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Fuzzy Filters Ofa Partial Ordered Ternary Semigroup

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Abstract: In this paper we introduced the concepts of fuzzy left filter ,fuzzy(rightlateral) filters of a poternarysemigroup and also the concepts of proper fuzzy filter,fuzzy left (right,lateral) filters of a po ternary semigroup generated by a fuzzy subset are also introduced.It is proved that the non empty intersection of two fuzzy left(right ,lateral) filters of poternarysemigroup is also a fuzzy left (right ,lateral) filter.

Keywords: fuzzy left filter ,fuzzy(right lateral), filtersproper fuzzy filter Completely semiprime, completely fuzzy prime, completely fuzzy semiprime,fuzzysemiprime.

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I. INTRODUCTION

The algebraic theory of semigroups was widely studied by Clifford[2,3]. The ideal theory in general semigroups was developed by Anjaneyulu[1]. Since then a series of researchers have been extending the concepts and results of abstract algebra. Padmalatha, A. Gangadhara Rao and A.Anjaneyulu[10] introduced posubsemigroup, posubsemigroup generated by a subset, two sided identity of a posemigroup, zero of a posemigroup, po ideal ,po ideal generated by a subset. On the other hand, P.M.Padmalatha , A.Gangadahara Rao, P.RamyaLatha [12] introduced completely prime, prime ideal of a posemigroupV.Sivaramireddy studied on ideals in partial ordered ternary semi groups [16].

The concept of a fuzzy set was introduced by Zadeh in 1965[6]. This idea opened up new thoughts and applications in a wide range of scientific fields. A. Rosenfeld applied the notion of fuzzy subset to several areas of mathematics, among other disciplines. N. Kuroki, J N Mordeson developed the fuzzy semigroups concept. N.Kehayopulu, M.Tsingelis introduced the notion of fuzzy subset of a posemigroups[7-9]. Motivated by the study of N.Kehayopulu, M.Tsingelis work in posemigroups we attempt in the paper to study the concept of fuzzy filters of poternarysemigroups.

II. PRELIMINARIES

Definition 2.1: [5] A semigroup T with an ordered relation ≤ is said to be po Ternarysemigroupif T is a partially ordered set such that $a \leq b \Rightarrow a a_1 a_2 \leq b a_1 a_2$, $a_1 a\, a_2 \leq a_1 b a_2$, $a_1 a_2\, a \leq a_1 a_2 b$ for all $a, b, a_1 a_2 \in T$.

Definition 2.2:A function f from T to the closed interval [0,1] is called a fuzzy subset of T. The po ternary semigroup T itself is a fuzzy subset of T such that T(x) =1, $\forall x \in T$. It is denoted by T or 1.

Definition 2.3: Let A be a non-empty subset of T. We denote f_A, the characteristic mapping of A. i.e., The mapping of T into [0,1] defined by

f_A(x)=$\begin{cases} 1 & if\ x \in A \\ 0 & if\ x \notin A \end{cases}$ Then f_A is a fuzzy subset of T

Definition 2.4:[5]**:** A fuzzy subset f of a po ternary semigroup T is called fuzzy Ternarysub semigroupof T if $f(xyz) \geq f(x) \wedge f(y) \wedge f(z) \forall x, y, z \in T$.

Definition 2.5: Let T be a po ternary semigroup. For H⊆ T
we define **(H]**={t∈T / t≤ h for some h∈H}. For H={a} we write (a]= ({a}] = { t∈T / t ≤a}

Definition 2.5: Let T be a po ternary semigroup. For H⊆ T
we define [H)={t∈T / h ≤ t for some h∈H}. For H={a} we write (a]= ({a}] = { t∈T / t ≤a}
Definition 2.6: Let f be a fuzzy subset of a po ternary semigroup T. We define $(f]$by

$(f](x) = \vee_{x \leq y} f(y), \forall x \in T.$

Note 2.7: Clearly$f \subseteq (f]$.

Note 2.8:The set of all fuzzy subsets of T is denoted by F(T).

Definition 2.9: Let (T, ≤) be a po ternary semigroup and f,g,h be fuzzy subsets of T. For $x \in T$ the product fogoh is defined by *(fogoh)(x)* = $\begin{cases} \vee_{x \leq pqr} f(p) \wedge g(q) \wedge h(r) & if\ x \leq pqr \text{ exists} \\ 0 & \text{otherwise} \end{cases}$

Definition 2.10:[11] A nonempty subset A of a po ternary semigroup T is said to be po left ternary ideal or po left ideal of T if i) b, c ∈ T, a ∈ A ⇒bca∈ A ii) a ∈ A and t ∈ T such that t ≤ a ⇒ t ∈ A.
NOTE :A nonempty subset A of a po ternary semigroup T is a po left ternary ideal of T if and only if i) TTA ⊆ A ii) (A] ⊆ A.

Definition 2.11:A nonempty subset A of a po ternary semigroup T is said to be po lateral ternary ideal or po lateral ideal of T if i) b, c ∈ T, a ∈ A ⇒bac∈ A ii) a ∈ A and t ∈ T such that t ≤ a ⇒ t ∈ A.
NOTE **2.12:** A nonempty subset A of a po ternary semigroup T is a po lateral ternary ideal of T if and only if i) TAT UTTATT ⊆ A ii) (A] ⊆ A.

Definition 2.13: A nonempty subset A of a po ternary semigroup T is said to be po right ternary ideal or po right ideal of T if i) b, c ∈ T, a ∈ A ⇒abc∈ A ii) a ∈ A and t ∈ T such that t ≤ a ⇒ t ∈ A
NOTE **2.14**: A nonempty subset A of a po ternary semigroup T is a po right ternary ideal of T if and only if i) ATT ⊆ A ii) (A] ⊆ A.

Definition 2.15:A nonempty subset A of a po ternary semigroup T is said to be po ternary ideal or po ideal of T if i) b, c ∈ T, a ∈ A ⇒bca∈A, bac∈A, abc∈ A ii) a ∈ A and t ∈ T such that t ≤ a ⇒ t ∈ A.
NOTE **2.16**: A nonempty subset A of a po ternary semigroup T is a po ternary ideal of T if and only if i) TTA ⊆ A, TAT ⊆ A, ATT ⊆ A ii) (A] ⊆ A.

Definition 2.17:[11]LetT be a po ternary semigroup. A fuzzy subset f of T is called a fuzzy poleft idealof T if (i) x ≤ y then f(x) ≥ f(y) (ii) f(xyz) ≥ f(z), ∀ x,y, z ∈ T

Lemma 2.18: [10] Let T be a po ternary semigroup and f be a fuzzy subset of T. Then f is a fuzzy po left ideal of T if and only if f satisfies that (i) x ≤ y then f(x) ≥ f(y) ∀x,y, z ∈T (ii) *Tofof⊆ f.*

Definition 2.19: [11]Let T be a po ternary semigroup. A fuzzy subset f of T is called a fuzzy po right idealof T if (i) x ≤ y then f(x) ≥ f(y) (ii) f(xyz) ≥ f(x), ∀ x,y,z ∈T.

Lemma 2.20 [10] Let T be a po ternary Semigroup and f be a fuzzy subset of T. Then f is a fuzzy right ideal of T if and only if f satisfies that (i) x ≤ y then f(x) ≥ f(y) ∀ x,y, z ∈T (ii)fo foT⊆ f.

Definition 2.21: [11] Let T be a po ternary semigroup. A fuzzy subset f of T is called apo lateral idealfuzzyof T if (i) x ≤ y then f(x) ≥ f(y) (ii) f(xyz) ≥ f(y), ∀ x,y,z ∈T

Lemma 2.22: [10] Let T be a po ternary Semigroup and f be a fuzzy subset of T. Then f is a fuzzy lateral ideal of T if and only if f satisfies that (i) x ≤ y then f(x) ≥ f(y) ∀ x,y, z ∈T
(ii)foTof⊆ f.

Definition 2.23: [11]Let T be a po ternary semigroup. A fuzzy subset f of Tis called a fuzzy idealof T if (i) x ≤ y then f(x) ≥ f(y) (ii) f(xyz) ≥ f(z), f(xyz) ≥ f(x) , f(xyz)≥ f(y)∀ x,,y, z ∈T.

Lemma 2.24 :[10] Let T be a po ternary semigroup and f be a fuzzy subset of T. Then f is a fuzzy ideal of T if and only if f satisfies that (i) x ≤ y then f(x) ≥ f(y) ∀ x,y, z ∈T (ii) fo foT⊆ f and Tofof⊆ f and foTof⊆ f.

Lemma 2.25:[7]Let T be a po ternary semigroup and ∅ ≠ A ⊆ T. Then A is a left ideal of T if and only if the characteristic mapping $f_A$of A is a fuzzy left ideal of T.

Lemma 2.26:[7] LetT be a po ternary semigroup and $\emptyset \neq A \subseteq T$. Then A is a right ideal of Tif and only if the characteristic mapping f_A of A is a fuzzy right ideal of T

Lemma 2.27:[7]Let T be a poternarysemigroup and $\emptyset \neq A \subseteq T$ Then A is an ideal of T if and only if the characteristic mapping f_A of A is a fuzzy ideal of T.

Proposition 2.28:[13]Let f,g,h be fuzzy subsets of T. Then the following statements are true.
a. $f \subseteq (f], \forall f \in F(T)$ b. If $f \subseteq g$ then $(f] \subseteq (g]$
c .$(f]o(g] \subseteq (fog], \forall f, g \in F(T)$d. $(f] = ((f]], \forall f \in F(T)$
e. For any fuzzy ideal f of T $f = (f]$
f. If f,g are fuzzy ideals of T, then fog ,f∪g are fuzzy ideals of T.
g . $fo(g \cup h] \subseteq (fog \cup foh]$ h.$(g \cup h]of \subseteq (gof \cup hof]$.
i. If a_λ is an ordered fuzzy point of T, then $a_\lambda = (a_\lambda]$.

Definition 2.29: [13]Let T be a po ternary semigroup, $a \in T$ and $\lambda \in (0,1]$. An ordered fuzzypoint $\mathbf{a}_\lambda$,$\boldsymbol{a}_\lambda: T \to [0,1]$ defined by $a_\lambda(x) = \begin{cases} \lambda \; if \; x \in (a] \\ 0 \; if \; x \notin (a] \end{cases}$
clearlya_λ is a fuzzy subset of T. For every fuzzy subset f of T, we also denote $a_\lambda \subseteq f$ by $a_\lambda \in f$

Definition 2.30:[5] Let f be a fuzzy subset of X. Let $t \in [0,1]$. Define$f_t = \{x \in X / f(x) \geq t\}$. We call f_t a t-cut or a level set.

Definition 2.31:[12]A po (left/right/lateral) ideal of A of a po ternary semigroup T is said to be completely prime (left/right lateral) ideal of T provided $x, y, z \in T$ and $xyz \in A$ implies either $x \in A$ or $y \in A$ or $z \in A$..

Definition 2.32: A po (left/right/lateral) ideal of A of a po ternary semigroup T is said to be completely prime (left/right/ lateral) idealof T provided $x, y, z \in T$and $xyz \in A$ implies either $x \in A$ or $y \in A$ or $z \in A$.

Definition 2.33: A fuzzy ideal$f$ of a po ternary semigroup T is said to be a completely fuzzysemiprimeidealif for any fuzzy point a_t of T such that $a_t{}^n \subseteq f$ for some odd natural number $n \in N$ then $a_t \subseteq f$.
Definition2.34: A fuzzy po subset f of T is said to be a fuzzy d-system if $x_t \subseteq f \Rightarrow x_t{}^n \subseteq f$ for all odd natural number $n \in N$.

Definition 2.35: A fuzzy po ideal$f$ of a po ternary semigroup T is said to be fuzzysemiprimeif g is a fuzzy po ideal of T and $g^n \subseteq f$ for some odd natural number n
then $g \subseteq f$.

Definition 2.36: A fuzzy ideal f of a po ternary semigroup T is called completely primefuzzy idealif∀ three ordered fuzzy points $x_t, y_r, z_s \in T$ $(\forall t, r, s \in (0,1])$ such that $x_t o\, y_r\, o\, z_s \subseteq f$ then $x_t \subseteq f$ or $y_r \subseteq f$ or $z_s \subseteq f$.

Definition 2.37: Let T be a po ternary semigroup. A fuzzy ideal f of T is said to be fuzzy prime if ∀ three fuzzy ideals g, h and iof Tgohoi ⊆ f then either g ⊆ for h ⊆ f or i ⊆ f.

3FUZZY FILTERS :

Definition 3.1: A poternary subsemigroup F of a po ternarysemigroupT is said to be po left filter of Tif $(i)\; a, b, c \in T, abc \in F \Rightarrow a \in F (ii) a, b \in T, a \leq b and a \in F \Rightarrow b \in F$.

Note 3.2: A poternarysubsemigroup F of a poternarysemigroupT is a po left filter of Tiff$(i) a, b, c \in T, abc \in F \Rightarrow a \in F (ii) (F] \subseteq F$.

Definition 3.3: LetT be a poternarysemigroup. A fuzzy subsemigroup f of T is called a fuzzy left filterof Tif (i)$x \leq y \Rightarrow f(x) \leq f(y)(ii)f(xyz) \leq f(z), \forall x, y, z \in T$.

Theorem 3.4: Let T be a poternarysemigroup and A be a non-empty subset of T. Then A is a po left filter of T iff the characteristic function f_A is a fuzzy left filter of T.

Theorem 3.5: The non-empty intersection of two fuzzy left filters of a poternarysemigroupT is also a fuzzy left filter of T.

Proof: Let f, g be two fuzzy left filters of poternarysemigroupT. Let $x \le y$,
Consider $(f \cap g)(x) = f(x) \wedge g(x) \le f(y) \wedge g(y) = (f \cap g)(y) \Rightarrow (f \cap g)(x) \le (f \cap g)(y)$.
Consider $(f \cap g)(xyz) = f(xyz) \wedge g(xyz) \le f(z) \wedge g(z) = (f \cap g)(z)$.
Therefore $f \cap g$ is a fuzzy left filter of **T.**

Theorem 3.6: The non-empty intersection of a family of fuzzy left filters of a poternarysemigroup**T** is also a fuzzy left filter of T.

Proof: Let $\{f_\alpha\}_{\alpha\in\Delta}$ be a family of fuzzy left filters of a poternarysemigroup**T** and let $F = \underset{\alpha\in\Delta}{\cap} f_\alpha = f_1 \cap f_2 \cap \ldots$
Let $x, y, z \in T$such that $x \le y$.
Consider $F(x) = \underset{\alpha\in\Delta}{\cap} f_\alpha(x) = f_1(x) \wedge f_2(x) \wedge f_3(x) \wedge \ldots \ldots$
$\le f_1(y) \wedge f_2(y) \wedge f_3(y) \wedge \ldots$
$= \underset{\alpha\in\Delta}{\cap} f_\alpha(y) = F(y) \Rightarrow F(x) \le F(y)$.
Consider $F(xyz) = \underset{\alpha\in\Delta}{\cap} f_\alpha(xyz) = f_1(xyz) \wedge f_2(xyz) \wedge f_3(xyz) \wedge \ldots \ldots$
$\le f_1(z) \wedge f_2(z) \wedge f_3(z) \wedge \ldots$
$= \underset{\alpha\in\Delta}{\cap} f_\alpha(z) = F(z)$
$\Rightarrow F(xyz) \le F(z)$.
Therefore Fis a fuzzy left filter of T.

Theorem 3.7: Let T be a po ternary semigroup. A fuzzy subsemigroupf of T is a fuzzy left filter of Tifff$'$ = (1-f) is a completely prime fuzzy right ideal of T.

Proof: Let f be a fuzzy left filter of T.
Let $x, y, z \in T$ such that $x \le y \Rightarrow f(x) \le f(y) \Rightarrow f'(x) \ge f'(y)$.
Consider $f'(xyz) = 1 - f(xyz) \ge 1 - f(x) = f'(x) \Rightarrow f'(xyz) \ge f'(x)$.
$\Rightarrow$ $f'$is a fuzzy right ideal of T.
Let $x_t, y_r, , z_s,$ be two ordered fuzzy points such that $t, r, s \in (0,1]$
suppose $x_t o\, y_r o z_s \subseteq f'$. Let $x_t \not\subseteq f' y_r \not\subseteq f'andz_s \not\subseteq f' \Rightarrow x_t \supset 1 - f, y_r \supset 1 -$ f$and, z_s \supset 1 - f$
$\Rightarrow 1 - x_t \subseteq f, 1 - y_r \subseteq$ fand $, 1 - z_s \subseteq f \Rightarrow (1 - x_t) \vee (1 - y_r) \vee (1 - z_s) \subseteq f \Rightarrow 1 - (x_t \wedge y_r \wedge z_s) \subseteq f$
But $(x_t o\, y_r o z_s) \subseteq f' = 1 - f \Rightarrow 1 - (x_t o\, y_r o z_s) \supset f$
$\Rightarrow f \subset 1 - (x_t\ o\ y_r\ o z_s) \subseteq 1 - (x_t \wedge y_r \wedge z_s)$,which gives a contradiction.
Therefore either $x_t \subseteq f'$ or $y_r \subseteq f'$or$z_s \subseteq f'$
$\Rightarrow f'$ is a completely prime fuzzy right ideal of T.
Conversely assume that f' is a completely prime fuzzy right ideal of T.
Let $x \le y$ then $f'(x) \ge f'(y) \Rightarrow f(x) \le f(y)$
Since $f'(xyz) \ge f'(x) \Rightarrow f(xyz) \le f(x)$.
Therefore f is a fuzzy left filter of T.

Corollary 3.8: Let T be a poternarysemigroup and f is a fuzzy left filter of T. Then $f'(= 1 - f)$ is a fuzzy prime ight ideal of T if $f' \neq \emptyset$.

Proof: By Theorem 3.7, f' is a completely prime fuzzy right ideal of T.
Every completely prime fuzzy ideal of T is a fuzzy prime ideal of T
Therefore if f is a fuzzy left filter of T then f' is a fuzzy prime right ideal of T.

Definition 3.9: Let T be a poternarysemigroup. A fuzzy ternary subsemigroup$f$ of T is called a fuzzyright filterof T if
(a) $x \le y \Rightarrow f(x) \le f(y)$ (b) $f(xyz) \le f(x), \forall x, y, z \in T$.

Theorem 3.10: Let T be a poternarysemigroup and A be a non-empty subset of T.Then A is a poright filter of T iff the characteristic function f_A is a fuzzy right filter of T.

Theorem 3.11: The non-empty intersection of two fuzzy right filters of a poternarysemigroup**T** is also a fuzzy right filter of T.

Proof: Let f, g be two fuzzy right filters of poternarysemigroup T. Let $x \le y$,
Consider $(f \cap g)(x) = f(x) \wedge g(x) \le f(y) \wedge g(y) = (f \cap g)(y) \Rightarrow (f \cap g)(x) \le (f \cap g)(y)$.
Consider $(f \cap g)(xyz) = f(xyz) \wedge g(xyz) \le f(x) \wedge g(x) = (f \cap g)(x)$.
Therefore $f \cap g$is a fuzzy right filter of **S.**

Theorem 3.12: The non-empty intersection of a family of fuzzy right filters of a poternarysemigroup**T** is also a fuzzy right filter of T.

Proof: Let $\{f_\alpha\}_{\alpha\in\Delta}$ be a family of fuzzy right filters of a po semigroup **T** and let $F = {}_{\alpha\in\Delta}^{\cap} f_\alpha = f_1 \cap f_2 \cap \ldots.$
Let $x, y, z \in T$ such that $x \le y$.
Consider $F(x) = {}_{\alpha\in\Delta}^{\cap} f_\alpha(x) = f_1(x) \wedge f_2(x) \wedge f_3(x) \wedge \ldots \ldots..$
$\le f_1(y) \wedge f_2(y) \wedge f_3(y) \wedge \ldots ....$
$= {}_{\alpha\in\Delta}^{\cap} f_\alpha(y) = F(y) \Rightarrow F(x) \le F(y)$.
Consider $F(xyz) = {}_{\alpha\in\Delta}^{\cap} f_\alpha(xyz) = f_1(xyz) \wedge f_2(xyz) \wedge f_3(xyz) \wedge \ldots \ldots..$
$\le f_1(x) \wedge f_2(x) \wedge f_3(x) \wedge \ldots ....$
$= {}_{\alpha\in\Delta}^{\cap} f_\alpha(x) = F(x)$
$\Rightarrow F(xyz) \le F(x)$.
Therefore F is a fuzzy right filter of T.

Theorem 3.13: Let$T$ be a poternarysemigroup. A fuzzy subsemigroupf of T is a fuzzy right filter of Tiff $f' = (1\text{-}f)$ is a completely prime fuzzy left ideal of T.

Proof: Let f be a fuzzy right filter ofT.
Let $x, y, z \in T$such that $x \le y \Rightarrow f(x) \le f(y) \Rightarrow f'(x) \ge f'(y)$.
Consider $f'(xyz) = 1 - f(xyz) \ge 1 - f(z) = f'(z) \Rightarrow f'(xyz) \ge f'(z)$.
$\Rightarrow$ $f'$is a fuzzy left ideal of T.
Let x_t, y_r, z_s be ordered fuzzy points such that $t, r, s \in (0,1]$
suppose $x_t o\, y_r o z_s \subseteq f'$. Let $x_t \nsubseteq f'$,$y_r \nsubseteq f'$and$z_s \nsubseteq f'$ $\Rightarrow$ $x_t \supset 1 - f, y_r \supset 1 - f, z_s \supset 1 - f$
$\Rightarrow 1 - x_t \subseteq f$,$1 - y_r \subseteq fand(1 - z_s) \subseteq f \Rightarrow (1 - x_t) \vee (1 - y_r) \vee (1 - z_s) \subseteq f \Rightarrow 1 - (x_t \wedge y_r \wedge z_s) \subseteq f$
But $(x_t o\, y_r o z_s) \subseteq f' = 1 - f \Rightarrow 1 - (x_t o\, y_r o z_s) \supset f$
$\Rightarrow f \subset 1 - (x_t o\, y_r o z_s) \subseteq 1 - (x_t \wedge y_r \wedge z_s)$,which gives a contradiction.
Therefore either $x_t \subseteq f'$ or $y_r \subseteq f'$or $z_s \subseteq f'$.
$\Rightarrow f'$ is a completely prime fuzzy left ideal of S.
Conversely assume that f' is a completely prime fuzzy left ideal of S.
Let $x \le y$ then $f'(x) \ge f'(y) \Rightarrow f(x) \le f(y)$
Since $f'(xyz) \ge f'(z) \Rightarrow f(xyz) \le f(y)$.
Therefore f is a fuzzy right filter of T.

Corollary 3.14: Let T be a poternarysemigroup and f is a fuzzy right filter of T. Then $f' = (1 - f)$ is a fuzzy prime left ideal of T if $f' \neq \emptyset$.

Proof: By Theorem 3.13, f' is a completely prime fuzzy left ideal of T.
Every completely prime fuzzy ideal of T is a fuzzy prime ideal of T.
Therefore if f is a fuzzy left filter of T then f' is a fuzzy prime left ideal of T.

Definition 3.15: Let T be a poternarysemigroup. A fuzzy subsemigroupf of T is called a fuzzy filterof T if (a) $x \le y \Rightarrow f(x) \le f(y)$ (b) $f(xyz) \le f(x) \wedge f(y) \wedge f(z), \forall x, y, z \in T$.

Theorem 3.16: Let T be a poternarysemigroup and A be a non-empty subset of T.Then A is a po filter of T iff the characteristic function f_A is a fuzzy filter of T.
Note 3.17: A fuzzy subsemigroupf of a poternary semigroup T is a fuzzy filter of T iff f is a fuzzy left filter, fuzzy right filter of T.

Definition 3.18: A fuzzy filter f of a poternarysemigroup T is said to be proper fuzzy filterif $f \neq T$.

Theorem 3.19: The non-empty intersection of two fuzzy filters of a poternarysemigroup T is also a fuzzy filter of T.

Proof: Let f, g be two fuzzy filters of poternarysemigroup T. Let $x \leq y$. Consider $(f \cap g)(x) = f(x) \wedge g(x) \leq f(y) \wedge g(y) = (f \cap g)(y) \Rightarrow (f \cap g)(x) \leq (f \cap g)(y)$.
Consider $(f \cap g)(xyz) = f(xyz) \wedge g(xyz)$

$$\leq f(x) \wedge f(y) \wedge f(z) \wedge g(x) \wedge g(y) \wedge g(z)$$
$$\leq f(x) \wedge g(x) \wedge f(y) \wedge g(y) \wedge f(z) \wedge g(z)$$
$$\leq (f \cap g)(x) \wedge (f \cap g)(y) \wedge (f \cap g)(z)$$

Therefore $f \cap g$is a fuzzy filter of **T.**
Theorem 3.20: The non-empty intersection of a family of fuzzy filters of a poternarysemigroupTis also a fuzzy filter of T.

Proof: Let $\{f_\alpha\}_{\alpha\in\Delta}$ be a family of fuzzy filters of a poternarysemigroup**T** and let $F = \underset{\alpha\in\Delta}{\cap} f_\alpha = f_1 \cap f_2 \cap \ldots.$
Let $x, y, z \in T$ such that $x \leq y$.
Consider $F(x) = \underset{\alpha\in\Delta}{\cap} f_\alpha(x) = f_1(x) \wedge f_2(x) \wedge f_3(x) \wedge \ldots \ldots.$

$$\leq f_1(y) \wedge f_2(y) \wedge f_3(y) \wedge \ldots \ldots$$

$= \underset{\alpha\in\Delta}{\cap} f_\alpha(y) = F(y) \Rightarrow F(x) \leq F(y)$.
Consider $F(xyz) = \underset{\alpha\in\Delta}{\cap} f_\alpha(xyz) = f_1(xyz) \wedge f_2(xyz) \wedge f_3(xyz) \wedge \ldots \ldots.$
$\leq f_1(x) \wedge f_1(y) \wedge f_2(x) \wedge f_2(y) \wedge f_3(x) \wedge f_3(y) \wedge f_1(z) \wedge f_2(z) \wedge f_3(z) \ldots \ldots$
$\leq (f_1(x) \wedge f_2(x) \wedge f_3(x) \wedge \ldots \ldots) \wedge (f_1(y) \wedge f_2(y) \wedge f_3(y) \wedge \ldots \ldots)$

$$\wedge \big(f_1(z) \wedge f_2(z) \wedge \qquad f_3(z)\big).$$

$= \underset{\alpha\in\Delta}{\cap} f_\alpha(x) \wedge \underset{\alpha\in\Delta}{\cap} f_\alpha(y) \wedge \underset{\alpha\in\Delta}{\cap} f_\alpha(z) = F(x) \wedge F(y) \wedge F(z)$

$$\Rightarrow F(xyz) \leq F(x) \wedge F(y) \wedge F(z)..$$

Therefore the nonempty intersection of fuzzy filters of a poternarysemigroup T is a fuzzy filter of T.

Theorem 3.21: Let*T* be a poternarysemigroup. A fuzzy subsemigroupf of T is a fuzzy filter of T iff f'(=1-f) is a completely prime fuzzy ideal ofT.

Proof: Let f be a fuzzy filter of T.
Let $x, y, z \in T$ such that $x \leq y \Rightarrow f(x) \leq f(y) \Rightarrow f'(x) \geq f'(y)$.
Consider $f'(xyz) = 1 - f(xyz) \geq \big(1 - f(x)\big) \wedge (1 - f(y)) = f'(x) \wedge f'(y)$.
$\Rightarrow$ $f'$is a fuzzy ideal of T.
Let $x_t, y_r, , z_s,$ be ordered fuzzy points such that $t, r, s \in (0,1]$
suppose $x_t o\, y_r \subseteq f'$. Let $x_t \nsubseteq f'$ and $y_r \nsubseteq f' \Rightarrow x_t \supset 1 - f$ and $y_r \supset 1 - f$
$\Rightarrow 1 - x_t \subseteq f$and $1 - y_r \subseteq f \Rightarrow (1 - x_t) \vee (1 - y_r) \subseteq f \Rightarrow 1 - (x_t \wedge y_r \wedge z_s) \subseteq f$
But $(x_t o\, y_r o z_s) \subseteq f' = 1 - f \Rightarrow 1 - (x_t o\, y_r o z_s) \supset f$
$\Rightarrow f \subset 1 - (x_t o\, y_r o z_s) \subseteq 1 - (x_t \wedge y_r \wedge z_s)$,which gives a contradiction.
Therefore either $x_t \subseteq f'$ or $y_r \subseteq f'$.
$\Rightarrow f'$is a completely prime fuzzy ideal of T.
Conversely assume that f' is a completely prime fuzzy ideal of T.
Let $x \leq y$ then $f'(x) \geq f'(y) \Rightarrow f(x) \leq f(y)$
Since $f'(xyz) \geq f'(x)$ and $f'(xyz) \geq f'(y) \Rightarrow f(xyz) \leq f(z)$and $f(xyz) \leq f(z)$
$\Rightarrow f(xyz) \leq f(x) \wedge f(y) \wedge f(z)..$
Therefore f is a fuzzy filter of T.

Corollary 3.22: Let T be a poternarysemigroup. If f is a fuzzy filter then $f =' (1 - f)$ is a fuzzy prime ideal of if $f' \neq \emptyset$.

Proof: Let f be a fuzzy filter ofT.
By cor 3.8 and cor 3.14, f' is a fuzzy prime ideal of T.

Corollary 3.23: Let f be a fuzzy subset of a commutative poternarysemigroup T is a filter iff$f' = (1 - f)$ is a fuzzy prime ideal of T.

Proof: Let f be a fuzzy filter of commutative poternarysemigroup T.
By cor 3.22, f' is a fuzzy prime ideal of T.
conversely, assume that f' is a fuzzy prime ideal of T.
we know f' is completely fuzzy prime ideal of T
By theorem 3.21, f is a fuzzy filter of T.

Theorem 3.25: Every fuzzy filter f of a poternarysemigroup T is a fuzzy m-system of T.

Corollary 3.26: Let T be a poternarysemigroup. If f is a fuzzy filter of T then $f' = (1 - f)$ is a completely fuzzy semiprime ideal of T
Proof: Let f be a fuzzy filter of T
By Theorem 3.21, f′ is a completely fuzzy prime ideal of T.
we know f′ is a completely fuzzy semiprime ideal of T.

Corollary 3.27: Every fuzzy filter f of a poternarysemigroup T is a fuzzy d-system of T.

Proof: Suppose that f is a fuzzy filter of a poternarysemigroupT.
By Cor 3.26, f′ is a completely fuzzy semiprime ideal of T.
we know $(f')' = f$ is a fuzzy d-system of T.

Corollary 3.28: let T be a poternarysemigroup. If f is fuzzy filter of T then $f' = (1 - f)$ is a fuzzy semi prime ideal of T

Proof: Let f be a fuzzy filter of poternarysemigroup T.
By Th 3.21, f′ is a completely fuzzy prime ideal of T.
we knowf′is completely fuzzy semi prime ideal of T
we know f′ is fuzzy semiprime ideal of T.
Corollary 3.29: Every fuzzy filter f of a poternarysemigroupT is a poternarysemigroup T is a fuzzy n-system of T.
Proof: Let f be a fuzzy filter of poternarysemigroup T
By Cor 3.28, f′ is fuzzy semiprime ideal of T.
we know $(f')' = f$is a fuzzy n-system ofT.

Definition 3.30: Let T be a poternarysemigroup and f be a fuzzy subset of T. The smallest fuzzy left filter of T containing f is called a fuzzy left filter of T generated by f and is denoted by $< f_l >$.

Theorem3.31: The fuzzy left filter of a poternarysemigroup T generated by f is the intersection of all fuzzy left filters of T containing f.

Proof: Let Δbe the set of all fuzzy left filters of T containing f.
Since T itself is a fuzzy left filter of T containing f, $T \in \Delta$ so $\Delta \neq \emptyset$.
Let$F^* = \bigcap_{g \in \Delta} g$, where g is the fuzzy left filter of T containing f.
since$f \subseteq g, \forall g \in \Delta \Rightarrow f \subseteq F^* \Rightarrow F^* \neq \emptyset$
By Th 3.6, F^* is the fuzzy left filter ofE
Let K be another fuzzy left filter of T containing f, clearly $f \subseteq K$ and K is the fuzzy left filter of T. $\Rightarrow K \in \Delta \Rightarrow F^* \subseteq K$. Therefore F^* is the smallest fuzzy left filter of T containing f. Hence F^* is the fuzzy left filter of T generated by f.

Definition 3.32: Let T be a poternarysemigroup and f be a fuzzy subset of T. The smallest fuzzy right filter of T containing f is called a fuzzy right filter of T generated by f and is denoted by $< f_r >$.

Theorem3.33: The fuzzy right filter of a poternarysemigroup T generated by f is the intersection of all fuzzy right filters of T containing f.

Proof: Let Δ be the set of all fuzzy right filters of T containing f.
Since T itself is a fuzzy right filter ofTcontaining f, $T \in \Delta$ so $\Delta \neq \emptyset$
Let$F^* = \bigcap_{g \in \Delta} g$, where g is the fuzzy right filter of T containing f.
since$f \subseteq g, \forall g \in \Delta \Rightarrow f \subseteq F^* \Rightarrow F^* \neq \emptyset$
By Th 3.12, F^* is the fuzzy right filter of T
Let K be another fuzzy right filter of T containing f, clearly $f \subseteq K$ and K is the fuzzy right filter of T.
$\Rightarrow K \in \Delta \Rightarrow F^* \subseteq K$. Therefore F^* is the smallest fuzzy right filter of T containing f.
Hence F^* is the fuzzy right filter of T generated byf.

Definition 3.34: Let T be a poternarysemigroup and f be a fuzzy subset of T. The smallest fuzzy filter of T containing f is called a fuzzy filter of T generated by f and is denoted by $< f >$.

Theorem 3.35: The fuzzy filter of a poternarysemigroup T generated by f is the intersection of all fuzzy filters of T containing f.

Proof: Let Δ be the set of all fuzzy filters of T containing f.
Since T itself is a fuzzy filter of T containing f, $T \in \Delta$ so $\Delta \neq \emptyset$
Let$F^* = \underset{g \in \Delta}{\cap} g$, where g is the fuzzy filter of T containing f.
since$f \subseteq g, \forall g \in \Delta \Rightarrow f \subseteq F^* \Rightarrow F^* \neq \emptyset$
By Th 3.20, F^* is the fuzzy filter of T
Let K be another fuzzy filter of T containing f, clearly $f \subseteq K$ and K is the fuzzy filter of T.
$\Rightarrow K \in \Delta \Rightarrow F^* \subseteq K$. Therefore F^* is the smallest fuzzy filter of T containing f.
Hence F^* is the fuzzy filter of T generated by f.

III. CONCLUSION

In this paper we studied fuzzy filters and proved theorems on relation with completely prime fuzzy and prime fuzzy ideals.we hope to study more and prove many concepts in the near future.

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