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Effect of a Long Dip-Slip Fault on Displacement, Stress and Strain in Viscoelastic Half Space of Burger's Rheology

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ABSTRACT: In the North-East India, China, mid Atlantic- ridge, Pacific seismic belt and Japan, most of the earthquake faults are dip slip in nature. In this work a long, surface breaking, dip-slip fault is assumed situated in a half space of linear viscoelastic medium of Burger's Rheology. Tectonic forces due to mantle convection and other associated phenomena are acting on the system. The nature of the fault movement is assumed to be slipping. The displacement, stresses and strains are obtained analytically at any field point in an isotropic, homogeneous, viscoelastic half-space using integral transformation, modified Green's function technique and correspondence principle. A close inspection of these expressions may give some clue about the nature of stressaccumulation in the lithosphere-asthenosphere system.

Keywords:*Aseismic Period, Dip-slip fault, Mantle Convection, Burger's Rheology.*

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I. INTRODUCTION

Rocky Mountains of Himalayas, Atlantic fault of central Greece-steeply dipping faults with dip 60 degree, 80 degree where the surface level changes during the motion i.e. the faults are dip-slip in nature. Therefore, it is necessary to understand the mechanism of plate motion in the dip direction with a displacement dislocation and the nature of stress-strain accumulation/release in spatial and temporal co-ordinate to predict the future event in space and time. The work involving static ground deformation in elastic media was initiated by(Steketee,1958a),(Steketee,1958b), (Chinnery,1961),(Maruyama,1964),(Maruyama,1964).

(Savage et al.,1966) did wonderful work in analyzing the displacement, stress and strain for dip-slip movement of the fault. Some theoretical models in this field have been studied by a numberof authors like (Freund et al., 1976), (Ghosh et al., 1992),(Rybicki,1986),(Rybicki,1971),(Sing et al., 1996),(Tomar et al., 2003). (Segall,2010) developed various aspects of fault movements in his book.

(Mukhopadhyay,1979),(Mukhopadhyayetal., 1980)have discussed stress accumulation near infinite fault in lithosphere-asthenosphere system. The work of (Debnath,2013), (Debnath,2014),(Sen et al., 2012) have been discussed about long dip-slip fault in the viscoelastic medium of Maxwell type material whereas the work of (Mondal et al., 2018) discussed about the representation of dip slip fault in standard linear solid (SLS) type material. In the earlier works, most of the cases elastic or viscoelastic half space or layered medium wereconsidered to represent the lithosphere-asthenosphere system. The study of (Hu et al., 2016) and Observations in seismicallyactive regions suggest that linear viscoelastic material of Burger's Rheology may be a suitable representation of the system. Inthis paper, we consider an infinite sudden dip-slip movement situated in a linear viscoelastic solid of Burger'sRheology.

The system is under the action of tectonic forces which forces are taken to be constant, generated due to mantle convection or similar other processes and displacements, stressesare analyzed.

II. FORMULATION

Considering a long, dip-slip fault F, with width D situated in a viscoelastic half space of linear Burger's Rheology. A Cartesian coordinate system is used with a suitable point O as origin on the fault, the strike of the fault is taken along $\boldsymbol{y_1}$ axis, $\boldsymbol{y_3}$axis pointing downwards so that the free surface is given by $\boldsymbol{y_3}$ = 0 and $\boldsymbol{y_2}$ axis is perpendicular to$\boldsymbol{y_1 y_3}$plane. We choose another coordinate system with $\boldsymbol{y_1'}$, $\boldsymbol{y_2'}$, $\boldsymbol{y_3'}$axes as shown in Figure 1 so that the fault can be given by F : ($\boldsymbol{y_2'}$= 0; 0 ≤$\boldsymbol{y_3'}$≤D)

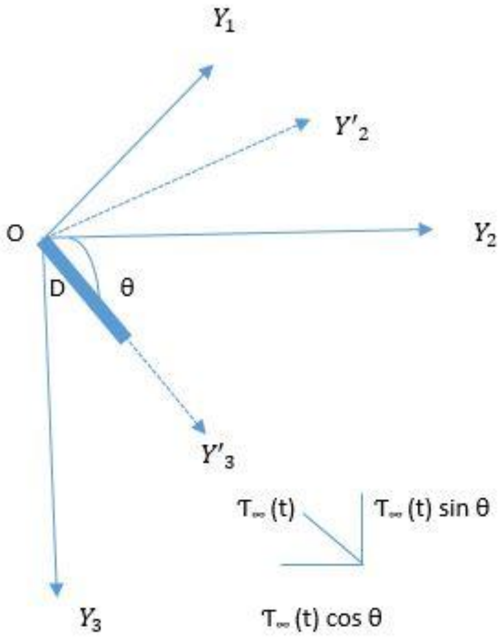

Fig-1

Coordinate system describing the location of the fault.

Let θ be the inclination of the fault F with the free surface. We consider the section of the model by the plane y_1 = 0. The displacement, stress and strain components separate out into two independent groups. One group containing displacementu, stress component (τ_{12}, τ_{13}) and strain components $(E_{12}, E_{13})$is associated with the strike- slip movement while the other group containing displacement component (v,w), stress components $(\tau_{22}, \tau_{23}, \tau_{33})$ and strain components (E_{22}, E_{23}, E_{33}) is associated with dip-slip movement. We consider the dip-slip movement across the fault F. Let v,w be the displacement components along the y_2, y_3 axes and $\tau_{22}, \tau_{23}, \tau_{33}$are the stress components and E_{22}, E_{23}, E_{33} are the strain components respectively. We take t = 0 as an instant when the medium is in aseismic state.

2.1 Constitutive Equation:

The constitutive laws provide the relation between stress and strain possibly including time derivatives. We consider dip-slip movement across the fault when the medium is in aseismic state (t=0) for which the displacements v and w,stressesτ_{22},τ_{23}, τ_{33} and strains $E_{22}, E_{23}, E_{33}$are present. The stress-strain relations for Burger's Rheology model of viscoelastic material are taken as follows(Segall,2010)

$$\left.\begin{aligned}
\tau_{22} + p_1 \frac{\partial \tau_{22}}{\partial t} + p_2 \frac{\partial^2 \tau_{22}}{\partial t^2} &= 2q_1 \frac{\partial E_{22}}{\partial t} + 2q_2 \frac{\partial^2 E_{22}}{\partial t^2} \\
\tau_{23} + p_1 \frac{\partial \tau_{23}}{\partial t} + p_2 \frac{\partial^2 \tau_{23}}{\partial t^2} &= 2q_1 \frac{\partial E_{23}}{\partial t} + 2q_2 \frac{\partial^2 E_{23}}{\partial t^2} \\
\tau_{33} + p_1 \frac{\partial \tau_{33}}{\partial t} + p_2 \frac{\partial^2 \tau_{33}}{\partial t^2} &= 2q_1 \frac{\partial E_{33}}{\partial t} + 2q_2 \frac{\partial^2 E_{33}}{\partial t^2}
\end{aligned}\right] \quad (1)$$

Where$p_1 = \frac{\eta_1}{\mu_1} + \frac{\eta_2}{\mu_2} + \frac{\eta_1}{\mu_2}$,$p_2 = \frac{\eta_1 \eta_2}{\mu_1 \mu_2}$,

$q_1 = \eta_1$, $q_2 = \frac{\eta_1 \eta_2}{\mu_2}$

Here $\eta_1, \eta_2$are the respective effectiveviscosities andμ_1, $\mu_2$are the respective effective rigidities of the materials.

*2.2**Stress Equation of Motion**:*

For the small deformations, if the inertial forces are very small so that the acceleration can be taken to be negligible and if there are no body forces acting in the system during our consideration, the quasi-static equilibrium equation is

$$\left.\begin{array}{l}\frac{\partial \tau_{22}}{\partial y_2}+\frac{\partial \tau_{23}}{\partial y_3}=0\\ \frac{\partial \tau_{32}}{\partial y_2}+\frac{\partial \tau_{33}}{\partial y_3}=0\\ (-\infty<y_2<\infty, y_3\geq 0, t\geq 0)\end{array}\right\} \quad (2)$$

2.3 **Boundary Conditions**:
For the fault F
$\tau_{22}(y_2,y_3,t)\to\tau_\infty(t)\cos\theta$ as $|y_2|\to\infty$, $\tau_{23}(y_2,y_3,t)$=0, $\tau_{33}(y_2,y_3,t)$=0 on y_3=0. (3)
Alsoas $y_3\to\infty$, $\tau_{23}(y_2,y_3,t)\to 0$
and $(y_2,y_3,t)\to\tau_\infty(t)\sin\theta$
$(-\infty<y_2<\infty,\ y_3\geq 0, t\geq 0)$

Where $\tau_\infty(t)$isthe value oftectonic forces which may or may not vary with time but is taken to be independent of y_1.

2.4 **Initial Conditions:**
We assume the time t from a suitable instant when thesystem is in an aseismic state and there is no seismic disturbancein it. Let $v=v_0$, $w=w_0$at the time t = 0 and$v_t=\frac{\partial v}{\partial t}=0andw_t=\frac{\partial w}{\partial t}=0$ at time t = 0. We alsoassume that $\tau_{22}=(\tau_{22})_0\tau_{23}=(\tau_{23})_0$, $\tau_{33}=(\tau_{33})_0$ and $\frac{\partial \tau_{22}}{\partial t}=0$, $\frac{\partial \tau_{23}}{\partial t}=0$, $\frac{\partial \tau_{33}}{\partial t}=0$at time t=0 and$E_{22}=(E_{22})_0$, $E_{23}=(E_{23})_0$, $E_{33}=(E_{33})_0$, at time t=0.
The above initial values satisfy all the relations given in (1) to (3).

III. SOLUTION

Now differentiating 1 st and 2ndequation of (1) with respect to y_2 and y_3 respectively, adding them and using 1st equation of (2). We obtain,

$$\left.\begin{array}{l}\nabla^2 V(y_2,y_3,t)=0 \text{ where } V=v-v_0\\ \text{Similarly,}\\ \nabla^2 W(y_2,y_3,t)=0 \text{ where } W=w-w_0\end{array}\right\} \quad (4)$$

3.1 Solution Before Any Fault Movement
Taking Laplace transformation with respect to time t of all the constitutive equations and the boundary conditions,the boundary value problem given by equation (1) to (4) can be solved.
On taking Laplace inverse transformation the solutions for displacement, stresses and strain are given below:

$$v_a=v_0+y_2\tau_\infty[\{\frac{t}{q_1}-\frac{q_2}{q^2{}_1}\left(1-e^{-\frac{q_1t}{q_2}}\right)\}\cos\theta\ +\{\frac{p_1}{q_1}\left(1-e^{-\frac{q_1t}{q_2}}\right)+\frac{p_2}{q_2}e^{-\frac{q_1t}{q_2}}\}(\cos\theta-1)],$$

$$w_a=w_0+y_3\tau_\infty[\{\frac{t}{q_1}-\frac{q_2}{q^2{}_1}\left(1-e^{-\frac{q_1t}{q_2}}\right)\}\sin\theta\ +\{\frac{p_1}{q_1}\left(1-e^{-\frac{q_1t}{q_2}}\right)+\frac{p_2}{q_2}e^{-\frac{q_1t}{q_2}}\}(\sin\theta-1)],$$

$$(\tau_{22})_a=\frac{(\tau_{22})_0}{A}[(p_1-p_2r_1)e^{-r_1t}-(p_1-p_2r_2)e^{-r_2t}+\tau_\infty[\cos\theta-\frac{1}{A}\{(p_1-p_2r_1)e^{-r_1t}$$
$$-(p_1-p_2r_2)e^{-r_2t}\}],$$

$$(\tau_{23})_a=\frac{(\tau_{23})_0}{A}[(p_1-p_2r_1)e^{-r_1t}-(p_1-p_2r_2)e^{-r_2t}],$$

$$(\tau_{33})_a=\frac{(\tau_{33})_0}{A}[(p_1-p_2r_1)e^{-r_1t}-(p_1-p_2r_2)e^{-r_2t}+\tau_\infty[\sin\theta-\frac{1}{A}\{(p_1-p_2r_1)e^{-r_1t}$$
$$-(p_1-p_2r_2)e^{-r_2t}\}], \quad (5)$$

$$(E_{22})_a=(E_{22})_0+\tau_\infty[\{\frac{t}{q_1}-\frac{q_2}{q^2{}_1}\left(1-e^{-\frac{q_1t}{q_2}}\right)\}\cos\theta\ +\{\frac{p_1}{q_1}\left(1-e^{-\frac{q_1t}{q_2}}\right)+\frac{p_2}{q_2}e^{-\frac{q_1t}{q_2}}\}(\cos\theta-1)],$$

$$(E_{23})_a=(E_{23})_0,$$

$$(E_{33})_a=(E_{33})_0+\tau_\infty[\{\frac{t}{q_1}-\frac{q_2}{q^2{}_1}\left(1-e^{-\frac{q_1t}{q_2}}\right)\}\sin\theta\ +\{\frac{p_1}{q_1}\left(1-e^{-\frac{q_1t}{q_2}}\right)+\frac{p_2}{q_2}e^{-\frac{q_1t}{q_2}}\}(\sin\theta-1)].$$

Where,

$$\boldsymbol{r_1} = \frac{(\boldsymbol{p_1} - \boldsymbol{A})}{2\boldsymbol{p_2}}, \boldsymbol{r_2} = \frac{(\boldsymbol{p_1} + \boldsymbol{A})}{2\boldsymbol{p_2}}, \boldsymbol{A} = (\boldsymbol{p_1}^2 - 4\boldsymbol{p_2})^{\frac{1}{2}}$$

and$\boldsymbol{p_1}$ and $\boldsymbol{p_2}$ are same as given in equation (1)

Above solution shows that $\boldsymbol{\tau_{22}}$increases with time and $\boldsymbol{\tau_{22}} \rightarrow \boldsymbol{\tau_\infty}$(t) $\mathbf{cos}\,\boldsymbol{\theta}$as t→∞ while $\boldsymbol{\tau_{23}}$→0 but$\boldsymbol{\tau_{33}} \rightarrow \boldsymbol{\tau_\infty}$(t) $\mathbf{sin}\,\boldsymbol{\theta}$as,where $\boldsymbol{\tau_\infty}$(t)= constant. The geological conditions as well as the characteristic of the fault is such that when the stress component $\boldsymbol{\tau_{23}}$across the fault reaches some critical value $\boldsymbol{\tau_c}$,$\boldsymbol{\tau_c} < \boldsymbol{\tau_\infty}(t)\mathbf{cos}\,\boldsymbol{\theta}$, the fault F starts slip and for bounded stresses and strains the dislocation function f($\boldsymbol{\xi'_3}$) say, should satisfy the conditions as discussed in (Ghosh et al., 1992)

(i) Its value will be maximum on the free surface. (ii) The magnitude of the dislocation will decrease with $\boldsymbol{y'_3}$ as we move downwards and ultimately tends to zero near the lower edge of the fault
$\boldsymbol{y'_2}$ = 0, $\boldsymbol{y'_3}$= D.

3.2 Solution After Any Fault Movement

The stresscomponent$\boldsymbol{\tau_{23}}$, which is the main driving force for the dip-slip motion of the fault, exceeds the critical value $\boldsymbol{\tau_c}$ = 200 barafter time T = 65 years (say) and the fault starts to slip.
An additional condition characterizing the dislocation of w due to the sudden movement is

$$\left.\begin{array}{l} [\boldsymbol{w}]_{\boldsymbol{F}} = \boldsymbol{U}\boldsymbol{f}(\boldsymbol{y'_3})\boldsymbol{H}(\boldsymbol{t_1}) \\ \text{Where } \boldsymbol{w}]_{\boldsymbol{F}} = \lim_{y'_2 \rightarrow 0^+} \boldsymbol{w} - \lim_{y'_2 \rightarrow 0^-} \boldsymbol{w} \\ \quad (y'_2 = 0, 0 \le y'_3 \le D). \end{array}\right\} \quad (6)$$

Here $\boldsymbol{H}(\boldsymbol{t_1})$ is the Heaviside function, Uis the slip magnitude and $[\boldsymbol{w}]_{\boldsymbol{F}}$is the discontinuity of w across F. It is to be noted that $[\boldsymbol{w}]_{\boldsymbol{F}}$ = 0 for$\boldsymbol{t_1} \le \boldsymbol{0}$, where $\boldsymbol{t_1} = \boldsymbol{t} - \boldsymbol{T}$.The fault F is located inthe region ($\boldsymbol{y'_2} = \boldsymbol{0}, \boldsymbol{0} \le \boldsymbol{y'_3} \le \boldsymbol{D}$). We note that v is continuouseven after the fault slip so that v = 0, while $\boldsymbol{w}$satisfies the dislocation condition given in equation (6). The modified boundary value problem is stated below:
$\boldsymbol{\nabla^2}\overline{\boldsymbol{W}} = \boldsymbol{0}$, where$\overline{\boldsymbol{W}} = \overline{\boldsymbol{w}} - \frac{\boldsymbol{w_0}}{\boldsymbol{p}}$.$\overline{\boldsymbol{w}}$is the L.T. of $\boldsymbol{w}$ with modified boundary conditions

$$\left.\begin{array}{l} \overline{\boldsymbol{\tau}}_{\boldsymbol{22}}(\boldsymbol{y_2}, \boldsymbol{y_3}, \boldsymbol{p}) \rightarrow 0 \text{ as } |\boldsymbol{y_2}| \rightarrow \infty, \boldsymbol{y_3} \ge \boldsymbol{0} \\ \overline{\boldsymbol{\tau}}_{\boldsymbol{23}}(\boldsymbol{y_2}, \boldsymbol{y_3}, \boldsymbol{p}) \rightarrow 0 \text{ as } \boldsymbol{y_3} \rightarrow \infty \\ (-\infty < \boldsymbol{y_2} < \infty) \quad (7) \\ \overline{\boldsymbol{\tau}}_{\boldsymbol{33}}(\boldsymbol{y_2}, \boldsymbol{y_3}, \boldsymbol{p}) \rightarrow 0 \text{ as } \boldsymbol{y_3} \rightarrow \infty \\ (-\infty < \boldsymbol{y_2} < \infty). \end{array}\right\}$$

In the absence of any fault movement,all other boundary conditions are same as stated.
We solve thisboundary value problem as shown in the appendix. Then the solution for displacements, stresses and strains after fault movement is in following equation

$$\left.\begin{array}{l} w_b = \frac{U}{2\pi}\varphi(y_2, y_3)H(t_1), \\ (\tau_{22})_b = 0, \\ (\tau_{23})_b = \frac{U}{2\pi A}[(q_1 - q_2 r_1)e^{-r_1 t} - (q_1 - q_2 r_2)e^{-r_2 t}]H(t_1)\varphi_1(y_2, y_3), \\ (\tau_{33})_b = \frac{U}{2\pi A}[(q_1 - q_2 r_1)e^{-r_1 t} - (q_1 - q_2 r_2)e^{-r_2 t}]H(t_1)\varphi_2(y_2, y_3), \\ (E_{22})_b = 0, \\ (E_{23})_b = \frac{U}{4\pi}H(t_1)\varphi_1(y_2, y_3), \\ (E_{33})_b = \frac{U}{2\pi}H(t_1)\varphi_2(y_2, y_3). \end{array}\right\} \quad (8)$$

$\boldsymbol{H}(\boldsymbol{t_1})$ is the Heaviside step function which gives the displacement at any point Q($\boldsymbol{y_2}$, $\boldsymbol{y_3}$) and φ($\boldsymbol{y_2}, \boldsymbol{y_3}$), $\boldsymbol{\varphi_1}(\boldsymbol{y_2}, \boldsymbol{y_3})$ and $\boldsymbol{\varphi_2}(\boldsymbol{y_2}, \boldsymbol{y_3})$are given in Appendix. We try to find the solutions in the following form:
$v = v_a + v_b,\ w = w_a + w_b$
$\tau_{22} = (\tau_{22})_a + (\tau_{22})_b, \tau_{23} = (\tau_{23})_a + (\tau_{23})_b,$
$\tau_{33} = (\tau_{33})_a + (\tau_{33})_b, E_{22} = (E_{22})_a + (E_{22})_b,$
$E_{23} = (E_{23})_a + (E_{23})_b, E_{33} = (E_{33})_a + (E_{33})_b$.
Where the suffix a, b represents solution before and after fault movement respectively.

4. **NUMECAL COMPUTATION**

Following (Cathles,1975), (Aki et al.,1980) and the recent studies on rheological behaviour of the crust and upper mantle by (Chiftet al., 2002), the value of the model parameters are taken as follows:
⍰$_1$ = 3×$\mathbf{10^{10}}$N / $\boldsymbol{m^2}$(Pascals),
⍰$_2$ = 3.5×$\mathbf{10^{10}}$N / $\boldsymbol{m^2}$(Pascals),
$\boldsymbol{\eta_1}$ = 3×$\mathbf{10^{19}}$Pa: s, $\boldsymbol{\eta_2}$ = 3.5×$\mathbf{10^{19}}$Pa: s
D=Width of the fault F=5km=5×$\mathbf{10^3}$meter,
$(\boldsymbol{\tau_{22}})_0$= 20×$\mathbf{10^5}$ N/$\boldsymbol{m^2}$(Pascals),
$(\boldsymbol{\tau_{23}})_0$= 20×$\mathbf{10^5}$ N/$\boldsymbol{m^2}$(Pascals),
$(\boldsymbol{\tau_{33}})_0$= 20×$\mathbf{10^5}$ N/$\boldsymbol{m^2}$(Pascals)
$\boldsymbol{\tau_c}$ = 200 bar = 2×$\mathbf{10^7}$N/$\boldsymbol{m^2}$(Pascals),
θ=π/6, π/4, π/3, π/2, $\boldsymbol{\tau_\infty}$(t) = 20 ×$\mathbf{10^5}$N/$\boldsymbol{m^2}$
R=2 meter, U=1.6 meter, 3.7 meter, 5.6 meter.
We consider different dislocation functionf($\boldsymbol{\xi'_3}$)in thefollowing form suggested by (Godara et al., 2017)
(i) Liner Slip Function (LSF): f ($\boldsymbol{\xi'_3}$) =R $(\mathbf{1}-\frac{\xi'_3}{D})$; (ii)Parabolic Slip Function (PSF): f ($\boldsymbol{\xi'_3}$) =R $(\mathbf{1}-\frac{\xi'^2_3}{D^2})$;
(iii) Elliptic Slip Function (ESF):f ($\boldsymbol{\xi'_3}$) =R$(\mathbf{1}-\frac{\xi'^2_3}{D^2})^{\frac{1}{2}}$, which are satisfied for all the conditions for bounded stresses and strains. We computed displacements, stresses and strains taking the above values of the parameter with new time origin$\boldsymbol{t_1 = t - T}$, where T = 65 years (say)
usingMATLAB.

The displacement component w against $\boldsymbol{y_2}$ for different slip function taking $\boldsymbol{y_3}$ = 5 km with average slip magnitude 3.7 meter and a fixed inclination $\boldsymbol{\theta = \frac{\pi}{3}}$just immediately after the fault movement has been shown in Fig-2. It is clear from this figure that the displacement is maximum for ESF and minimum for LSF and this displacement tends to zeroas we move far away from the fault. In Fig-3, with the value of $\boldsymbol{y_2}$ = 5 km, slip magnitude U = 3.7 meter for the ESF, displacement is found to be maximum on the freesurface$\boldsymbol{y_3}$= 0 and then it sharply decreases and gradually→ 0 with the increasing value of depth. The rate of decreaseof displacement is higher when the fault is inclined at an angle $\boldsymbol{\theta = \pi/6}$and this displacement falls off rapidly with the increase of inclination of the fault and finally tend to zero as $\boldsymbol{y_3} \rightarrow \infty$ for all θ. Thus for the Burger's Rheology and SLS type material, the effect on displacement component discussed in (Mondal et al., 2018) is almost same. It has been observed that for earthquake of smaller magnitude, the slip is small and this slip is higher for earthquake of higher magnitude.

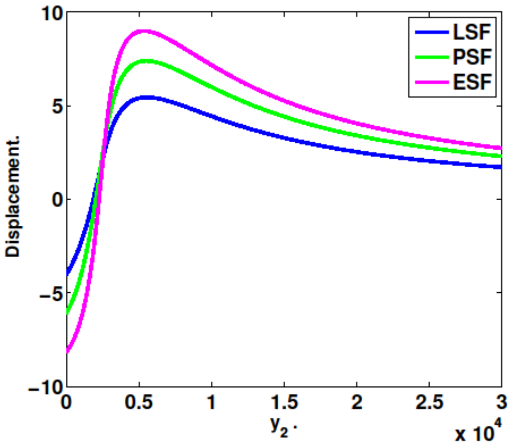

Fig-2
Displacement w with$\boldsymbol{y_2}$ for different slip function.

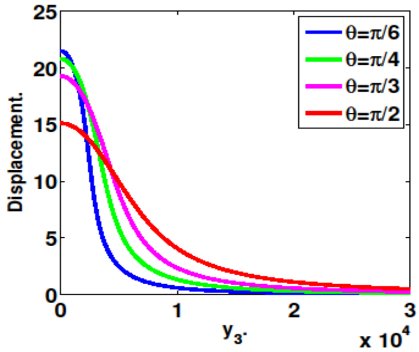

Fig-3

Displacement $\boldsymbol{w}$ against $\boldsymbol{y_3}$for various inclination.

In the Fig-4, shear stress $\boldsymbol{\tau_{23}}$ has been plotted against $\boldsymbol{y_2}$ for various slip magnitude with fixed inclination π/3 and ESF. This shows that stress $\boldsymbol{\tau_{23}}$ has different value near the free surface $\boldsymbol{y_3}$ = 0 for different slip magnitude and for small magnitude of slip, stress quickly tending to zero. Each plot has a brunch cut and stress $\boldsymbol{\tau_{23}}$vanishes far away from the fault $\boldsymbol{y_2}$=0 after few km along $\boldsymbol{y_2}$. Stress $\boldsymbol{\tau_{23}}$ with $\boldsymbol{y_2}$ for various slip function has been shown in the Fig-5, taking $\boldsymbol{y_3}$=5 km, with average slip magnitude U=3.7 meter and θ=π/3. In this figure we see that stress is maximum for ESF and minimumfor LSF. For different slip function stress $\boldsymbol{\tau_{23}}$increases slowly and after attending certain value of order $\mathbf{10^{-4}}$, it decreases rapidly and finally tending to zero as stress is releasing after the fault slip.

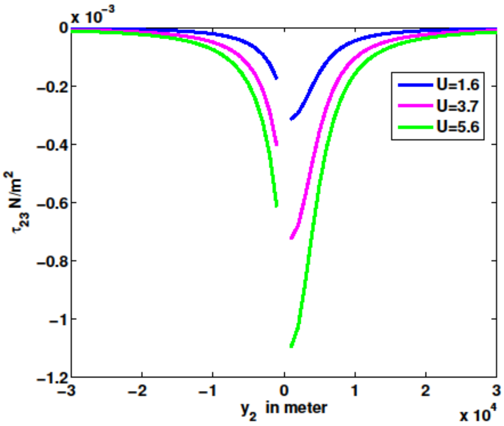

Fig-4

Stress $\boldsymbol{\tau_{23}}$ with $\boldsymbol{y_2}$ for different slip magnitude.

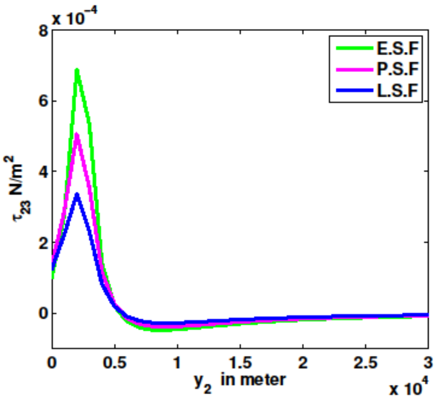

Fig-5

Stress $\boldsymbol{\tau_{23}}$ with $\boldsymbol{y_2}$ for various slip function.

In Fig-6, we plotted stress $\boldsymbol{\tau_{23}}$for various slip function against $\boldsymbol{y_3}$ taking the same value of $\boldsymbol{y_2}$, U and θthat (Mondal et al.,2018) uses for standard linear solid (SLS) type material. Here we found that the stress is maximum and minimum for ESF and LSF respectively but order of this magnitude is of $\mathbf{10^{-4}}$which is very less than the order of $\mathbf{10^{8}}$shown in Fig-7, which is drown by (Mondal etal., 2018) for standard linear solid (SLS) type material.

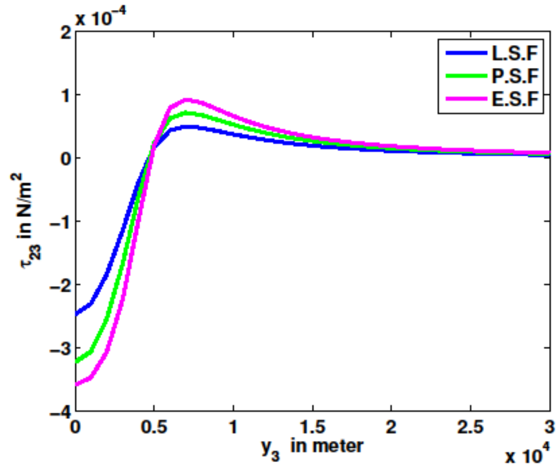

Fig-6

Stress $\boldsymbol{\tau_{23}}$ with $\boldsymbol{y_3}$ for Burger's Rheology of various slip function.

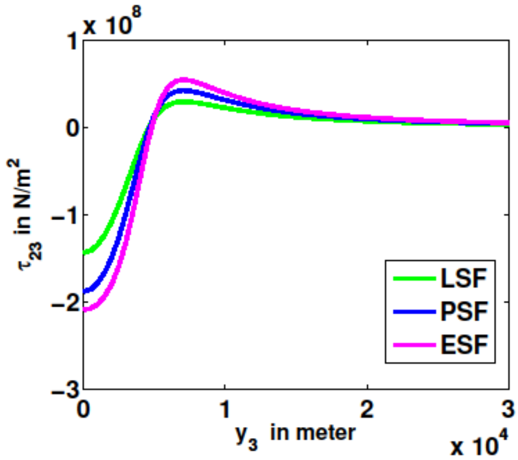

Fig-7

Stress $\boldsymbol{\tau_{23}}$ with $\boldsymbol{y_3}$for SLS of various slip function.

Normal stress for a fixed inclination with $\boldsymbol{y_2}$ for various$\boldsymbol{y_3}$, taking parabolic slip function and average slip magnitude has been shown in Fig-8. It is found that for $\boldsymbol{y_3}$ = 1 km, the normal stress $\boldsymbol{\tau_{33}}$increases sharply up to certain value of order $\mathbf{10^{-4}}$, after that it decreases rapidly and → 0 as expected. For $\boldsymbol{y_3}$= 3 km, there is gradual increase of stress and then it decreases. The rate of decrease of stress is higher for $\boldsymbol{y_3}$= 1 kmthan the other values of$\boldsymbol{y_3}$.

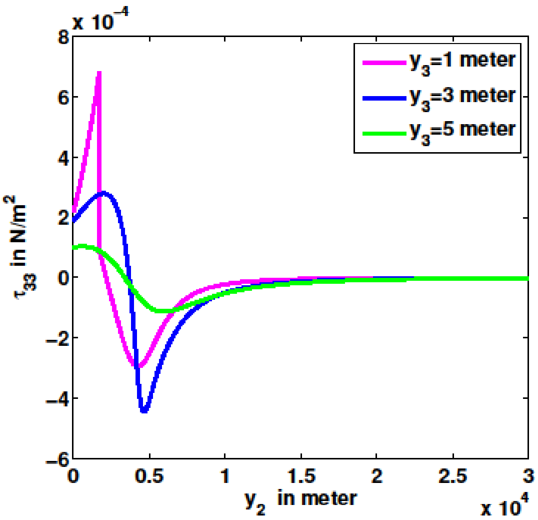

Fig-8
Normal stress against y_2 for Burger's Rheology of various y_3.

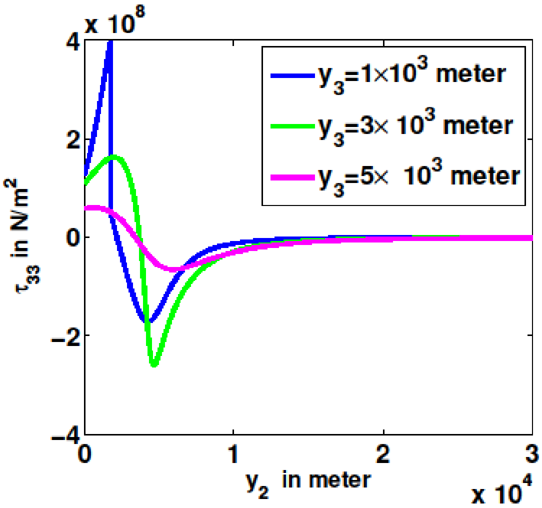

Fig-9
Normal stress against y_2 for SLS of various y_3.

The Fig-8 has been shown for Burger's Rheology but same normal stress has been plotted with the same fetcher for SLS in Fig-9. This shows that order of normal stress is of $\mathbf{10^8}$. Thus there is a clear effect on stress component of various medium.

IV. CONCLUSION

The contribution of the previous study (Huetal.2016)suggest that the Rheological properties of Burger's material can be a proper representation of the lithosphere-asthenosphere system.

Therefore, we derived analytical solutions for displacement, stress and strain due to faultmovement across an inclined, infinite, dip-slip fault situated in a viscoelastic half space. It is found from the above numerical computation hat these displacementand stresses depend on various inclinations and slip magnitude of the fault. In this article, a comparable discussion have been studied between SLS and Burger's Rheology type of material. The stress accumulation / release near the fault varies not only due to dip angle and slip magnitude but also on slip functions and viscoelastic material in the lithosphere- asthenosphere system. The values of the model parameters play an important role in determining the displacement and stress. The movement of fault causes stress accumulation /release near the fault which essentially depend on different positions of point on the fault for fixed width.

APPENDIX:
Taking the Laplace transform of all constitutive equations and boundary conditions, one can obtain

$$\overline{\tau_{22}} = \frac{(p_1+p_2p)(\tau_{22})_0}{(1+p_1p+p_2p^2)} + \frac{(q_1p+q_2p^2)\overline{\frac{\partial v}{\partial y_2}}}{(1+p_1p+p_2p^2)} - \frac{(q_1+q_2p)(\frac{\partial v}{\partial y_2})_0}{(1+p_1p+p_2p^2)} \qquad (9)$$

and similar other equation for $\tau_{23}$and τ_{33}.
Also we have the boundary condition in transform domain as:

$$\left.\begin{array}{l}\overline{\tau_{22}} = \frac{\tau_\infty \cos\theta}{p} \text{as } |y_2| \to \infty, \\ \overline{\tau_{23}} = 0 \text{as } y_3 \to \infty, \\ \overline{\tau_{33}} = \frac{\tau_\infty \sin\theta}{p} \text{as } |y_2| \to \infty, \\ (y_3 \geq 0, t \geq 0)\end{array}\right\} \qquad (10)$$

Here τ_∞ is the constant value of τ_∞(t), where $\overline{\tau_{22}}= \int_0^\infty \tau_{22} e^{-pt}\, dt$, p being Laplace transform variable. We have equation (4), in transform domain it can be written as,
$\nabla^2 \overline{V} = 0$, where $\overline{V} = \overline{v} - \frac{v_0}{p}$.
$\nabla^2 \overline{W} = 0$, where $\overline{W} = \overline{w} - \frac{w_0}{p}$.

We solve this governing Laplace equation with the boundaryconditions (3) and (7).
To solve this problem, one can assume that $\overline{v}$, $\overline{w}$ have the form $\overline{v} = \frac{v_0}{p}$+A y_2+By_3 and$\overline{w}= \frac{w_0}{p}$+C y_2+Dy_3. Using the initial and boundary conditions and taking inverse Laplace transform, the solution of displacement, stress and strain before any fault movement is given by the equation (5). After the fault movement, an additional uniform dislocation condition which characterizes the sudden movement across F is given by equation (6). Taking L.T. of (6), we get

$$\overline{[w]} = \frac{U}{p}\text{f}(y'_3) \qquad (11)$$

All the basic equations and initial conditions are same as before after fault movement. The modified boundary conditions are given in equation (7).
We solved the above boundary value problem by modified Green's function method developed by(Maruyama, 1964), (Maruyama, 1966), (Rybicki,1986),(Rybicki, 1971) and correspondence principle.Let Q(y_2, y_3) be any point in the medium andP(ζ_2, ζ_3) be any point on the fault F, then we have

$$\overline{w}(Q) = \int_F \overline{w}(P)\, G(P,Q) \qquad (12)$$

where G(P, Q) = $G_{32}{}^3(P,Q)d\zeta_3 - G_{33}{}^3(P,Q)d\zeta_2$and $G_{32}{}^3(P,Q), G_{33}{}^3(P,Q)$are given by

$$G_{32}{}^3(P,Q) = \frac{[\frac{y_2-\zeta_2}{L^2} + \frac{y_2-\zeta_2}{M^2}]}{2\pi}$$

$$G_{33}{}^3(P,Q) = \frac{[\frac{y_3-\zeta_3}{L^2} - \frac{y_2+\zeta_3}{M^2}]}{2\pi}$$

$L^2 = (y_2-\zeta_2)^2 + (y_3-\zeta_3)^2$,
$M^2 = (y_2-\zeta_2)^2 + (y_3+\zeta_3)^2$.

As(ζ_2, ζ_3)being a point on F, 0≤ $\zeta_2 \leq D\cos\theta$, 0≤ $\zeta_2 \leq D \leq \sin\theta$and$\zeta_2 = \zeta_3 \cot\theta$. A change in the coordinate axis from (ζ_2, ζ_3) to (ζ'_2, ζ'_3) is connected by the relation:
$\zeta_2 = \zeta'_2 \sin\theta + \zeta'_3 \cos\theta$,
$\zeta_3 = -\zeta'_2 \cos\theta + \zeta'_3 \sin\theta$,so that $\zeta'_2 = 0$, 0 ≤ ζ'_3 ≤D on F. Therefore,
$d\zeta_2 = \cos\theta\, d\zeta'_3$, $d\zeta_3 = \sin\theta\, d\zeta'_3$.
Thus,

$$\overline{w}(Q)= \frac{U}{2\pi p}\int_0^D [\frac{y_2\sin\theta - y_3\cos\theta}{L^2} + \frac{y_2\sin\theta + y_3\cos\theta}{M^2}]\text{f}(\zeta'_3)d\zeta'_3$$

Taking the inverse Laplace transform, we get

$$\text{w(Q)} = \frac{U}{2\pi}\varphi(y_2, y_3)\, \text{H}(t_1) \qquad (13)$$

where $\varphi(\boldsymbol{y_2},\boldsymbol{y_3})$
$=\int_{\boldsymbol{0}}^{\boldsymbol{D}}[\frac{y_2\sin\theta-y_3\cos\theta}{L^2}+\frac{y_2\sin\theta+y_3\cos\theta}{M^2}]f(\boldsymbol{\zeta'_3})\boldsymbol{d\zeta'_3}$
and$\boldsymbol{L^2}=\boldsymbol{\zeta'^2_3}-\boldsymbol{2\zeta'_3}(\boldsymbol{y_2}\cos\boldsymbol{\theta}+\boldsymbol{y_3}\sin\boldsymbol{\theta})+\boldsymbol{y^2_2}+\boldsymbol{y^2_3}$, $\boldsymbol{M^2}=\boldsymbol{\zeta'^2_3}-\boldsymbol{2\zeta'_3}(\boldsymbol{y_2}\cos\boldsymbol{\theta}-\boldsymbol{y_3}\sin\boldsymbol{\theta})+\boldsymbol{y^2_2}+\boldsymbol{y^2_3}$.
It is to be noted that $\boldsymbol{w}$ = 0 for $\boldsymbol{t_1}.=\boldsymbol{t}-\boldsymbol{T}\leq\boldsymbol{0}$.
From the equation (9), (12) and assuming displacement, stress and strain to be zero for $\boldsymbol{t_1}=\boldsymbol{t}-\boldsymbol{T}<0$, we have $\overline{\boldsymbol{\tau_{22}}}$ = 0. We note that v is continuous even after the fault slip so that v = 0 after fault movement. Taking inverse Laplace transform, $\boldsymbol{\tau_{22}}$ = 0. Similarly other equations are as follows:
$\tau_{23}=\frac{U}{2\pi A}[(q_1-q_2r_1)e^{-r_1t}-(q_1-q_2r_2)e^{-r_2t}]H(t_1)\varphi_1(y_2,y_3)$,
$\tau_{33}=\frac{U}{2\pi A}[(q_1-q_2r_1)e^{-r_1t}-(q_1-q_2r_2)e^{-r_2t}]H(t_1)\varphi_2(y_2,y_3)$,$E_{22}=0$,
$E_{23}=\frac{U}{4\pi}H(t_1)\varphi_1(y_2,y_3)$, $E_{33}=\frac{U}{2\pi}H(t_1)\varphi_2(y_2,y_3)$
Where$\boldsymbol{\varphi_1}(\boldsymbol{y_2},\boldsymbol{y_3})=\frac{\partial\varphi}{\partial y_2}$, $\boldsymbol{\varphi_2}(\boldsymbol{y_2},\boldsymbol{y_3})=\frac{\partial\varphi}{\partial y_3}$ which have the expression in the appendix of the paper of (Mondal et al.,2018)

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