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Mathcad: Teaching and Learning Tool for Biaxial Reinforced Column Design

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Abstract: Mathcad is a sophisticated computation and presentation tool, which is versatile, easy to use, and accessible. It holds strong potential as a learning aid for education and training. This paper demonstrates the use of Mathcad to supplement and enhance traditional teaching and learning methods both inside and outside the classroom. The paper focuses on the topic of reinforced concrete biaxial column design. Interactive teaching and learning devices in reinforced concrete biaxial column deign produced using the presentation and programming features available in Mathcad. This paper also compares the results of the Mathcad for the biaxial column with the computer software “SP Column” and the results obtained from the Mathcad program are quite close to the ones obtained from the computer software.

Keywords: Biaxial Column Design, learning methods, Mathcad, SP Column.

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I. INTRODUCTION

Mathcad [1] is an efficient learning environment for technical topics such as reinforced concrete design. It’s computational and presentation capabilities not only lend themselves to the solution of mathematically based problems, but also to the effective communication of both problem and solution. Mathcad contains powerful presentation capabilities, which include the use of charts, graphic objects, and animation effects. It can also easily import objects from other application programs, such as images and digital photographs. These capabilities offer significant learning enhancements to students of technical subjects [2].

Mathcad makes possible new learning strategies for students and teachers [3-4]. What-if discussions, trend analyses, trial and error analyses, and optimization are all valuable learning activities, which take more time than the traditional technical problem-solving approach permits. Taking advantage of the computational power and speed of Mathcad, instructors and students can quickly cycle through problem scenarios, observing trends in the design behavior of reinforced concrete components.

The proposed paper describes the use of Mathcad program as a teaching and learning tool in reinforced concrete design courses. A program for the design of reinforced concrete bi-axial columns discussed and demonstrated to show the attractive computational environment of Mathcad and compares the results for the biaxial column with the computer-aided software “SP Column” [5].This program will also help to illustrate its importance as a teaching and learning tool for Civil Engineering students.

1.1 Overview of Reinforced Concrete Column Design

Columns are vertical compression members, which transmit loads from the upper floors to the lower levels and to the soil through the foundations. Based on the position of the load on the cross section, columns are classified as concentrically loaded, Figure 1, or eccentrically loaded, Figure 2.

Eccentrically loaded columns are subjected to moments, in addition to axial force. The moments can be converted to a load **P** and eccentricity $\mathbf{e_X}$ and $\mathbf{e_Y}$. The moments can be uniaxial, as in the case when two adjacent panels are not similarly loaded, such as columns **A** and **B** in Figure 3.A column is considered as bi-axially loaded when the bending occurs about the X and Y axes, such as in the case of corner column **C** in Figure 3.

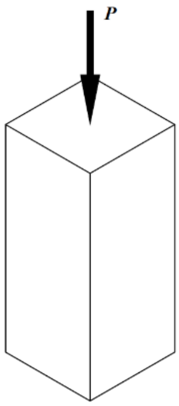

Figure 1: Concentrically Loaded columns

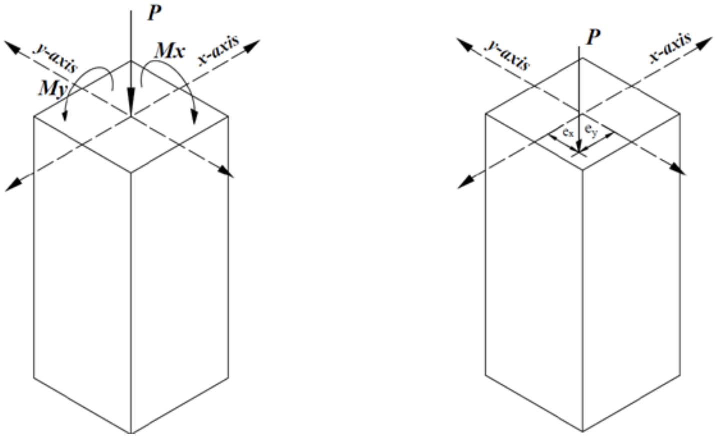

Figure 2: Eccentrically Loaded Column

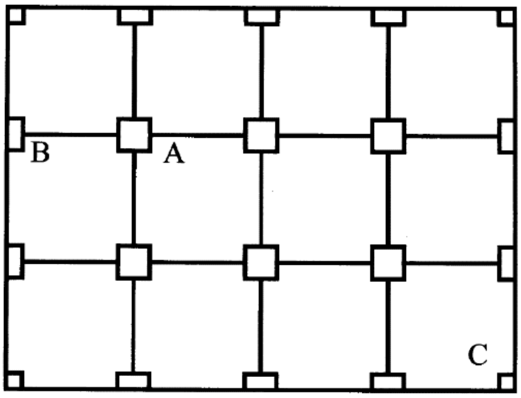

Figure 3: Uniaxially and Biaxially Loaded column

The strength of reinforced concrete columns is determined using the following principles:

1. A linear strain distribution exists across the thickness of the column
2. There is no slippage between the concrete and the steel
3. The concrete strain at failure for strength calculations is set equal to 0.003mm/mm.
4. The tensile resistance of the concrete is negligible and disregarded.

The strength of reinforced concrete columns is usually expressed using interaction diagrams to relate the design axial load $\emptyset P_n$ to the design bending moment $\emptyset M_n$. Figure 4 explains the control points for the column interaction curve ($\emptyset P_n - \emptyset M_n$).Each point on the curve represents one combination of design axial load $\emptyset P_n$ and design bending moment $\emptyset M_n$ corresponding to a neutral-axis location. The interaction diagram is separated into a tension control region and a compression control region. The balanced condition occurs when the failure develops simultaneously in tension (i.e., steel yielding) and in compression (concrete crushing).

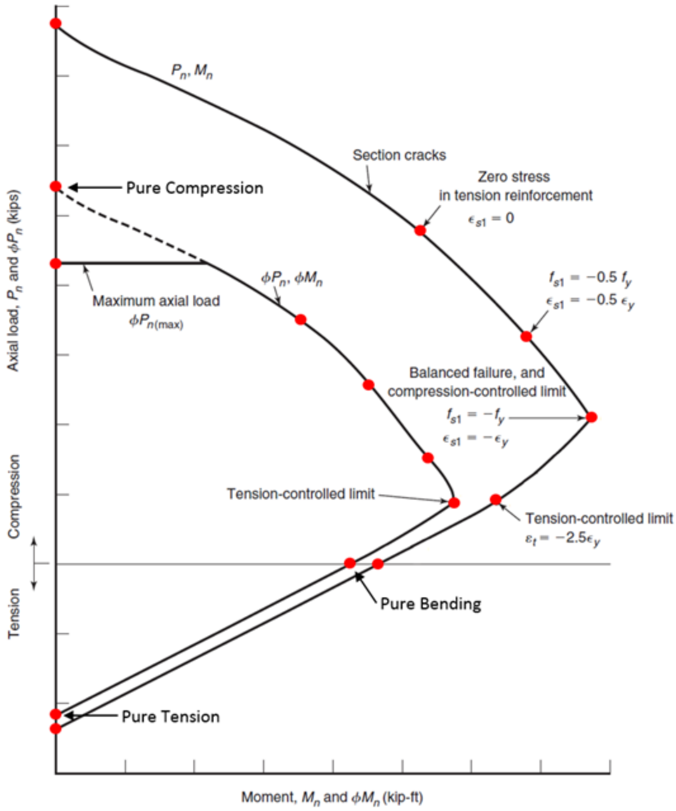

Figure 4: Control Points for Colum Interaction Curve ($\boldsymbol{\emptyset P_n - \emptyset M_n}$) [6]

Figure 5 displays the interaction curve for the biaxial column. The strength of the biaxial column can be analyzed by using Bresler's Formula[7](Equation-1) to check if the obtained values of $\emptyset P_n$ and $\emptyset M_n$ are larger or equal to the values of P_u *and* M_u respectively. [8-19] explained the design of reinforced concrete biaxial column in detail.

$$P_c = \frac{1}{\frac{1}{\emptyset P_{ny}} + \frac{1}{\emptyset P_{nx}} - \frac{1}{\emptyset P_N}} \qquad (1)$$

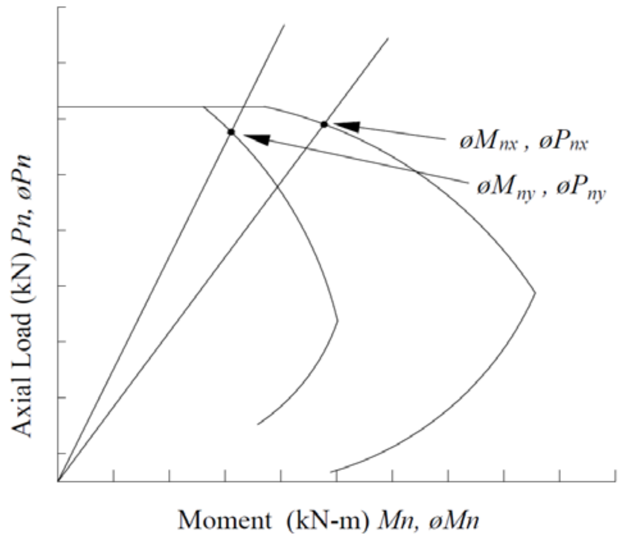

Figure 5: Biaxial Column Interaction Diagram

II. MATHCAD PROGRAM FOR REINFORCED CONCRETE DESIGN

A Mathcad program is written to automate the manual design of reinforced concrete columns.The program, which totally emulates the manual design procedure, consists of the following computational steps for X-X and Y-Y axis (Figure 6):

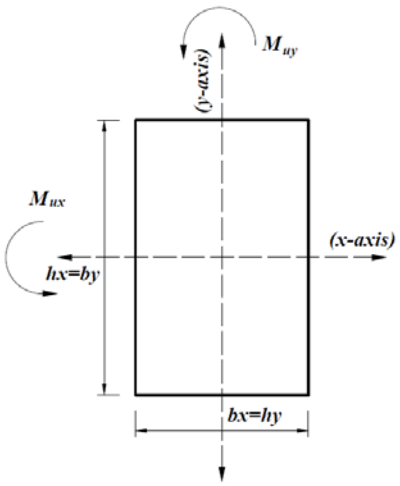

Figure 6: Biaxial column cross section

STEP-1:

The first step consists of reading the following input data (Figure 7):

1. The number of steel layers ***NSL***.
2. The area of steel in each layer A_{sj} (A_{sj}, j = 1..., *NS*L).
3. The distance d_j between each layer and the top column fiber (d_j, j = 1..., *NS*L).
4. The dimensions ***b*** and ***h***of the column.
5. The yield strength of steel ***fy***, the concrete compressive strength ***f'c***, and the steel modulus of elasticity ***Es***.
6. The factored load ***Pu*** and bending moment ***Mu***.
7. If the factored bending moment ***Mu*** is less than the minimum bending moment M_{min}, ***Mu*** is set equal to M_{min}. The minimum bending moment M_{min} is computed using the following equation:

$$M_{min} = 0.1hP_u \tag{2}$$

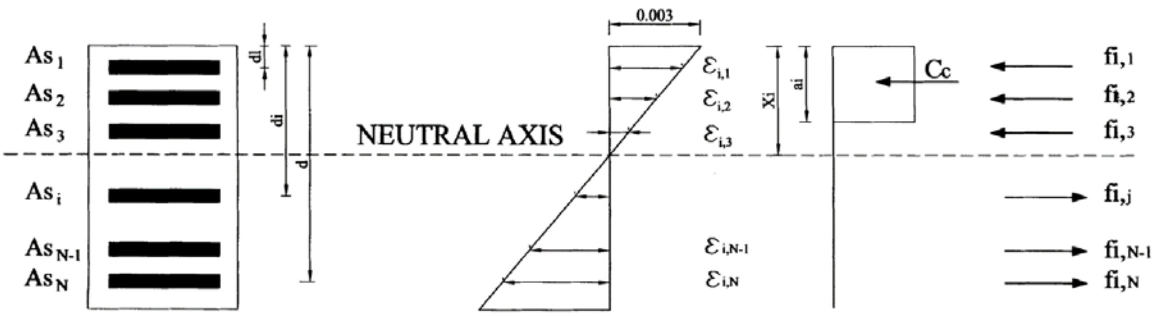

Figure 7: Reinforced concrete column strains and stresses

STEP-2:

In the second step, the plastic centroid ***Yp***, the reinforcement ratio ρ, and the parameter $\beta$are computed. The plastic centroid of the column cross section is computed using the following equation:

$$Y_p = \frac{\sum_{j=1}^{NSL}(As_j f_y d_j + 0.85 f_c' b \frac{h^2}{2})}{\sum_{j=1}^{NSL}(As_j f_y + 0.85 f_c' bh)} \tag{3}$$

$$y = if\left(Y_p \neq \frac{h}{2}, Y_p, \frac{h}{2}\right)$$

The reinforcement ratio $\rho$is determined using the following equation:

$$\rho = \frac{\sum_{j=1}^{NSL} As_j}{bd} \quad (4)$$

Finally, the parameter $\boldsymbol{\beta}$ is computed using the following equation:

$$\beta = 0.85 - 0.008(f_c' - 30) \leq 0.85 \quad (5)$$

$$\beta = if(\beta > 0.85, 0.85, \beta)$$

STEP-3:

The iterative procedure starts by selecting the first position of the neutral axis ***Xi*** (*Xi = i + d1, with i = 0*). Then, the parameter $\boldsymbol{a_i}$(depth of the compression block) is computed using the following equation:

$$a_i = \beta X_i \quad (6)$$

STEP-4:

The strain $\boldsymbol{\varepsilon_{i,j}}$in each reinforcing steel bar is determined by the linear strain distribution to ensure the strain compatibility (Figure 7). The strain $\boldsymbol{\varepsilon_{i,j}}$ is computed using the following equation:

$$\varepsilon_{i,j} = 0.003\,\frac{x_i - d_j}{x_i} \quad (7)$$

On the other hand, the stresses $\boldsymbol{f_{i,j}}$in each reinforcing steel bar is obtained using the expression:

$$f_{i,j} = E_s \times \varepsilon_{i,j} \quad (8)$$

where $\boldsymbol{f_{i,j}}$has to be less than or equal to the yield strength of steel ***fy.***

$$f_{i,j} = if\left(f_y < E.\left|\varepsilon_{i,j}\right|, \frac{\left|\varepsilon_{i,j}\right|}{\varepsilon_{i,j}}.f_y, f_{i,j}\right)$$

Using the equilibrium of the internal forces and moments, the design axial load $\emptyset P_{ni}$ and the design bending moment $\emptyset M_{ni}$ are, respectively, computed using the following equations:

$$\emptyset P_{ni} = 0.7 \times \left(0.85 f_c' a_i b + \sum_{j=1}^{NSL} As_j f_{i,j}\right) \quad (9)$$

$$\emptyset M_{ni} = \emptyset P_{ni}\, e$$

$$\emptyset M_{ni} = 0.7\left(0.85 f_c' a_i b\left(Y_p - \frac{a_i}{2}\right) + \sum_{j=1}^{NSL} As_j F_{i,j}\,(Y_p - d_j)\right) \quad (10)$$

$$\emptyset M_i = if(\emptyset M_i < 0, 0, \emptyset M_i)$$

$$\emptyset P_i = if(\emptyset P_i < 0, 0, \emptyset P_i)$$

The load eccentricity$\boldsymbol{eu_{ix}}$ is computed using the following expression:

$$eu_{ix} = \frac{\emptyset M_{nxi}}{\emptyset P_{ni}} \quad (11)$$

The values of $\emptyset P_{ni}$ and $\emptyset M_{nxi}$, represent a point on the interaction diagram ($\emptyset P_{ni} - \emptyset M_{ni}$).

STEP-5:

The number of iterations ***(i)***is incremented by 1. Then, ***STEP 3*** through ***STEP 5*** are repeated until the value of ***(i)*** reaches the value of ***h***.

STEP-6:

At the end of the computation process, the design bending moment $\emptyset M_{n(N)}$ is set equal to zero *(last point in* $\emptyset M - \emptyset P$ *diagram)*while the design axial loads $\emptyset P_{n(N-1)}$ and $\emptyset P_{n(N)}$*(located on the horizontal plateau of* $\emptyset M - \emptyset P$ *diagram)* are set equal to the following expression:

$$\emptyset P_{n(N)} = \emptyset P_{n(N-1)} = \emptyset 0.8 \times \left(0.85 f_c' hb + \sum_{j=1}^{NSL} As_j f_y\right) \quad (12)$$

The values of $\emptyset P_{n(N-1)}$ and $\emptyset P_{n(N)}$correspond to the design axial load of concentrically loaded columns (i.e., *en* =0 and $\emptyset M_{n(N)}$= 0)

$$sort(\emptyset P) = (\emptyset P_i > \emptyset P_N, \emptyset P_N, \emptyset P_i)$$

At this stage, the interaction diagram is fully determined.

Step ***7*** through ***9*** are concerned with the manual design reinforced concrete biaxial columns. In other words, the remaining computational steps deal with checking the strength adequacy of reinforced concrete columns.

STEP-7:
The eccentricity e_{uix} of the factored load *Pu* is computed using the following equation:

$$e_{uix} = \frac{\emptyset M_{ux}}{\emptyset P_u} \quad (13)$$

STEP-8:
MathCad Steps:
Draw a line that cuts the $\emptyset P_n - \emptyset M_n$ diagram in order to check the column strength.
Z=0…..2

$F_o = 0$	$m_o = 0$	*First point determined by (F_0, m_0)*
$F_1 = P_u$	$m_1 = P_u$	*Second point determined by (F_1, m_1)*
$F_2 = 1.7.F_1$	$m_2 = \frac{m_1}{F_1}.F_2$	*Third point determined by (F_2, m_2)*

The required column strength is represented by the second point (F_1, m_1). The second point has to be inside or on the $\emptyset P_n - \emptyset M_n$ diagram for the column to be safe.

STEP-9:
Another method for checking column strength. The method determines $\emptyset P_i$ and $\emptyset M_i$ which corresponds to the closest e_i to the ultimate load eccentricity e_u.
MathCad Steps:

Sort (e) — *Sort load eccentricities e_i*
eu = 0.16

$$t(e, eu) := \left| \begin{array}{l} j \leftarrow 0 \\ while\ e_j \geq e_u \\ \quad j \leftarrow j+1 \\ j \end{array} \right.$$

Find the closet e_i to e_u

t(e, eu)= Value
$\emptyset P_{value}$ — *Value of $\emptyset P_N$ which corresponds to e_{value}*
$\emptyset M_{value}$ — *Value of $\emptyset M_N$ which corresponds to e_{value}*

The axial load $\emptyset P_{nx}$ and the bending moment $\emptyset M_{nx}$, which corresponds to the ultimate load eccentricity e_{ux}, are first determined

STEP-10:
Repeat steps ***1*** through ***9***to determine the values $\emptyset P_{ny}$ and $\emptyset M_{ny}$, which corresponded to the ultimate load eccentricity e_{uiy}.
STEP-11:
The strength of the column is adequate if the obtained values satisfies the Bresler formula;

$$P_c = \frac{1}{\frac{1}{\emptyset P_{ny}} + \frac{1}{\emptyset P_{nx}} - \frac{1}{\emptyset P_N}} \quad (14)$$

The obtained design values ***Pc*** should be higher than the factored values ***Pu***. The strength of the column is adequate if the point defined by *Pu* and *Mu* is inside, or on the interaction diagram ($\emptyset P_n - \emptyset M_n$). The strength of the column is not adequate if the point is outside the curve $\emptyset P_n - \emptyset M_n$. The closet is the point to the curve $\emptyset P_n - \emptyset M_n$, the more economical is the design.

III. TRADITIONAL VERSUS MATHCAD ENHANCED INSTRUCTION

Traditional teaching methods usually involves the time-consuming task of the instructor writing detailed problem solutions on the board while students hurriedly copy the solutions into their notebooks. The learning process in the classroom is often suspended while the teacher and the students occupy themselves with transcribing information. This traditional classroom activity can discourage critical thinking and deprives both the students and the teachers of engaging exchanges with each other about the subject.

A Mathcad enhanced teaching method can be successfully integrated into a concrete design course. Steps 1 to 11 shows the complete Mathcad program developed for the design of reinforced concrete Bi-axial column.

The program is projected directly from the instructor's computer onto a large screen in an appropriately equipped classroom. In the program, different formatting, including various fonts, colors, patterns, and borders are used. The readability of the text exceeds what instructors can produce by hand on the classroom board. The equations look the same as they are written on a blackboard or in a reference book. To free student attention from transcription, students are given a hard copy for taking additional notes. An electronic copy of the Mathcad program is also made available for the student to review and practice later.

The photograph, which is shown in Figure 8, was easily digitized and imported into the program. Photographs and images are rich sources of visual information that can be shared among teachers and students. Images from the field or laboratory bring glimpses of the engineering world into the classroom where they can be shared quite easily. Existing photos and slides can be digitized using slide and film scanning processes. Digital photographs can be taken with digital cameras and downloaded directly to the computer without the use of film.

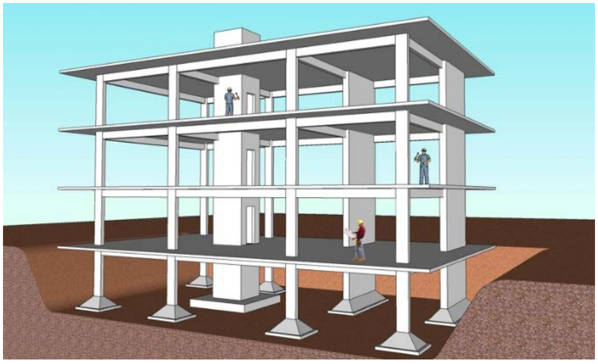

Figure 8: Program drawings and photographs [20]

The interaction diagram $\emptyset P_n - \emptyset M_n$, which is shown in Figure 5, was easily produced by the program like spreadsheets, as soon as a change is made in the input data, the results are updated, and the interaction diagram is redrawn. Other types of charts, such as pie and histogram charts, can also be easily generated. As was mentioned previously, interaction diagrams play an important role in the manual design of reinforced concrete columns. The Mathcad program allows for the determination of an optimum design simply by changing the input data and observing the changes in the interaction diagram.

There are several benefits of a Mathcad enhanced approach to teaching. The time saved from tedious transcription free student and teacher for the discussion of concepts, and exploration of alternate problem scenarios, observation of trends, and expansion of the discussion to related topics. Outside the classroom, the instructor uses the same program to quickly generate test questions and solution keys. Trial and error solutions are cycled through rapidly. The student can review the classroom material by changing input variables and observing results. Homework assignments can be developed to encourage students to use the program. Making the program available to students, encourages them to learn by exploring on their own. Visual changes of the interaction diagram give students a good control of the design. The time spent using the program to explore problem scenarios posted by the instructor, can lead students to a better understanding of the concepts involved in the problems. Students can learn to write Mathcad programs using their own way of problem solving.

II. ILLUSTRATIVE EXAMPLE

The manual design example of the biaxial column is demonstrated using the Mathcad program and the results were later compared with the computer software SP column. Selectthe most economical biaxial column section(400x 800) mm, (450 x 600) mm and (400 x 700) mm using the Mathcad program from the given ultimate load values and compare the results with the SP column.

Input Data:

Axial load = P_u= 4000 *kN*

Moment is X-direction = M_{ux} = 320 *kN-m.*

Compressive strength of concrete = f'_c = 30*MPa*

Cover to Rebar = d'= 80 mm

Area of Steel = A_s = 8 Ø24 =3619.1 mm^2

Moment is Y-direction = M_{uy} = 160 *kN-m.*

Steel yield strength = f_y = 400 *MPa*

Compression Control Failure = $\emptyset_c$ =0.7

Solution:
For the section 400mm x 800mm, the interaction diagram which is shown in Figure 9 demonstrates the section as uneconomical since both points are well inside the $\emptyset P_n - \emptyset M_n$ curve. The value of Pc obtained is 5000 kN which is much greater than the Pu= 4000 kN and it comes out to be an overdesigned section.

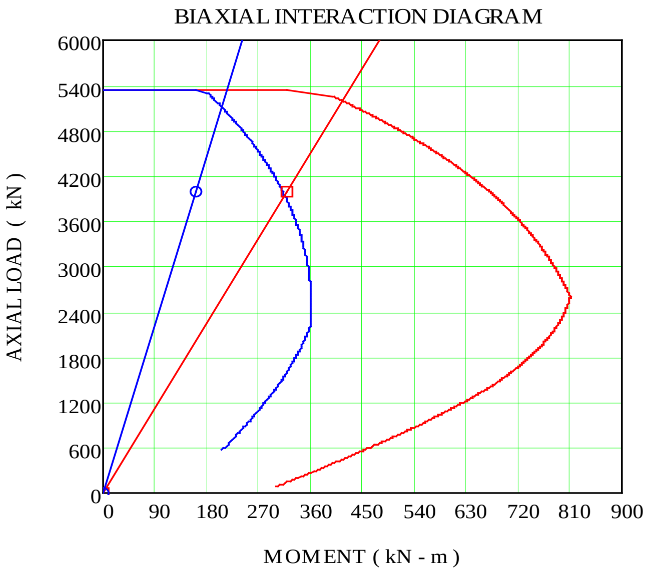

Figure 9: Interaction diagram for section (400x800) mm

The Mathcad program can be used easily to improve the first design trial either by reducing the coulmn cross section or by reducing the area of steel. For the second trial section, (450 mm x 600 mm), the interaction diagram is shown in Figure 10.

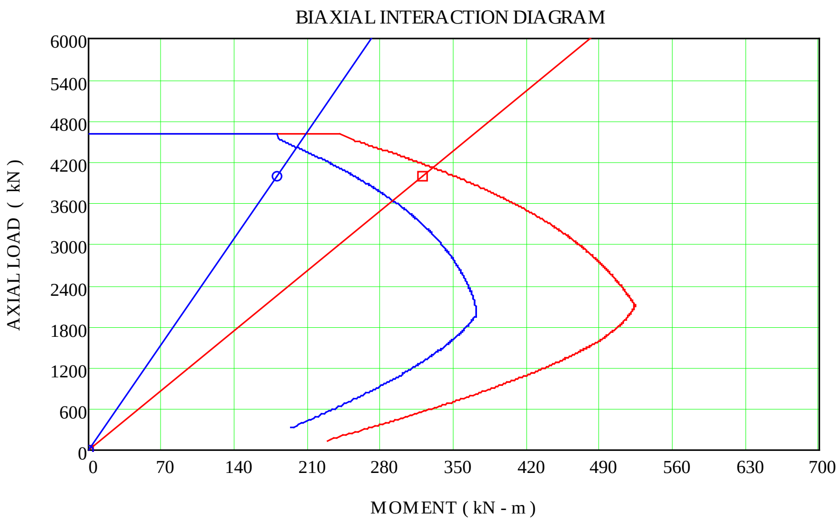

Figure 10: Interaction diagram for section (450x600) mm

The capacity of the section for this cross section is not adequate as the value of *Pc*= 3971 kN is lower than the *Pu*= 4000 kN.

Therefore, the selected cross section dimensions are not acceptable and section dimensions are revised to be 400 mm x 700 mm. This section gives an optimum design as the value of *Pc* obtained is 4284 and the interaction points obtained are quite closer to the $\emptyset P_n - \emptyset M_n$ curve (Figure 11).

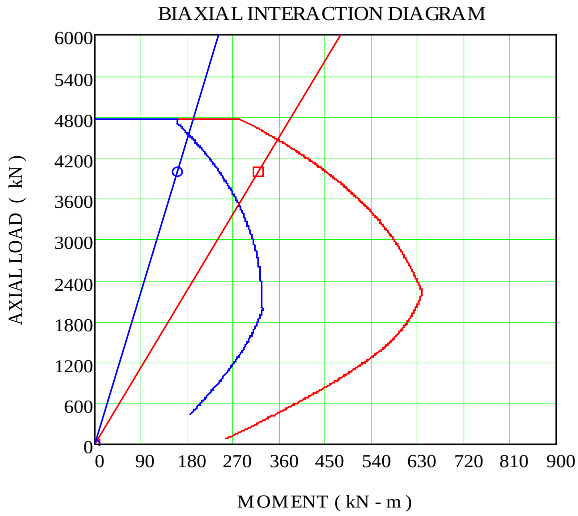

Figure 11: Interaction diagram for section (450x600) mm

The results obtained using Macthcad program were also compared with the Computer Software "SP Column". The comparison table illustrated in Table 1 really shows that the Mathcad results obtained are quite closer to the ones obtained from the software. Both the programs justifies the 400 mm x 700mm section as the most economical biaxial reinforced column section for the given loads. The results obtained are also depicted in the bar chart shown in Figure 12.

***Table 1:** Comparison Table between three selected biaxial column sections*

No.	**Section (mm x mm)**	**Mathcad *Pc (kN)***	**Computer Software "SP Column *Pc(kN)***	**Remarks**
1	400 x 800	5000	5316	Pc >> Pu over design
2	450 x 600	3971	3740	Pc <Pu Not adequate
3	400 x 700	4284	4228	Pc > Pu

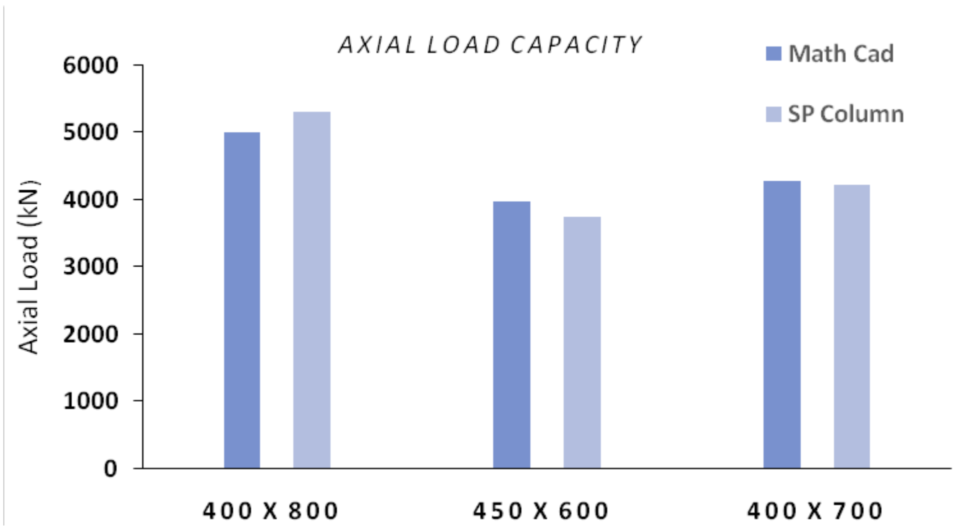

Figure 12: Axial load capacity comparison for selected column cross sections

IV. CONCLUSION

Mathcad contains tools which can enhance and supplement traditional methods of teaching and learning. The versatility, accessibility, and ease of use make Mathcad a platform for creating learning modules for technically based courses. Mathcad contains the capabilities for traditional classroom computation, but at more accuracy, reliability, and presentation quality. In addition, its speed at repetitive tasks, and its programmability, make new learning strategies possible. Mathcad programs take time for an instructor to develop, but with many benefits in return. By freeing the instructor and student from tedious computation and transcription, Mathcad programs create opportunities for meaningful understanding of technical material. However, a well-designed Mathcad program can engage both student and teacher, inviting their exploration and discovery of the subject, drawing them deeper into the secrets that it holds.

The design of biaxial reinforced concrete column can be done quite easily on Mathcad once the input file is ready and that file can be used for any section to design a biaxial column. An example for the biaxial column using Mathcad was also performed and the results were compared with the computer software "SP Column" and it shows a good agreement between the results obtained from the Mathcad to the ones obtained from the software.

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