IOSR Journal of Engineering (IOSRJEN) **www.iosrjen.org**
ISSN (e): 2250-3021, ISSN (p): 2278-8719
Vol. 09, Issue 1 (January. 2019), ||V (II) || PP 14-19

Casson Fluid Flow and Heat Transfer over a Permeable Vertical Stretching Surface with Magnetic Field and Thermal Radiation

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ABSTRACT: This study investigates the flow of Casson fluid and heat transfer over a permeable vertical stretching surface considering the effects of magnetic field and thermal radiation. Two dimensional boundary layer governing equations are transformed into a system of ordinary differential equations and then solved numerically by shooting method. The effects of various involved parameters on flow and temperature distributions are explained graphically while the skin friction coefficient and Nusselt number are tabulated. Results show that the effect of increasing radiation parameter tends to increase the velocity and temperature profiles.

KEYWORDS: Non-Newtonian fluid, Radiation, Heat transfer, Prandtl number

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Date of Submission: 28-12-2018 Date of acceptance: 14-01-2019

I. INTRODUCTION

The problem of flow and heat transfer in two-dimensional boundary layers over a porous stretching surface has attached considerable attention during last few decades. Such situations occur in many industrial processes such as polymer processing, paper production, manufacturing of food etc [1]. In all cases, non-Newtonian fluids are more appropriate than Newtonian. In order to know the characteristic of non-Newtonian fluid and their applications, it is necessary to study their behavior. But the mechanism of non-Newtonian fluid is complex due to the nonlinearity between the stress and the rate of strain. So these fluids present challenge to the researchers. As a result, no single constitutive equation is available which can show all the properties of such non-Newtonian fluids. Consequently, a lot of non-Newtonian fluid models are available in the literatures [2-3]. Casson fluid model is one kind of non-Newtonian fluid model proposed by Casson [4]. This fluid can be defined as shear thinning fluid. Honey, jelly, human blood [5-6] are considered as Casson fluids. This fluid is reduced to Newtonian fluid at very high shear stress.

Non-Newtonian Casson fluid flow between two rotating cylinders has been discussed by Eldabe et al. [7]. Nadim et al. [8] has investigated the MHD flow of a Casson fluid over an exponentially shrinking sheet. Numerical solutions for non-Newtonian Casson fluid flow over an exponentially stretching surface were studied by Mukhopadhyay and Gorla [9] and they found that the velocity field is decreased with the increase of Casson parameter. Heat and mass transfer in MHD Casson fluid over an exponentially permeable stretching surface has been solved numerically by Raju et al. [10]. Dual solution of non-Newtonian Casson fluid flow and heat transfer over an exponentially permeable shrinking sheet with viscous dissipation were discussed by Zaib et al. [11].

The radiation due to heat transfer effects on different flows is very important in technology and high temperature processes [12]. Thermal radiation plays an essential role in controlling heat transfer in polymer processing industry. Sajid and Hayat [13] discussed the influence of radiation on boundary layer flow over exponentially stretching sheet with the help of homotopy analysis method. From the above studies, we say that little work has been made to analysis the Casson fluid flow considering thermal radiation. The objective of the work is to study the Casson fluid flow and heat transfer over a permeable vertical surface in presence of magnetic field and thermal radiation effects numerically by shooting method. The effects of different involved parameter on the fluid velocity and temperature distributions are plotted and discussed.

II. MATHEMATICAL FORMULATION

Consider a steady two dimensional boundary layer flow of a viscous incompressible electrically conducting fluid along a permeable vertical stretching sheet with heat generation and thermal radiation. Two equal and opposite forces are introduced along the x-axis so that the sheet is stretched keeping the origin fixed as seen in Figure 1. A magnetic field B_0 of uniform strength is applied in y-direction. The effect of the induced magnetic field is neglected in comparison to the applied magnetic field. Here x-axis is taken along the direction of the plate and y-axis normal to it.

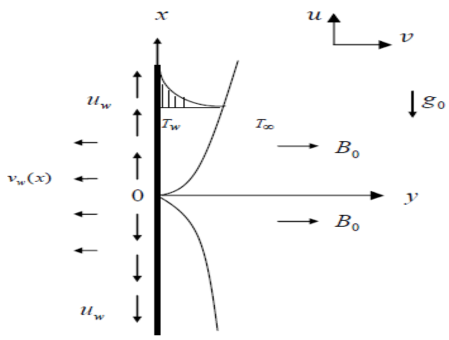

Fig 1: Physical configuration of the problem

Then the equation of state for an isotropic flow of a Casson fluid is [7]

$$\tau_{ij} = 2\left(\mu_b + \frac{P_y}{\sqrt{2\pi}}\right) e_{ij} \tag{1}$$

where e_{ij} is the (i,j)th component of deformation rate, τ_{ij} is the (i,j)th component of the stress tensor, π is the product of the component of deformation rate with itself, and μ_b is the plastic dynamic viscosity. The yield stress P_y is expressed as $P_y = \frac{\mu_b\sqrt{2\pi}}{\beta}$, where β Casson fluid parameter. For non-Newtonian Casson fluid flow $\mu = \mu_b + \frac{P_y}{\sqrt{2\pi}}$ which gives $\vartheta' = \vartheta\left(1 + \frac{1}{\beta}\right)$, where $\vartheta = \frac{\mu_b}{\rho}$ is the kinematic viscosity for Casson fluid. It is assumed that plate temperature is initially T_w, while the temperature far away the sheet is T_∞. If u and v are the velocity components in x and y-directions respectively, then the governing equations for steady boundary layer flow of non-Newtonian Casson fluid are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \vartheta\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} + g_0\beta^*(T - T_\infty) - \frac{\sigma B_0^2}{\rho}u \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p}(T - T_\infty) - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{4}$$

where $g_0$is the acceleration due to gravity, β^* is the volumetic co-efficient of themal expansion, , σ is the electric conductivity, B_0 is the uniform magnetic field strength, ρ is the fluid density, C_p is the specific heat at constant pressure, k is the thermal conductivity, Q_0 is the volumetric rate of heat generation and q_r is the radiative heat flux.

The corresponding boundary conditions are

$$\left.\begin{array}{lll} u = u_w, v = v_w(x), & T = T_w & \text{at } y = 0 \\ u = 0, & T = T_\infty & \text{as } y \to \infty \end{array}\right\} \tag{5}$$

where u_w is the tangential velocity and we consider $u_w = Dx$, $D(> 0)$ is a constant and v_w is the suction velocity.

Using Rosseland approximation for radiation we can get

$$q_r = -\frac{4\sigma^*}{3k'}\frac{\partial T^4}{\partial y} \tag{6}$$

where σ^* is the Stefan- Boltzman constant and $3k'$ is the absorption coefficient. Here we consider the temperature difference within the flow is very small such that T^4 may be expanded as a linear function of temperature. Using Taylor series and neglecting the higher order terms, we get, $T^4 \cong 4T_\infty^3 T - 3T_\infty^4$. Thus equation (4) implies

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p}(T - T_\infty) + \frac{16\sigma^* T_\infty^3}{3k'\rho C_p}\frac{\partial^2 T}{\partial y^2} \tag{7}$$

The governing equations (3) and (7) can be made dimensionless by introducing the following similarity variables.

$$u = Dxf'(\eta)\,, v = -\sqrt{D\vartheta}f(\eta)\,, \eta = y\sqrt{\frac{D}{\vartheta}}\,,\ \psi = \sqrt{D\vartheta}\,xf(\eta),\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{8}$$

where ψ is the stream function, η is the dimensionless distance normal to the sheet, θ be the dimensionless temperature .

Using equation (8) in equations (3) and (7), we get

$$\left(1 + \frac{1}{\beta}\right)f''' + ff'' - f'^2 + \gamma\theta - Mf' = 0 \tag{9}$$

$$\left(1 + \frac{4}{3}N\right)\theta'' + Prf\theta' + PrQ\theta = 0 \tag{10}$$

The reduced boundary conditions are

$$\left.\begin{array}{llll} f' = 1, & f = f_{\mathrm{w}}, & \theta = 1 & \text{at} \quad \eta = 0 \\ f' = 0, & & \theta = 0 & \text{as} \quad \eta \to \infty \end{array}\right\} \tag{11}$$

where $M = \frac{\sigma B_0^2}{\rho D}$ is the magnetic field parameter, $\gamma = \frac{g_0 \beta_T (T_w - T_\infty)}{D^2 x}$ is the buoyancy parameter, $Q = \frac{Q_0}{D \rho C_p}$ is the heat source parameter, $Pr = \frac{\mu C_p}{k}$ is the Prandtl number, $Ec = \frac{D^2 x^2}{C_p (T_w - T_\infty)}$ is the Eckert number, $f_{\mathrm{w}} = -\frac{v_w}{\sqrt{D\vartheta}}$ is the suction parameter and $N = \frac{4\sigma^* T_\infty^3}{kk'}$ is the radiation parameter.

Finally, skin friction coefficient(C_f), local Nusselt number (Nu_x) can be written as

$$Re_x^{\frac{1}{2}} C_f = \left(1 + \frac{1}{\beta}\right) f''(0)\,,\ Nu_x / Re_x^{\frac{1}{2}} = -\,\theta'(0) \tag{12}$$

II. NUMERICAL SOLUTION

The coupled non-linear differential equations (9)-(10) under the boundary conditions (11) have been solved numerically by shooting method namely Nachtsheim-Swigert iteration technique along with sixth order Runge-Kutta iteration scheme. The step size $\Delta\eta = 0.01$ is chosen to satisfy the convergence criterion of 10^{-6} in all cases. The value of η_∞ was found to each iteration loop by $\eta_\infty = \eta_\infty + \Delta\eta$. The maximum value of η_∞ to each group of parameters $\beta, fw, Q\ M, Pr$ and N determined when the value of the unknown boundary conditions at $\eta = 0$ not change to successful loop with error less than10^{-6}.

IV. RESULTA AND DISCUSSION

For the purpose of discussing the result, the numerical calculations are presented in the form of non-dimensional velocity, temperature and concentration profiles. Numerical computations have been carried out for different values of the Prandtl number (Pr), Magnetic field parameter (M), Heat source parameter (Q), Suction parameter (fw), non-Newtonian Casson fluid parameter (β) and Radiation parameter (N). They are chosen arbitrarily where $Pr = 0.71$ corresponds physically to air at $20^\circ C$ and $Pr = 7$ corresponds to water. The value of $\gamma = 1$ is taken fixed. The numerical results for the velocity and temperature profiles are displayed in Figures 2-13.

Figure (2)-(3) depicts the effects of Casson parameter β on velocity and temperature profiles. Velocity is seen to decrease with the increasing Casson parameterβ. Casson parameter is proportional to plastic dynamic viscosity which creates a resistance in the flow. On the other hand, the effect of β leads to enhance the temperature field as seen in Figure (3).

The behavior of suction parameter fw on velocity and temperature profiles can be found in the Figure (4)-(5). It is observed that when wall suction is increased, this causes a decrease in the boundary layer thickness and the velocity field is reduced. Again temperature field decreases with increasing suction. The temperature field becomes much more suppressed because of increased suction. So, this parameter has dominating effects on velocity and temperature profiles.

The effect of magnetic field parameter M has been exhibited in the Figure (6)-(7). It is noted that the flow profile is considerably reduced with the increase of M. The reason is that this M produces a force in the flow field and this force retards the fluid flow. Opposite behavior is seen for temperature field.

The impact of increasing radiation parameter N on velocity and temperature profiles for Casson fluid are discussed in the Figure (8)-(9). It is clear that N has an increasing effect on both velocity and temperature profiles. This is due to the fact that an increase in N enhances the thermal boundary layer.

The nature of Prandlt number Pr on dimensionless velocity and temperature profiles have been displayed in the Figure (10)-(11). The velocity and temperature fields decrease when we increase Pr. Prandtl number signifies the ration of momentum diffusivity to thermal diffusivity. This implies that fluids with lower Pr have higher thermal conductivity. So heat can defuse from the sheet faster than for higher Prandtl number fluids.

In Figure (12)-(13) we have illustrated non-dimensional velocity and temperature profiles against η for some representative values of the heat source parameter $Q = 0.2, 0.5, 0.7, 0.9$. The positive value of Q represents source i.e. heat generation in the fluid. We know that when heat is generated the buoyancy force increases, which induces the flow rate to increase, giving rise to increase in the velocity profiles seen in Figure 12. Again the temperatures increase rapidly as Q increases.

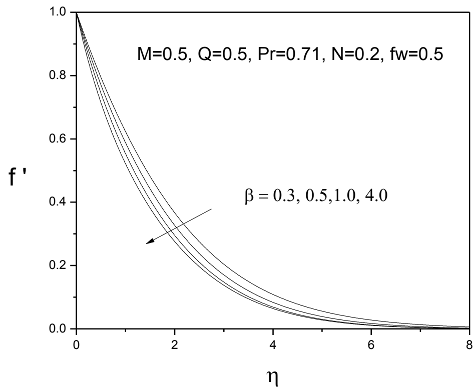

Fig 2: Effect of β on velocity profiles

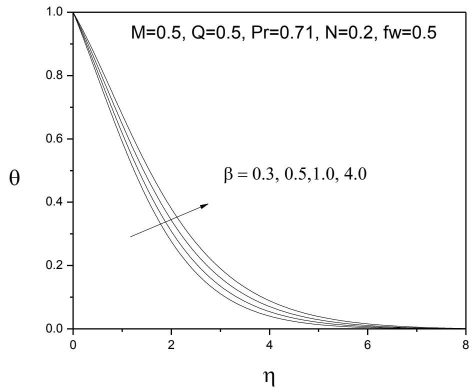

Fig 3: Effect of β on temperature profiles

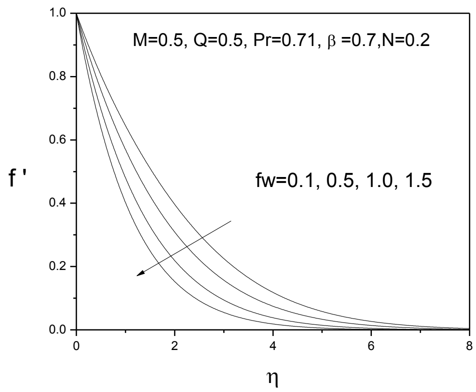

Fig 4: Effect of fw on velocity profiles

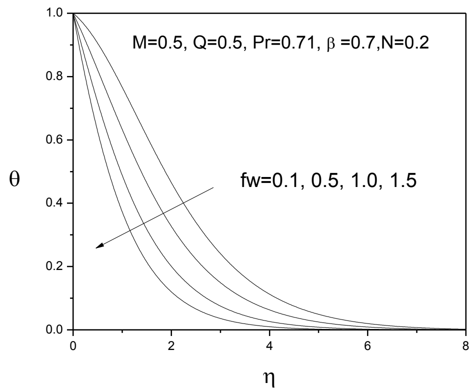

Fig 5: Effect of fw on temperature profiles

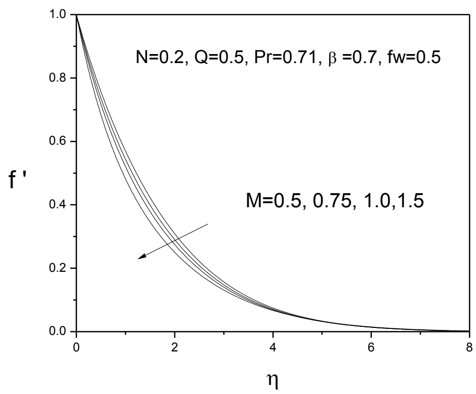

Fig 6: Effect of M on velocity profiles

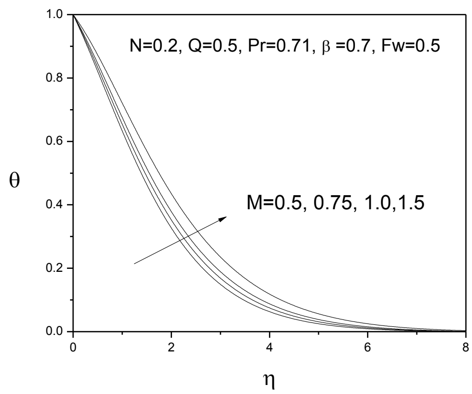

Fig 7: Effect of M on temperature profiles

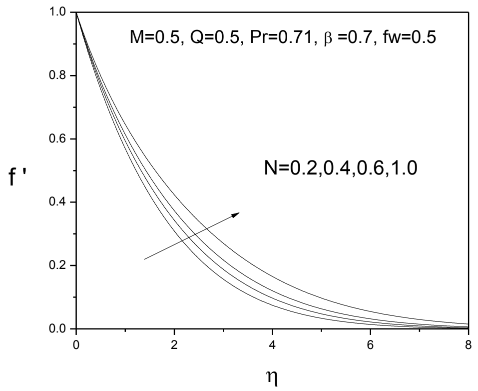

Fig 8: Effect of N on velocity profiles

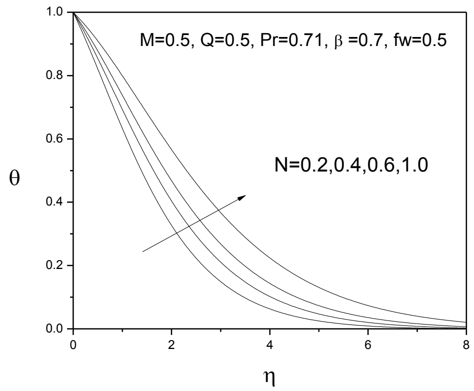

Fig 9: Effect of N on temperature profiles

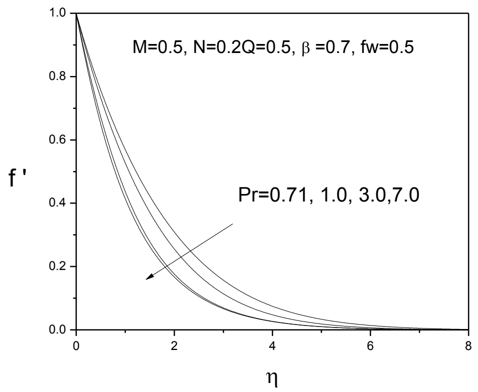

Fig 10: Effect of Pr on velocity profiles

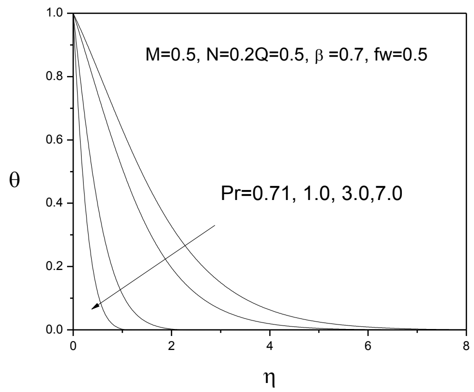

Fig11: Effect of Pr on temperature profiles

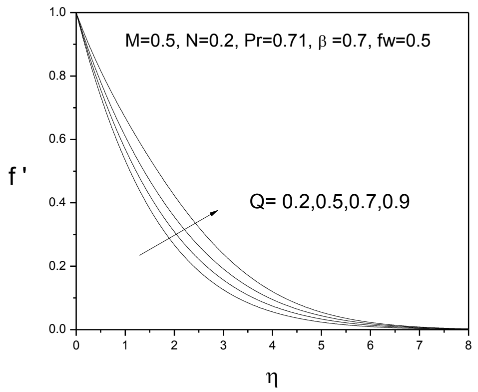

Fig 12: Effect of Q on velocity profiles

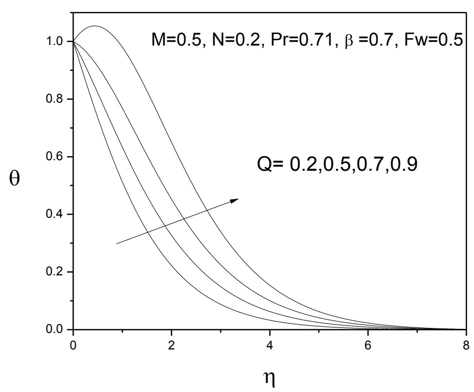

Fig 13: Effect of Q on temperature profiles

The influence of different parameter on skin friction coefficient and local Nusselt number is given below.

β	N	M	$(1+1/\beta)f''(0)$	$-\theta'(0)$
0.3	0.2	0.5	-1.94878	0.37963
0.5	0.2	0.5	-1.54515	0.3443
1.0	0.2	0.5	-1.19368	0.3065
4.0	0.2	0.5	-0.88727	0.26669
0.7	0.2	0.5	-1.35060	0.32416
0.7	0.4	0.5	-1.28005	0.25776
0.7	0.6	0.5	-1.22225	0.21158
0.7	1.0	0.5	-1.13363	0.15252

0.7	0.2	0.5	-1.35060	0.32416
0.7	0.2	0.75	-1.50945	0.29637
0.7	0.2	1.0	-1.65852	0.26838
0.7	0.2	1.5	-1.93095	0.20926

Table 1: Numerical valued of skin friction coefficient and Nusselt number for different values

V. CONCLUSION

A study of Casson fluid flow and heat transfer in presence of magnetic field over a permeable vertical stretching surface with thermal radiation has been presented. Here the conclusion includes that

1. Velocity profile decreases with increasing Casson parameter but temperature increases in this case.
2. Magnetic field parameter decreases the velocity boundary layer thickness
3. Radiation has significant effect on temperature profiles. So we can control temperature distribution by using Radiation.
4. Thermal boundary layer thickness decreases with the increasing Prandtl number. So it can be used to increase the rate cooling in conducting fluid.
5. Velocity and temperature profiles decrease with increasing suction.
6. The magnitude of heat transfer rate at the surface decreases for N and β.

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