

Control Chart Performance in Monitoring Manufacturing Processes

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ABSTRACT: Manufacturing processes that consist of time series data are frequently monitored by forecast-based quality control schemes. These control schemes are based on the application of a time series forecast to the process and monitoring the resultant forecast errors with a control chart. This study compares the performance of the Individuals control chart, the Cumulative Sum (CUSUM) and the Exponentially Weighted (EWMA) chart in their ability to detect the presence of changes in the process mean (step shift) and additive outliers in an autocorrelated process. The criteria used are the Average Run Length (ARL) and Cumulative Distribution Function (CDF) of the run lengths. The CDF is offered as an alternative performance evaluation criterion, for forecast-based schemes. The Individuals chart offers the greatest probability of early detection of a step shift and an additive outlier in an autocorrelated process, based on the CDF criterion.

Keywords: *control charts, process control, quality engineering.*

I. INTRODUCTION

The occurrence of large unusual observations is not uncommon in time series data. These outliers may be due to recording errors or to one-time unique situations such as an unexpected change in demand for a product or a change in a production system. Fox (1972) defines two types of outliers may occur in practice. An additive outlier corresponds to an external disturbance that affects the value of a single observation. An innovational outlier refers to an internal disturbance that changes the value of an observation and all other successive observations. Typically, in process control environments, monitoring schemes are compared based on their ability to detect step shifts or innovational outliers in the level of a process. However, which monitoring scheme detects the presence of an additive outlier most quickly is also of interest.

Autocorrelation implies the existence of a relationship between consecutive observations and can be of two types. A process that tends to drift over time is characteristic of positive autocorrelation and results when successive observations are similar in value. Negative autocorrelation is depicted by a sawtooth pattern and results when consecutive observations are dissimilar. High volume manufacturing processes along with an increased frequency of sampling by automated gages, gives rise to autocorrelated data.

The presence of autocorrelation creates unique problems for process monitoring schemes. Positive autocorrelation tends to increase the frequency of out-of-control signals that are detected by monitoring schemes. Positive autocorrelation occurs most often in production environments and chemical operations (Woodall and Faltin (1993)).

Alwan and Roberts (1988) have proposed a method for monitoring autocorrelated data that involves the application of a time-series forecast to the process and monitoring the forecast errors. Unusual behavior in the process should result in a large error that is reflected as a signal on a control chart or tracking signal.

Traditionally, monitoring tools have been compared on the basis of Average Run Lengths (ARLs). The ARL is the expected number of observations required to detect an out-of-control situation. However, simple exponential smoothing forecasts recover quickly from step increases in the time series process that it monitors. This would suggest that the performance of forecast-based schemes should be based on the probability of "early detection". As an average measure that is inflated by long run lengths, the ARL is an inadequate measure of quick recovery, that is characterized by short run lengths. Hence the cumulative distribution function (CDF) of the run lengths is offered as an alternative criterion to the average run length (ARL) for the selection of an appropriate monitoring scheme. The CDF provides the cumulative probability of a signal occurring by the i th time period after a disturbance.

This paper compares the performance of control charts in monitoring forecast errors from exponential smoothing forecasts applied to autoregressive process data of order one, denoted by AR (1), in the presence of a changes in the process mean and additive outliers. Superville and Yorke (2012) compared the performance on control charts in detecting additive outliers. The study shows that the ETS tracking signal offers the highest probability of early detection of a shift in the mean and an additive outlier in AR (1) manufacturing processes.

II. LITERATURE REVIEW

The presence of autocorrelation creates unique problems for process monitoring schemes. Positive autocorrelation tends to increase the frequency of out-of-control signals that are detected by monitoring schemes. Positive autocorrelation occurs most often in production environments and chemical operations (Woodall and Faltin (1993)).

The performance of control charts in the presence of autocorrelation has been explored by several authors. Superville and Adams (1994) compared the performance of an Individuals Chart, a Cumulative Sum (CUSUM) and Exponentially Weighted Moving Average (EWMA) Chart in their ability to detect step shifts in autocorrelated process. Superville and Adams (1995) compared the performance of these charts to tracking signals in their ability to detect step shifts in autocorrelated process. Lu and Reynolds (1999) suggest the use of a combined Shewhart-EWMA for autocorrelated data. Lianjie, Daniel and Fugee (2002) suggest the use of a triggered CUSCORE on residuals. Lee et al. (2009) propose distribution-free charts for monitoring shifts in the mean of autocorrelated processes. Wu and Yu (2010) advocate a neural network approach for monitoring the mean and variance of an autocorrelated process. Chang and Wu (2011) suggest a Markov Chain approach to calculating the ARL for control charts on autocorrelated process data. Superville and Yorke (2012) compare the performance of Individuals, CUSUM and EWMA Charts in detecting additive outliers from an autocorrelated process.

In the field of statistical process control (SPC), control charts have traditionally been used to monitor production processes. In the forecasting and time series fields, tracking signals perform a similar function, the monitoring of forecasting systems. The statistical tools are similar in that both are designed to monitor systems and provide information concerning changes in the systems.

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Traditionally, monitoring tools have been compared on the basis of Average Run Lengths (ARLs). The ARL is the expected number of observations required to detect an out-of-control situation. However, simple exponential smoothing forecasts recover quickly from step increases in the time series process that it monitors. This would suggest that the performance of forecast-based schemes should be based on the probability of "early detection". As an average measure that is inflated by long run lengths, the ARL is an inadequate measure of quick recovery, that is characterized by short run lengths. Hence the cumulative distribution function (CDF) of the run lengths is offered as an alternative criterion to the average run length (ARL) for the selection of an appropriate monitoring scheme. The CDF provides the cumulative probability of a signal occurring by the i th time period after a disturbance.

III. A MODEL FOR AUTOCORRELATED DATA

A time series model that has been found to be useful in production and quality control environments is the ARIMA(1,0,0), referred to as the first-order autoregressive model and denoted by AR(1). It is represented by

$$X_t = \xi + \phi X_{t-1} + \varepsilon_t \quad (1)$$

Without loss of generality it is assumed that $\varepsilon_t \sim N(0,1)$. It is also assumed that an AR (1) model is applicable in this article. Montgomery and Mastrangelo (1991) show that a number of chemical and manufacturing processes conform to this model.

The simple exponential smoothing forecast, also known as an exponentially weighted moving average (EWMA) forecast is given by

$$F_{t+1} = \alpha_F X_t + (1-\alpha_F)F_t, \quad 0 \leq \alpha_F \leq 1 \quad (2)$$

where X_t represents the process observation at time period t , and F_{t+1} represents the one-step-ahead forecast for observation X_{t+1} at time period t . The forecast error at time period t , denoted by e_t , is defined as

$$e_t = X_t - F_t \quad (3)$$

Alwan and Roberts (1988) have observed that processes that do not drift too rapidly are well modeled by simple exponential smoothing. For the AR(1) model, Cox (1961) has shown that optimal simple exponential smoothing in terms of minimum mean square forecast error is given by

$$\alpha_F = 1 - \frac{1}{2}[(1-\phi)/\phi], \quad 1/3 < \phi \leq 1 \quad (4)$$

where ϕ is the parameter of the AR(1) process. The simulation study on which this article is based, rely on this result.

IV. FORECAST-BASED QUALITY CONTROL SCHEMES

In this study, the Individuals, Cumulative Sum (CUSUM) and EWMA control charts are applied to exponential smoothing forecast errors and their performances evaluated.

The Individuals Control Chart

The Individuals control chart applied to forecast errors requires an estimate of the variance of the forecast errors. Defining the i th moving range to be

$$MR_i = |e_i - e_{i-1}|, \quad i = 2, 3, \dots, m \quad (5)$$

and

$$\overline{MR} = \frac{1}{m-1} \sum_{i=2}^m MR_i, \quad (6)$$

the control limits are

$$\bar{X} \pm C_1 MR/d_2 \quad (7)$$

where the constant C_1 is set to achieve a desired in-control ARL. Montgomery (1991) has tabulated values for C_1 and d_2 .

The Cumulative Sum Control Chart

An alternative to the Shewhart control chart is the Cumulative Sum (CUSUM) control chart. The CUSUM control chart may be represented by either a V-mask representation or equivalently by the use of two one-sided cumulative sums.

The V-mask form of the CUSUM applied to forecast errors requires plotting the quantity

$$S_i' = \sum_{j=1}^i e_j, \quad i = 1, 2, \dots \quad (8)$$

against the sample number i .

The 'two one-sided cumulative sums' procedure also known as the Tabular CUSUM requires calculating:

$$S_i = \max 0, (e_i/\sigma_e) - K + S_{i-1} \quad (9)$$

$$T_i = \min 0, (e_i/\sigma_e) + K + T_{i-1} \quad (10)$$

where $S_0 = w$ and $T_0 = -w$ ($0 \leq w < K$). The value σ_e represent the standard deviation of the forecast errors, which is typically estimated in practice. A head start value, w , is recommended for earlier detection of out of control situations. In this study $w = 0$ is used. The reference value K is usually set to be $\delta/2$, where δ is the smallest shift in the mean (measured in forecast error standard deviations) considered important to be detected quickly. If $S_i > h$ or $T_i < -h$ (where h is a critical value) the chart signals. The critical values, h used in this study were determined through simulation.

The Exponentially Weighted Moving Average Control Chart

The Exponentially Weighted Moving Average (EWMA) control chart is another alternative to the Shewhart control chart that has been found to be more sensitive to small process disturbances. Also known as a Geometric Moving Average control chart, the EWMA applied to forecast errors is a weighted average of past and present data given by

$$Z_i = \lambda e_i + (1-\lambda)Z_{i-1}, \quad i = 1, 2, \dots \quad (11)$$

where λ ($0 < \lambda < 1$) is a smoothing constant and $Z_0 = 0$ usually. Assuming that the process is in control and the observations are independent then

$$\sigma^2_{Z_i} = \frac{\sigma^2}{n} \frac{\lambda}{2-\lambda} [1-(1-\lambda)^{2i}] \quad (12)$$

with control limits determined by

$$\bar{X} \pm c\sigma_{Z_i} \quad (13)$$

where c is a constant designed to achieve a desired in-control ARL. The value of c required for this study was determined through simulation. Note if $\lambda=1$, the EWMA control chart becomes a Shewhart control chart.

V. EVALUATION CRITERIA: ARL vs. CDF

The ARL is a criterion on which the relative performance of both tracking signals has been based. However, exponentially smoothed forecasts tend to recover quickly from disturbances in the time series that it monitors. In general, the rate of forecast recovery depends on the type of shift, the underlying model and the forecasting tool in use. In most cases, forecast recovery is shown to significantly impact ARLs.

The necessity of quick detection of process shifts leads one to the cumulative probability of a signal following a process shift as a meaningful criterion for the comparison of forecast-based monitoring schemes. The use of the cumulative distribution functions (CDF) as an evaluation criterion is not new. Barnard (1959), Bissell (1968) and Gan (1991) recommend its use on independent observations. Referred to as a 'response to a change in demand', McClain (1988) advocates its use for forecast-based schemes which incorporate tracking signals. The CDF measures the cumulative percentage of disturbances in a time series that are detected early.

VI. DESIGN OF THE SIMULATION STUDY

In this simulation study, the control charts are compared. They are the Individuals, CUSUM and EWMA control charts. ARLs and CDFs are provided for each monitoring scheme for step shifts of size 0.0 , $1\sigma_p$ and $3\sigma_p$ and outliers of size $3.0\sigma_p$, where $\sigma^2_p = \sigma^2 / (1-\phi^2)$, is the variance of an AR(1) process. The simulation study was conducted as follows:

- i) AR(1) series with autoregressive parameter values of $\phi = 0.0, 0.5, 0.7, \text{ and } 0.9$ and $N(0,1)$ are generated by the IMSL (1991, p.1350-1351) subroutine RNARM / DRNARM. The parameter ξ was set equal to zero, without loss of generality,
- ii) the first fifty observations are used to allow for a burn-in period,
- iii) the forecast is started at time period 2 with its initial value set equal to the first observed data point,
- iv) fifty (50) preliminary sequences of forecast errors are used to estimate the variance of the forecast errors for a step increase of zero (the in-control state),
- v) control charts are constructed based on the estimates obtained in step (iv). The initial MAD values are set to $0.8\sigma_e$ (σ_e is the standard deviation of the forecast errors) as suggested by Montgomery, Johnson and Gardiner (1990),
- vi) the monitoring schemes are applied to the forecast errors,
- vii) steps (i)-(iii), (v) and (vi) are repeated 1000 times. For each monitoring scheme, the run length for each simulation iteration is recorded. These run lengths are used to obtain the ARLs and CDFs after a shift of size 0.0 , $1\sigma_p$ and $3\sigma_p$ and then an additive outlier of size $3.0\sigma_p$.

VII. SIMULATION RESULTS

Table I displays simulated ARLs and CDFs for Individuals, CUSUM and EWMA control charts applied to the optimal exponential smoothing forecast errors from an AR(1) process with ϕ ranging from 0.0 to 0.9 , after a shift of size 0.0 , $1\sigma_p$ and $3\sigma_p$. Table 2 displays simulated ARLs and CDFs for Individuals, CUSUM and EWMA control charts applied to the optimal exponential smoothing forecast errors from an AR(1) process with ϕ ranging from 0.0 to 0.9 , with outliers simulated as $3\sigma_p$.

TABLE I. Average Run Lengths and Percentage of Signals detected by the i th observation after a shift of size Δ . Forecast errors from AR(1) processes with autoregressive parameters ϕ and an in-control ARL of 250.

ϕ	Δ	Monitoring Scheme	ARL	Number of time periods after an outlier						
				1	2	3	4	5	6	
0	0.0	Individuals	250	0.3	0.7	1.1	1.4	1.7	2.3	
		CUSUM	252	1.0	1.6	2.7	3.4	3.9	4.1	
		EWMA	250	0.9	1.5	1.7	2.4	2.8	3.3	
	1.0	Individuals								
		CUSUM	33.8	3.5	5.3	7.4	10.6	13.1	14.9	
		EWMA	8.4	2.8	6.2	13.6	23.5	33.1	43.3	
	3.0	Individuals								
		CUSUM	8.7	1.7	5.0	8.9	17.2	26.1	35.4	
		EWMA								
	0.5	1.0	Individuals	242.4	2.2	3.0	3.4	3.8	3.9	4.0
			CUSUM	235.9	1.5	3.4	4.4	4.9	6.1	6.6
			EWMA	234.7	1.7	3.8	6.0	7.5	9.1	10.2
3.0		Individuals								
		CUSUM	133.2	45.5	48.5	49.8	50.1	50.4	50.6	
		EWMA	85.1	17.6	42.1	53.8	61.7	65.5	67.2	
0.7		1.0	Individuals	241.3	2.6	3.2	3.6	3.8	4.0	4.5
			CUSUM	237.6	1.4	2.5	3.3	3.7	4.9	5.4
			EWMA	220.2	1.9	3.7	5.1	6.6	7.6	8.0
		3.0	Individuals							
			CUSUM	129.7	47.4	48.2	48.2	48.4	48.6	48.8
			EWMA	142.7	19.6	33.6	39.3	42.5	43.6	44.5
	0.9	1.0	Individuals	240.9	3.0	3.7	4.2	4.7	5.4	5.7
			CUSUM	239.5	1.5	2.6	3.5	4.4	5.0	5.2
			EWMA	236.7	1.1	2.2	3.1	3.7	4.3	4.4
		3.0	Individuals							
			CUSUM	123.7	50.0	50.2	50.4	50.6	50.8	50.9
			EWMA	171.1	19.0	24.7	26.7	27.6	28.7	29.1
3.0		Individuals	147.1	13.4	17.3	19.2	20.6	22.8	23.4	

For step shifts (Table 1), the results may be summarized as follows:

1. For step shifts of 1σ and 3σ , the ARLs for the autocorrelated cases ($\phi > 0$) are substantially larger for the independent case ($\phi = 0$). This is a result of the quick recovery of the forecast illustrated by short run length and the inability of ARLs to adequately reflect these short run lengths. In the calculation of the ARL, longer run lengths mask shorter run lengths.
2. Based on CDFs, the Individuals chart provides a higher probability of early detection of a step shift than the CUSUM or EWMA charts for the autocorrelated cases ($\phi > 0$). The detection of a step shift early is critical, since the forecast recovers quickly. This suggests the use of the Individuals chart for autocorrelated cases.
3. As with the independent case, when the control charts are applied to the forecast residuals for the autocorrelated cases ($\phi > 0$), the CUSUM and EWMA charts perform better for detecting small process shifts while the Individuals chart performs better for larger shifts. The differences in the abilities of the CUSUM and EWMA charts and the Individuals chart to detect small shifts is negligible.

For outliers (Table II), the results may be summarized as follows:

1. With the exception of the case where $\phi = 0.9$, the magnitude of the ARLs for the autocorrelated cases ($\phi > 0$) are significantly larger than for the independent case ($\phi = 0$). The difference in ARL magnitudes can be attributed to the quick recovery of the EWMA forecast. Recall that the ARL, as an average measure, is inflated by long run lengths. It is unable to adequately reflect short run lengths that are indicative of quick forecast recovery. For forecast-based schemes, ARLs are not informative.

2. Based on CDFs, the Individuals control chart provides a higher probability of early detection of an outlier for the autocorrelated cases where $\phi=0.5$ and 0.7 . This occurs although the Individuals control chart may have a longer ARL than any other monitoring scheme. As an example, consider the case where $\phi=0.5$. The Individuals control chart provides a higher probability of early detection on the first observation after the outlier (60.5%) despite having a longer ARL (92.7) than the other monitoring schemes. The detection of an outlier early, that is, within the first few observations after the occurrence of an outlier is critical since the forecast recovers quickly. This suggests the use of the Individuals control chart for the autocorrelated cases.

TABLE II. Average Run Lengths and Percentage of Signals detected by the i th observation after an outlier of size $3\sigma_p$. Residuals are from AR(1) processes with autoregressive parameters ϕ and an in-control ARL of 250.

ϕ	Monitoring Scheme	ARL	Number of time periods after an outlier					
			1	2	3	4	5	6
0	Individuals	1.8	54.2	79.0	90.5	96.1	98.2	98.9
	CUSUM	2.2	11.5	73.3	97.4	99.9	100	100
	EWMA	2.5	12.9	52.5	86.7	97.5	99.4	100
0.5	Individuals	92.7	60.5	64.0	64.3	64.8	64.9	65.2
	CUSUM	49.4	27.0	60.6	72.5	77.0	78.4	80.4
	EWMA	12.4	26.8	64.6	77.7	83.4	88.3	90.8
0.7	Individuals	42.9	84.9	85.2	85.4	85.4	85.5	85.5
	CUSUM	47.9	56.3	72.1	76.3	79.4	80.8	81.3
	EWMA	19.8	49.3	68.2	75.8	79.7	82.9	85.0
0.9	Individuals	1.0	100	100	100	100	100	100
	CUSUM	2.5	99.3	99.4	99.5	99.6	99.6	99.6
	EWMA	10.5	85.0	88.4	88.8	89.4	89.8	90.1

VIII. CONCLUSIONS

This paper has compared forecast-based quality control schemes for monitoring autocorrelated observations in the presence of additive outliers. The quick recovery property of forecasting tools suggests that comparisons of control charts and tracking signals applied to forecast errors be based on the CDF on the run lengths and not on the ARL. The Individuals control chart is recommended over the CUSUM and EWMA control charts as it offers the highest probability of early detection of a step shift and an additive outlier in an AR(1) process.

REFERENCES

- [1]. Alwan, L, and Roberts, H. V. (1988). "Time Series Modeling for Statistical Process Control," *Journal of Business and Economic Statistics*, 6, 87-95.
- [2]. Barnard, G. A. (1959). "Control Charts and Stochastic Processes," *Journal of the Royal Statistical Society*, B21, 239-271.
- [3]. Bissell, A. F. (1968). "CUSUM Techniques for Quality Control," *Applied Statistics*, 18, 1-30.
- [4]. Brown, R. G. (1959). *Statistical Forecasting for Inventory Control*, McGraw-Hill, New York, NY.
- [5]. Change, Y.M and Wu, T.L. (2011). "On Average Run Length of Control Charts for Autocorrelated Processes," *Methodology and Computing in Applied Probability*, 13(2), 419-431.
- [6]. Cox, D. R. (1961). "Prediction by Exponentially Weighted Moving Average and Related Methods," *Journal of the Royal Statistical Society*, B23, 414-422.
- [7]. Fox, A. J. (1972). "Outliers in Time Series," *Journal of the Royal Statistical Society*, Series B, 3, 350-363.
- [8]. Gan, F. F. (1991). "Computing the Percentage Points of the Run Length Distribution of an EWMA Control Chart," *Journal of Quality Technology*, 23, 359-365.
- [9]. Gardner, E. S. Jr. (1983). "Automatic Monitoring of Residuals," *Journal of Forecasting*, 2, 1-21.
- [10]. Gardner, E. S. Jr. (1985). "CUSUM vs. Smoothed-Error Forecast Monitoring Schemes: Some Simulation Results," *Journal of the Operation Research Society*, 36, 43-47.
- [11]. International Mathematical and Statistical Libraries, Inc. (IMSL) (2010). *User's Manual Stat/Library*, (Version 7.0), Houston, TX.
- [12]. Lee, J, Alexopoulos, C, Goldsman, D, Kim, S.H, Tsui, K.L, and Wilson, J.R. (2009). "Monitoring Autocorrelated Processes Using A Distribution-Free tabular CUSUM Chart with Automated Variance Estimation," *IIE Transactions*, 41(11), 979-994.

- [13]. Lianjie, S, Daniel, W.A and Fugee, T. (2002). "Autocorrelated Process Monitoring Using Triggered CUSCORE Charts," *Quality and Reliability Engineering International*, 18, 411-421.
- [14]. Lu, C.W and Reynolds, M.R, Jr. (1999). "Control Charts for Monitoring the Mean and Variance of Autocorrelated Process," *Journal of Quality Technology*, 31(3), 259-274.
- [15]. McClain, J. O. (1988). "Dominant Tracking Signals," *International Journal of Forecasting*, 4, 563-572.
- [16]. McKenzie, E. (1978). "The Monitoring of Exponentially Weighted Forecasts," *Journal of Operation Research Society*, 29, 449-458.
- [17]. Montgomery, D. C. (1991). *Introduction to Statistical Quality Control*, 2nd Edition. John Wiley and Sons, Inc., New York, NY.
- [18]. Montgomery, D. C., Johnson, A. L., and Gardiner, J. A. (1990). *Forecasting and Time Series Analysis*, 2nd Edition. McGraw-Hill, Inc., New York, NY.
- [19]. Montgomery, D. C., and Mastrangelo, C. M. (1991). "Some Statistical Process Control Methods for Autocorrelated Data," (with discussion), *Journal of Quality Technology*, 23, 179-204.
- [20]. Superville, C. R., and Adams, B. M. (1994). "An Evaluation of Forecast-Based Quality Control Schemes," *Communications in Statistics - Simulation and Computation*, 23(3), 645-661.
- [21]. Superville, C. R., and Adams, B. M. (1995). "The Performance of Control Charts and Tracking Signals in Process Monitoring" *International Journal of Operations and Quantitative Management*, 1(2),131-143.
- [22]. Superville, C. R., and Yorke, G. (2012). "Outliers and Production Forecasts," *Global Education Journal*, 2012(1), 98-106.
- [23]. Trigg, D. W. (1964). "Monitoring a Forecasting System," *Operational Research Quarterly*, 15, 271-274.
- [24]. Wardell, D.G, Moskowitz, H. and Plante, R. D. (1994). "Run Length Distributions of Special Cause Control Charts for Correlated Data," *Technometrics*, 36(1), 2-19.
- [25]. Woodall, W. H., and Faltin, F. W. (1993). "Autocorrelated Data and SPC," *American Society for Quality Control (ASQC) Statistics Division Newsletter*, 13, 18-21.
- [26]. Wu, B. and Yu, J-bo. (2010). "A Neural Network Ensemble Model for On-Line Monitoring of Process Mean and Variance Shifts in Correlated Process," *Expert Systems with Applications*, 37(6), 4058-4056.

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