classical part and shear deformation part. They are:

$$u = u_c + u_s \qquad (1)$$

$$v = v_c + v_s \qquad (2)$$

Where the classical parts are:

$$u_c = -z\theta_{cx} = -z\frac{dw}{dx} \qquad (3)$$

$$v_c = -z\theta_{cy} = -z\frac{dw}{dy} \qquad (4)$$

Ibearugbulem et al. (2016) gave the shear deformation parts as:

$$u_s = F(z)\theta_{sx} \qquad (5)$$

$$v_s = F(z)\theta_{sy} \qquad (6)$$

Where F(z) is the shear deformation function profile across the thickness of the plate. They gave it in dimensional form as:

$$F(z) = \frac{3z}{2}\left(1 - \frac{4}{3}\left[\frac{z}{t}\right]^2\right) \qquad (7a)$$

This is written in non dimensional form as:

$$F(S) = \frac{3St}{2}\left(1 - \frac{4}{3}S^2\right) \qquad (7b)$$

Where S = z/t

The total potential energy functional for thick plate in pure bending is given as:

$$\Pi = \int_x \int_y \left[\int_{-\frac{t}{2}}^{\frac{t}{2}} \frac{1}{2}\left(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz}\right) dz - qw\right] dxdy \qquad (8)$$

The five engineering strain components are defined as:

$$\varepsilon_x = \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2} + F(z)\frac{\partial \theta_{sx}}{\partial x} \qquad (9)$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = -z\frac{\partial^2 w}{\partial y^2} + F(z)\frac{\partial \theta_{sy}}{\partial y} \quad (10)$$

$$\gamma_{xy} = \frac{du}{dy} + \frac{dv}{dx}. \; That\ is:$$

$$\gamma_{xy} = -2\frac{\partial^2 w}{\partial x \partial y} + F(z)\frac{\partial \theta_{sx}}{\partial y} + F(z)\frac{\partial \theta_{sy}}{\partial x} \quad (11)$$

$$\gamma_{xz} = \frac{du}{dz} + \frac{dw}{dx}. \; That\ is:$$

$$\gamma_{xz} = \frac{\partial F(z)}{\partial z}.\theta_{sx} \quad (12)$$

$$\gamma_{yz} = \frac{\partial F(z)}{\partial z}.\theta_{sy} \quad (13)$$

The corresponding five stress components are:

$$\sigma_x = \frac{E}{1-\mu^2}\left[\varepsilon_x + \mu\varepsilon_y\right] \quad (14)$$

$$\sigma_y = \frac{E}{1-\mu^2}\left[\mu\varepsilon_x + \varepsilon_y\right] \quad (15)$$

$$\tau_{xy} = \frac{E(1-\mu)}{1-\mu^2}\gamma_{xy} \quad (16)$$

$$\tau_{xz} = \frac{E(1-\mu)}{1-\mu^2}\gamma_{xz} \quad (17)$$

$$\tau_{yz} = \frac{E(1-\mu)}{1-\mu^2}\gamma_{yz} \quad (18)$$

Substituting equations (14) to (18) into equation (8) gave:

$$\Pi = \int_x \int_y \left\{\int_{-\frac{t}{2}}^{\frac{t}{2}} \frac{E}{2(1-\mu^2)}\left(\varepsilon_{xx}^2 + 2\mu\varepsilon_{xx}.\varepsilon_{yy} + \varepsilon_{yy}^2 + (1-\mu)\left[\frac{\gamma_{xy}^2}{2} + \frac{\gamma_{xz}^2}{2} + \frac{\gamma_{yz}^2}{2}\right]\right) dz - qw\right\} dxdy \quad (19)$$

Substituting equations (9) to (13) into equation (19) gave:

$$\Pi = \frac{D}{2}\iint \{\left[\left(\frac{d^2w}{dx^2}\right)^2 + 2\left(\frac{d^2w}{dxdy}\right)^2 + \left(\frac{d^2w}{dy^2}\right)^2\right] - 2g_2\left[\frac{d^2w}{dx^2}\frac{d\theta_{sx}}{dx} + \frac{d^2w}{dx^2}.\frac{d\theta_{sy}}{dy} + \frac{d^2w}{dy^2}.\frac{d\theta_{sx}}{dx} + \frac{d^2w}{dy^2}\frac{d\theta_{sy}}{dy}\right]$$

$$+g_3\left[\left(\frac{d\theta_{sx}}{dx}\right)^2 + (1+\mu)\frac{d\theta_{sx}}{dx}\frac{d\theta_{sy}}{dy} + \left(\frac{d\theta_{sy}}{dy}\right)^2 + \frac{(1-\mu)}{2}\left(\left[\frac{d\theta_{sx}}{dy}\right]^2 + \left[\frac{d\theta_{sy}}{dx}\right]^2\right)\right] + \frac{(1-\mu)}{2}g_4\left[{\theta_{sx}}^2 + {\theta_{sy}}^2\right]$$

$$-\frac{2qw}{D}\} \, dx \, dy \quad (20)$$

Where:

$$D = \frac{Et^3}{12(1-\mu^2)}; \; g_2 = \frac{12}{t^3}\int_{-\frac{t}{2}}^{\frac{t}{2}} zF(z)\, dz = 1.2$$

$$g_3 = \frac{12}{t^3}\int_{-\frac{t}{2}}^{\frac{t}{2}} [F(z)]^2 \, dz = \frac{51}{35}; \; g_4 = \propto^2 g_5$$

$$g_5 = \frac{12}{t^3}\int_{-\frac{t}{2}}^{\frac{t}{2}} \left[\frac{dF(z)}{dz}\right]^2 dz = 14.4$$

Writing eequation (20) in terms of the non-dimensional coordinates gave:

$$\Pi = \frac{abD}{2}\int_0^1\int_0^1 \{\frac{1}{a^4}\left[\left(\frac{d^2w}{dR^2}\right)^2 + \frac{2}{\beta^2}\left(\frac{d^2w}{dRdQ}\right)^2 + \left(\frac{d^2w}{dQ^2}\right)^2\right]$$

$$-\frac{2g_2}{a^3}\left[\frac{d^2w}{dR^2}\frac{d\theta_{sx}}{dR} + \frac{1}{\beta}\frac{d^2w}{dR^2}.\frac{d\theta_{sy}}{dQ} + \frac{1}{\beta^2}\frac{d^2w}{dQ^2}.\frac{d\theta_{sx}}{dR} + \frac{1}{\beta^3}\frac{d^2w}{dQ^2}\frac{d\theta_{sy}}{dQ}\right]$$

$$+\frac{g_3}{a^2}\left[\left(\frac{d\theta_{sx}}{dR}\right)^2 + \frac{(1+\mu)}{\beta}\frac{d\theta_{sx}}{dR}\frac{d\theta_{sy}}{dQ} + \frac{1}{\beta^2}\left(\frac{d\theta_{sy}}{dQ}\right)^2 + \frac{(1-\mu)}{2\beta^2}\left(\left[\frac{d\theta_{sx}}{dQ}\right]^2 + \left[\frac{d\theta_{sy}}{dR}\right]^2\right)\right]$$

$$+\frac{(1-\mu)}{2}g_4\left[{\theta_{sx}}^2 + {\theta_{sy}}^2\right] - \frac{2qw}{D}\} \, dR \, dQ \quad (21)$$

Minimizing equation (20) with respect to deflection and the shear deformation rotations (w, θ_{sx} and θ_{sy}) gave the governing eqation and two compatibility equations respectively as:

$$\frac{q}{D} = \left(\frac{d^4w}{dx^4} + 2\frac{d^4w}{dx^2dy^2} + \frac{d^4w}{dy^4}\right) - g_2\left(\frac{d^3\theta_{sx}}{dx^3} + \frac{d^3\theta_{sx}}{dxdy^2} + \frac{d^3\theta_{sy}}{dx^2dy} + \frac{d^3\theta_{sy}}{dy^3}\right) \qquad (23a)$$

$$D[g_2\left(\frac{d^3w}{dx^3} + \frac{d^3w}{dxdy^2}\right) - g_3\left(\frac{d^2\theta_{sx}}{dx^2} + \frac{1-\mu}{2}\frac{d^2\theta_{sx}}{dy^2}\right) - g_3\frac{(1+\mu)}{2}\frac{d^2\theta_{sy}}{dxdy} - \frac{1-\mu}{2}g_4\theta_{sx}] = 0 \qquad (23b)$$

$$D[g_2\left(\frac{d^3w}{dy^3} + \frac{d^3w}{dx^2dy}\right) - g_3\left(\frac{d^2\theta_{sy}}{dy^2} + \frac{1-\mu}{2}\frac{d^2\theta_{sy}}{dx^2}\right) - g_3\frac{(1+\mu)}{2}\frac{d^2\theta_{sx}}{dxdy} - \frac{1-\mu}{2}g_4\theta_{sy}] = 0 \qquad (23c)$$

III. SOLUTIONS OF GOVERNING EQUATIONS

Solving equations (23b) and (23c) simultabeously gave:

$$\theta_{sx} = \frac{g_2}{g_3}\frac{dw}{dx} = \frac{2g_2}{g_3(1-\mu)}\frac{dw}{dx} = c\frac{dw}{dx} \qquad (24)$$

$$\theta_{sy} = \frac{g_2}{g_3}\frac{dw}{dy} = \frac{2g_2}{g_3(1-\mu)}\frac{dw}{dy} = c\frac{dw}{dy} \qquad (25)$$

$$\frac{d^2}{dxdy} = -\frac{(1-\mu)g_4}{(1+\mu)g_3} \qquad (26)$$

Where, c is a yet to be determined constant.

Substituting equations (24), (25) and (26) into equation 23a and rearranging gave:

$$\frac{d^4w}{dx^4} + 2\frac{d^4w}{dx^2dy^2} + \frac{d^4w}{dy^4} = \frac{PP}{D(1-g_2c)} \qquad (27)$$

The ready solution to equation (27) after integration in terms of non dimensional coordinates is:

$$w = h\,A_1 = [h_x][a_x].[h_y][a_y] \qquad (28)$$

where:

$[a_x]^T = [a_0a_1a_2a_3a_4]$;

$[a_y]^T = [b_0b_1b_2b_3b_4]$

$[h_x] = [1\ \ R\ \ R^2R^3R^4]$; $[h_y] = [1\ \ Q\ \ Q^2Q^3Q^4]$

Substituting equation (28) into equations (24) and (25) gave (in terms non dimensional coordinates):

$$\theta_{sx} = c\frac{dw}{dx} = \frac{A_2}{a}\left(\frac{dh}{dR}\right) \qquad (29)$$

$$\theta_{sy} = c\frac{dw}{dy} = \frac{A_3}{b}\left(\frac{dh}{dQ}\right) \qquad (30)$$

The boundary conditions for cccc plate are:

$$At\ edge\ of\ plate;\ x = 0\,, a:\ \ w = \frac{dw}{dx} = 0$$

$$At\ edge\ of\ plate;\ y = 0\,, b:\ \ w = \frac{dw}{dy} = 0$$

Satisfying these boubdary conditions gave:

$$w = A_1(R^2 - 2R^3 + R^4).(Q^2 - 2Q^3 + Q^4)$$

IV. DETERMINATION OF DISPLACEMENT COEFFICIENTS

Substituting equations (28), (29) and (30) into equation (21) and minimizing the outcome with respect to A_1, A_2 and A_3 respectively gave:

$$\frac{qa^4}{D}.k_qA_1 = (A_1{}^2 - g_2A_1A_2)k_x + (2A_1{}^2 - g_2A_1A_2 - g_2A_1A_3)\frac{1}{\beta^2}k_{xy} + (A_1{}^2 - g_2A_1A_3)\frac{1}{\beta^4}k_y \qquad (31)$$

$$\left[g_3k_x + g_3\frac{1-\mu}{2\beta^2}k_{xy} + \frac{1-\mu}{2}g_5\left(\frac{a}{t}\right)^2 k_{Nx}\right]A_2 + g_3\frac{1+\mu}{2\beta^2}k_{xy}A_3 = \left[k_x + \frac{k_{xy}}{\beta^2}\right]g_2A_1 \qquad (32)$$

$$\left[\frac{g_3}{\beta^4}k_y + g_3\frac{1-\mu}{2\beta^2}k_{xy} + \frac{1-\mu}{2\beta^2}g_5\left(\frac{a}{t}\right)^2 k_{Ny}\right]A_3 + g_3\frac{1+\mu}{2\beta^2}k_{xy}A_2 = \left[\frac{k_y}{\beta^4} + \frac{k_{xy}}{\beta^2}\right]g_2A_1 \qquad (33)$$

Where:

$$k_x = \int_0^1\int_0^1\left(\frac{d^2h}{dR^2}\right)^2 dR\,dQ$$

$$k_{xy} = \int_0^1 \int_0^1 \left(\frac{d^2h}{dR\,dQ}\right)^2 dR\,dQ$$
$$k_y = \int_0^1 \int_0^1 \left(\frac{d^2h}{dQ^2}\right)^2 dR\,dQ$$
$$k_{Nx} = \int_0^1 \int_0^1 \left(\frac{dh}{dR}\right)^2 dR\,dQ$$
$$k_{Ny} = \int_0^1 \int_0^1 \left(\frac{\mathrm{dh}}{\mathrm{dQ}}\right)^2 dR\,dQ$$
$$k_q = \int_0^1 \int_0^1 \mathrm{h}\,dR\,dQ$$

Solving equations (32) and (33) simultaneously gave:

$$A_2 = T_2A_1 \qquad (34)$$
$$A_3 = T_3A_1 \qquad (35)$$

Where:

$$T_2 = \frac{(\mathrm{c}_{12}\mathrm{c}_{23} - \mathrm{c}_{13}\mathrm{c}_{22})}{(\mathrm{c}_{12}\mathrm{c}_{12} - \mathrm{c}_{11}\mathrm{c}_{22})} \qquad (36)$$
$$T_3 = \frac{(\mathrm{c}_{12}\mathrm{c}_{13} - \mathrm{c}_{11}\mathrm{c}_{23})}{(\mathrm{c}_{12}\mathrm{c}_{12} - \mathrm{c}_{11}\mathrm{c}_{22})} \qquad (37)$$
$$\mathrm{c}_{11} = \mathrm{g}_3 k_x + \mathrm{g}_3 \frac{1-\mu}{2\beta^2} k_{xy} + \frac{1-\mu}{2}\mathrm{g}_5 \left(\frac{\mathrm{a}}{\mathrm{t}}\right)^2 k_{Nx}$$
$$\mathrm{c}_{12} = \mathrm{g}_3 \frac{1+\mu}{2\beta^2} k_{xy};\quad \mathrm{c}_{13} = \left[k_x + \frac{k_{xy}}{\beta^2}\right]\mathrm{g}_2$$
$$\mathrm{c}_{22} = \frac{g_3}{\beta^4} k_y + g_3 \frac{1-\mu}{2\beta^2} k_{xy} + \frac{1-\mu}{2\beta^2}\mathrm{g}_5 \left(\frac{\mathrm{a}}{\mathrm{t}}\right)^2 k_{Ny}$$
$$\mathrm{c}_{23} = \left[\frac{k_y}{\beta^4} + \frac{k_{xy}}{\beta^2}\right] g_2$$

Substituting equations (36) and (37) into equation (31) and reaaranging the out-come gave:

$$\frac{A_1 D}{q a^4} = \frac{k_q}{k_T} \qquad (38)$$

Where:

$$k_T = (1 - g_2T_2)k_x + (2 - g_2T_2 - g_2T_3)\frac{1}{\beta^2}k_{xy} + (1 - g_2T_3)\frac{1}{\beta^4}k_y$$

V. DEFINITION OF PARAMETERS

The following definitions for parameters were made:

$$w = \bar{w}\left(\frac{qa^4}{D}\right) = A_1 h \left(\frac{qa^4}{D}\right)$$
$$u = \bar{u}\left(\frac{qa^4}{D}\right) = \frac{1}{\propto}(-A_1 S + A_2 F(S))\frac{dh}{dR}.\left(\frac{qa^4}{D}\right)$$
$$v = \bar{v}\left(\frac{qa^4}{D}\right) = \frac{1}{P\propto}(-A_1 S + A_3 F(S))\frac{dh}{dQ}\left(\frac{qa^4}{D}\right)$$
$$\sigma_x = \overline{\sigma_x}.q = 12q \propto^2 \left\{[-A_1 S + A_2 F(S)]\frac{d^2h}{dR^2} + \frac{\mu}{P^2}[-A_1 S + A_3 F(S)]\frac{d^2h}{dQ^2}\right\}$$
$$\sigma_y = \overline{\sigma_y}.q = 12q \propto^2 \left\{\mu[-A_1 S + A_2 F(S)]\frac{d^2h}{dR^2} + \frac{1}{P^2}[-A_1 S + A_3 F(S)]\frac{d^2h}{dQ^2}\right\}$$
$$\tau_{xy} = \overline{\tau_{xy}}.q = \frac{6q\propto^2}{P}\left\{[-2A_1 S + A_2 F(S) + B_3 F(S)]\frac{d^2h}{dRdQ}\right\}(1-\mu)$$
$$\tau_{xz} = \overline{\tau_{xz}}.q = 6q\propto^3 \left\{A_2 \frac{dF(S)}{dS}\frac{dh}{dR}\right\}(1-\mu)$$
$$\tau_{yz} = \overline{\tau_{yz}}.q = 6q\propto^3 \left\{\frac{A_3}{P}\frac{dF(S)}{dS}\frac{dh}{dQ}\right\}(1-\mu)$$

VI. NUMERICAL PROBLEM

Determine the in-plane displacements (u and v) at (R = 0.5; Q = 0.5; S = 0.2) of cccc thick plate. Determine also the in-plane normal stresses σ_x and σ_y at (R = 0.5; Q = 0.5; S = 0.5), in-plane shear stress (τ_{xy}) at (R = 0; Q = 0; S = 0.5) and the vertical shear stress (τ_{xz}) at (R = 0.2; Q = 0.5; S = 0.2) of the cccc thick plate. Polynomial displacement function for cccc plate used in this analysis is:

$h = (R^2 - 2R^3 + R^4).(Q^2 - 2Q^3 + Q^4)$.

The stiffness coefficients (k values) from this displacement function are:

$k_1 = 0.00127;\ k_2 = 0.000363;\ k_3 = 0.00127$

$k_4 = 0.0000302;\ k_4 = 0.0000302$

Frq $= 0.00111$

VII. RESULTS AND DISCUSSIONS

The value of vertical shear stress from classical plate theory (CPT) analysis is zero. Any plate whose span-to-depth ratio is such that the value of vertical shear stress from thick plate analysis is approximately zero can be idealized as thin plate. Analyzing such plate with classical plate theory will not introduce significant errors. From Table 1, it is apparent that for span-to-depth ratio between 60 and 100, the value of vertical shear stress is significant when corrected to 6 decimal places. Hence, for such span-to-depth ratio, the plate is classified as thin plate. For span-to-depth ratio between 20 and 50, the value of vertical shear stress is significant when corrected to 5 decimal places. Thus, these plates can be classified as moderately thick plate. Hence, analyzing them with classical plate theory will introduce significant errors. When the span-to-depth ratio is less than 20, the value of vertical shear stress is significant when corrected to 4 decimal places. This range of span-to-depth ratio produces plate classified as thick plate.

To determine the correctness of the results from the present studies, comparison was made between values from the present study and those from past scholars. These comparisons were presented on Table 2, Table 3 and Table 4. Table 2 shows the values of centroidal deflection (which was multiplied by 100) for square cccc plate at various span-to-depth ratios from the present study and those from past scholars. The percentage differences between the values from the present study and those of past scholars were presented on Table3 and Table 4. A critical look at Table 3 reveals that maximum recorded percentage difference is 4.07 % (Li et al., 2014; Sheng and He,1995; Liu and Liew, 1998; Lok and Cheng, 2001; Zhong and Xu, 2017). This implies that at 96 % confidence level, the values from the present study are the same with those of previous studies. Furthermore, it is evident from Table 4 that the maximum percentage difference between the values from the present study and those from Xiao et al. (2007) is 5.28 %. Again, at 94 % confidence level, the values from the present study are the same with those from Xiao et al. (2007).

The recorded differences between value from the present study and earlier works may be attributed to difference in deflection functions used. While some of the earlier scholars used trigonometric deflection function in variational methods, Naviers approach, Levy's approach and Timoshenko approach, others used polynomial deflection numerical methods like finite element method. In this present work, polynomial deflection function used in a variational method.

Table 1: Displacements and stresses of square cccc thick plate of various span-to-depth ratio (a/t)

a/t	$\bar{w}$	$\bar{u}$	$\bar{v}$	$\overline{\sigma_x}$	$\overline{\sigma_y}$	$\overline{\tau_{xy}}$	$\overline{\tau_{xz}}$
2	0.006231	-0.003297	-0.003297	0.02232	0.02232	-0.01013	0.009412
2.5	0.004517	-0.002858	-0.002858	0.019349	0.019349	-0.00878	0.00612
3	0.003562	-0.002613	-0.002613	0.017694	0.017694	-0.00803	0.004288
3.333333	0.003145	-0.002506	-0.002506	0.016971	0.016971	-0.00770	0.003486
4	0.002596	-0.002366	-0.002366	0.01602	0.01602	-0.00727	0.002433
5	0.002144	-0.00225	-0.00225	0.015235	0.015235	-0.00691	0.001564
6	0.001896	-0.002187	-0.002187	0.014806	0.014806	-0.00672	0.001088
7	0.001746	-0.002148	-0.002148	0.014547	0.014547	-0.00660	0.000801
8	0.001649	-0.002123	-0.002123	0.014378	0.014378	-0.00652	0.000613
9	0.001582	-0.002106	-0.002106	0.014262	0.014262	-0.00647	0.000485
10	0.001534	-0.002094	-0.002094	0.014179	0.014179	-0.00643	0.000393
11	0.001498	-0.002085	-0.002085	0.014117	0.014117	-0.00641	0.000325
12	0.001471	-0.002078	-0.002078	0.01407	0.01407	-0.00638	0.000273
13	0.00145	-0.002073	-0.002073	0.014034	0.014034	-0.00637	0.000233
14	0.001434	-0.002068	-0.002068	0.014005	0.014005	-0.00635	0.000201
15	0.00142	-0.002065	-0.002065	0.013982	0.013982	-0.00634	0.000175
16	0.001409	-0.002062	-0.002062	0.013963	0.013963	-0.00633	0.000154
17	0.0014	-0.00206	-0.00206	0.013947	0.013947	-0.00633	0.000136
18	0.001392	-0.002058	-0.002058	0.013933	0.013933	-0.00632	0.000121
19	0.001386	-0.002056	-0.002056	0.013922	0.013922	-0.00632	0.000109
20	0.00138	-0.002055	-0.002055	0.013913	0.013913	-0.00631	0.000098
30	0.001352	-0.002047	-0.002047	0.013863	0.013863	-0.00629	0.000044
40	0.001342	-0.002045	-0.002045	0.013846	0.013846	-0.00628	0.000025
50	0.001337	-0.002044	-0.002044	0.013838	0.013838	-0.00628	0.000016
60	0.001335	-0.002043	-0.002043	0.013834	0.013834	-0.00628	0.000011
70	0.001333	-0.002043	-0.002043	0.013831	0.013831	-0.00628	0.000008
80	0.001332	-0.002042	-0.002042	0.013829	0.013829	-0.00627	0.000006
90	0.001332	-0.002042	-0.002042	0.013828	0.013828	-0.00627	0.000005
100	0.001331	-0.002042	-0.002042	0.013827	0.013827	-0.00627	0.000004

$Legend: \bar{w} = \bar{w}(R = 0.5, Q = 0.5, S = 0.5);\ \bar{u} = \bar{u}(R = 0.2, Q = 0.5, S = 0.5)$
$\bar{v} = \bar{v}(R = 0.5, Q = 0.2, S = 0.5);\ \overline{\sigma_x} = \overline{\sigma_x}(R = 0.5, Q = 0.5, S = 0.5)$
$\overline{\sigma_y} = \overline{\sigma_y}(R = 0.5, Q = 0.5, S = 0.5);\ \overline{\tau_{xy}} = \overline{\tau_{xy}}(R = 0.2, Q = 0.2, S = 0.5)$
$\overline{\tau_{xz}} = \overline{\tau_{xz}}(R = 0, Q = 0.5, S = 0);\ *\overline{\tau_{xz}} = \overline{\tau_{xz}}(Present) - \overline{\tau_{xz}}\ at\ (a/t = 100)$

Table 2: Centroidal deflection of square cccc thick plate multiplied by hundred

$100\,\overline{w}(R = 0.5, Q = 0.5, S = 0)$						
Span-to-depth ratio (a/t)	Present	Li et al. (2014)	Sheng and He (1995)	Liu and Liew (1998)	Lok and Cheng (2001)	Zhong and Xu (2017)
3	0.3562	*	*	*	*	0.3611
5	0.2144	0.2172	0.2204	0.2172	0.2147	0.2114
10	0.1534	0.1505	0.1513	0.1505	0.1495	0.1483
20	0.1381	0.1327	0.1329	0.1327	*	*

Table 3: Percentage difference between the values of centroidal deflection from present and past studies

$\%Diff = \frac{/Present\ value - past\ value/}{past\ value} \times 100$					
a/t	Li et al. (2014)	Sheng and He (1995)	Liu and Liew (1998)	Lok and Cheng (2001)	Zhong and Xu (2017)
3	*	*	*	*	1.36
5	1.29	2.72	1.29	0.14	1.42
10	1.93	1.39	1.93	2.61	3.44
20	4.07	3.91	4.07	*	*

Table 4: Percentage difference between the values of centroidal deflection from present and work of Xiao et al. (2007)

$100\,\overline{w}(R = 0.5, Q = 0.5, S = 0)$			
a/t	Present	Xiao et al. (2007)	% Diff
2	0.6231	0.6079	2.50
2.5	0.4517	0.4434	1.87
3.3333	0.3145	0.3092	1.71
5	0.2144	0.2089	2.63
10	0.1534	0.1457	5.28

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