IOSR Journal of Engineering (IOSRJEN) **www.iosrjen.org**
ISSN (e): 2250-3021, ISSN (p): 2278-8719
Vol. 09, Issue 2 (February. 2019), ||V (I) || PP 80-86

Study of Heat Transfer through Porous Fin with Uniform Magnetic Field by Application of Modified Least Square Method

Shatendra ku. Sahu[1*], Dr. Arun Arora[2], Dr D.S.Bal[3]
[12]*Department of Mechanical Engineering Bhilai Institute of Technology Durg Chhattisgarh India 491001*
[3]*Ex Professor Department of Mechanical Engineering Government Engineering College Raipur Chhattisgarh India*
Corresponding Author: Shatendra ku. Sahu

Abstract: - Least square method is widely used for optimization of both linear and nonlinear problems with high accuracies. In this study modified least square method is used for the analysis of porous fin in the presence of uniform magnetic field and the obtained result is compared with the Least Square Method.

Keywords: - Least Square Method modified least square method, uniform magnetic field, porous fin.

Date of Submission: 26-01-2019 Date of acceptance:09-02-2019

I. INTRODUCTION

Fins plays vital role in increasing rate of heat transfer. It should be small in size for high heat transfer rate. The heat transfer rate through fins depends on various parameters such as surface area, surrounding fluid, thermal conductivity of material, length of fin etc. They find their application in heat-exchangers, furnaces, super heaters, and turbine. Fins are also used in airplanes, power plants, bikes etc. Number of researches has been done to optimize the effectiveness of the fin. (Hoshyar et al, 2016) used least square method for optimization of porous fin in uniform magnetic field and compared it with the numerical result. It shows that former gives superior result over latter one. (Hatami et al, 2013) applied various methods to study porous fin with temperature dependent internal heat generation. This Analysis shows that the differential transformation method, least square method and collocation method are more effective compared to numerical method here passage velocity and Darcy's model are used for heat transfer simulation. (Aziz et al, 2011) also uses least square method and compared with the obtained numerical result for longitudinal fin. (Akbari et al, 2016) studied the efficiency of straight fin with different methods and developed Akbari's- Ganji's method which is quite effective for nonlinear equations. (Aziz and Hug, 1975) applied the regular perturbation method to obtain a closed form solution for a straight convecting fin using temperature dependent thermal conductivity. (Razani & Ahmadi, 1977) considered circular fins with both nonlinear temperature-dependent thermal conductivity and an arbitrary heat source distribution and obtained the results for the optimum fin designing. (Yu & Chen 1999) assumed the linear variation of the thermal conductivity and its exponential function with the interval of the heat transfer coefficient and later solved the nonlinear conducting-convecting-radiating heat transfer equation by the differential transformation method. (Bouaziz and Aziz, 2010) introduced a double optimal linearization method (DOLM) to get a simple and precise solution for the temperature distribution in a straight rectangular convective–radiative fin with temperature dependent thermal conductivity. (Bouaziz et al, 2001) presented the efficiency of longitudinal fins with temperature-dependent thermo physical properties. Likewise, the effects of temperature-dependent thermal conductivity of a moving fin in addition to radiative component to the surface heat loss have been studied with (Aziz & Khani ,2011). (Ghasemi et al, 2014) solved the nonlinear temperature repartition equation in a longitudinal fin with temperature dependent internal heat generation and thermal conductivity using Differential Transformation Method (DTM).Recently numerous methods have been proposed to solve nonlinear problems; Optimal Homotopy Analysis Method (OHAM) is one of them (Joneidi et al,2009).

(Rao and Bal,1986,1982) developed modified least square method for curve fitting which has proved it to be more advantageous over least square method when applied to many engineering problems. (Bal & Bal 2015) used modified least square method for optimization of space curve generation mechanism whose equations are nonlinear is compared with least square method. Hence modified least square method is powerful tool to solve nonlinear differential equation. In this paper we have adopted modified least square method for analysis of temperature distribution in pours fin in the presence of uniform magnetic field.

II. DESCRIPTION OF PROBLEM

The problem is studied with the help of porous fin having rectangular profile in presence of uniform magnetic field. (Hoshyar et al , 2016) used porous fin with rectangular profile having cross section area A and length L, perimeter p, was width of fin, d as diameter of fin, h as convective heat transfer coefficient, h0 as convective heat transfer coefficient at the base, k is thermal conductivity of material, µ is permeability of fin, qi is ideal heat transfer rate and q is fin heat transfer rate. For the problem to be less complex few assumptions have been made such as porous medium is homogeneous as well as isotropic in nature and both fluid and solid have same phase. The temperature inside the fin is only a function of length and also uniform magnetic field is applied in y-direction. The intercommunication between the porous medium and the clear fluid can be simulated by the Darcy formulation. The fin is of finite length and the tip is insulated. In order to reduce the complexity of problem due to radiative heat flux, the porous medium is assumed to behave as an optically thick gas (Hoshyar et al 2016). Now applying energy balance equation by considering small segment of fin of length ΔX (Taklifi et al. 2010; Khani et al, 2009)

$$q_x - q_{x+\Delta x} = \dot{m}c_p(T_x - T_\infty) + hp\Delta x(1-\emptyset)(T_x - T_\infty) + \frac{I+I}{\sigma} + p\Delta x\sigma_{st}\varepsilon(T_x^4 - \frac{\alpha}{\varepsilon}T_\infty^4) \quad \text{(i)}$$

Where I is conduction current intensity that can be written as

$$I = \sigma(E + V * B) \quad \text{(ii)}$$

And total current intensity can be given as

$$J = I + \rho_\varepsilon V \quad \text{(iii)}$$

Where J is total current intensity.

The fluid is passing through the porous fin and mass flow rate is given by

$$\dot{m} = \rho\overline{\vartheta_w}\Delta xw \quad \text{(iv)}$$

Where,

$$\overline{\vartheta_w} = \frac{gk\beta}{v}(T_x - T_\infty) \text{ from Darcy's model} \quad \text{(v)}$$

The energy flux vector of combined radiation and conduction at the base of the fin can be expressed as

$$q_{finbase} = q_{conduction} + q_{radiation} \quad \text{(vi)}$$

From Fourier's law of conduction $q_{conduction}$ is given by,

$$q_{conduction} = k_{pp}A_b\frac{dT}{dx} \quad \text{(vii)}$$

And based on the Rosseland diffusion approximation proposed the radiation heat flux term is expressed, as

$$q_{radiation} = -\frac{4\sigma_{st}}{3\beta_R}\frac{dT^4}{dx} \quad \text{(viii)}$$

Put equation (vi),(vii),(viii) in to equation (i)gives

$$\frac{d}{dx}\left[\frac{dT}{dx} + \frac{4\sigma}{3\beta_R k_{pp}}\frac{dT^4}{dx}\right] = \frac{\rho c_p gk\beta}{bvk_{pp}}(T_x - T_\infty)^2 + \frac{hp(1-\emptyset)}{k_{pp}}(T_x - T_\infty) + \frac{I*I}{\sigma k_{pp}A_b} + \frac{\sigma_{st}\varepsilon p}{k_{pp}A_b}\left(T_x{}^4 - T_\infty{}^4\right) \quad \text{(ix)}$$

Where

$$\frac{I*I}{\sigma} = \sigma H^2u^2 \quad \text{(x)}$$

Situation where temperature difference is small the term T^4 may be expressed as linear function of temperature.

$$T^4 = T_\infty{}^4 + 4T_\infty{}^3(T - T_\infty) + 6T_\infty{}^2(T - T_\infty)^2 + \cdots \cong 4T_\infty{}^3T - 3T_\infty{}^4 \quad \text{(xi)}$$

Using some simplifications and introducing the following dimensionless parameters:

$$\theta = \frac{(T_x - T_\infty)}{(T_b - T_\infty)} \qquad X = \frac{x}{b} \qquad \theta_b = \frac{T_b}{T_\infty} \quad \text{(xii)}$$

Substituting Dimensionless parameters in to equation ix and from equation xi

$$(1+4R)\theta''(X) - R_m\theta^2 - N_c(1-\emptyset)\theta(X) - S_r\theta(X) - H_n\theta(X) = 0 \quad \text{(xiii)}$$

Where, Modified Rayleigh number is

$$R_m = \frac{gk\beta(T_b - T_\infty)b}{\alpha vk_r}$$

$$convection\text{–}conduction\ parameter\ N_c = \frac{pb\hbar}{k_{pp}A_b}$$

$$Surface\ ambient\ radiation\ parameter S_r = \frac{4\sigma_{st}bT_\infty{}^3}{k_{pp}}$$

$$Hartman\ parameters\ H_n = \frac{\sigma H^2u^2}{k_{pp}A_b}$$

$$Radiation\text{–}conduction\ parameter R = \frac{4\sigma_{st}T_\infty{}^3}{3\beta_R k_{pp}} \quad \text{(xiv)}$$

As per above assumptions the boundary condition will be,

$$\theta(0) = 1, \qquad \theta'(0) = 0 \quad \text{(xv)}$$

III. MODIFIED LEAST SQUARE METHOD

Least square method is used to solve linear and nonlinear differential equation approximately and very conveniently. Since it gives approximate solution to problem it must be close to the exact value. Modified least square method is (Rao and Bal 1986, 1982) developed over least square method to solve nonlinear differential equation and gives accurate solution. (Rao and Bal, 1986) solved some examples like rigid body dynamics, problems of chemical solution, with modified least square method and compared with the result of least square method. Modified least square method is given by(Rao and Bal 1986, 1982)

$$f = \sum_{i=1}^{n} y_i^2(\overline{y_i} - y_i)^2 \qquad \text{(xvi)}$$

Between the fixed value and approximated value where y_i as experimental data and $\overline{y_i}$as approximated value. To achieve minimum value the function f, derivative of f with respect to all unknown parameter must be zero.

Application of modified least square method

The boundary conditions for the porous fin are according to assumptions made are$\theta(0) = 1, \theta'(0) = 0$. To satisfy the boundary condition the solution of given differential equation, the trail function can be approximated as$\left[X - \left(\frac{1}{n+1}\right).X^{n+1}\right]$.It is to be noted that other polynomial equation which satisfy above boundary condition can also be considered. So it is taken as,

$$\theta(X) \ = \ 1 - A\left(x - \tfrac{1}{2}x^2\right) + B\left(x - \tfrac{1}{3}x^3\right) - C\left(x - \tfrac{1}{4}x^4\right) + D\left(x - \tfrac{1}{5}x^5\right) \qquad \text{(xvii)}$$

Where A, B, C, D are constants. By putting equation (xvii) in to (xiii) and (xvii) the residual function for modified least square method would be,

$S := \sum_{i=1}^{n}\Bigg(X^2\Bigg((1+4R)(-4DX^3 - 3CX^2 - 2BX - A) - Rm\left(1 - A\left(x - \frac{1}{2}x^2\right) + B\left(x - \frac{1}{3}x^3\right) -\right.$

$Cx\!-14x4\!+\!Dx\!-15x52\!-\!Nc1\!-\!\emptyset(1\!-\!Ax\!-12x2\!+\!Bx\!-13x3\!-\!Cx\!-14x4\!+\!Dx\!-15x5)\!-\!Sr(1\!-\!Ax\!-12x2\!+\!Bx\!-13x3\!-\!Cx\!-14x4\!+\!Dx\!-15x5)\!-\!Hn1\!-\!Ax\!-12x2\!+\!Bx\!-13x3\!-\!Cx\!-14x4\!+\!Dx\!-15x52$

(xviii)

By differentiating xviii with respect to unknowns, the temperature distribution equation is found as where Rm=0.2, R=0.5, ε=0.2, Nc=0.3, Sr=0.8, Hn=0.9

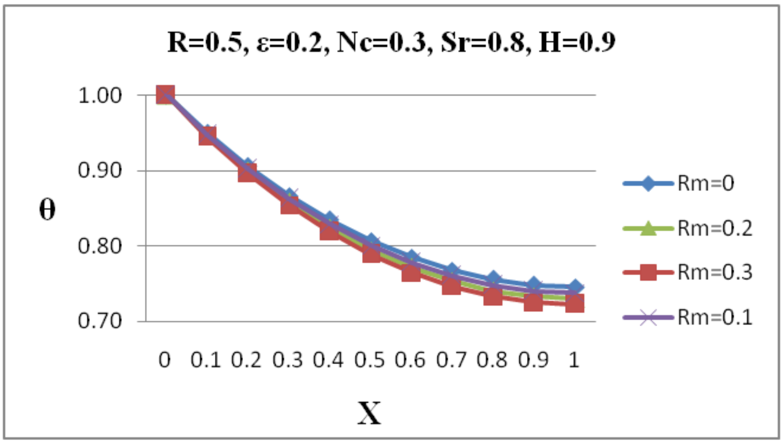
$$\theta(X) = 1 - .5737093790X + .3557922014X^2 - 0.07199664273X^3 + 0.02111756904X^4 - 0.001271070060X^5 \qquad \text{(xix)}$$

Whereas temperature distribution equation by solving equation (xiii) considering same trial function by least square method is (Hoshyar et al (2016))

$$\theta(X) = 1 - 0.5738228590X + 0.3565147734X^2 - 0.07356149367X^3 + 0.02257507249X^4 - 0.001764499350X^5 \qquad \text{(xx)}$$

IV. RESULT AND DISCUSSION

The temperature distribution has been simulated using modified least square method. Result has been plotted for $\theta(X)$ and$\theta'(X)$. Since there are some variables like Hn, g,ϕ and m, the temperature distribution is as shown accordingly.



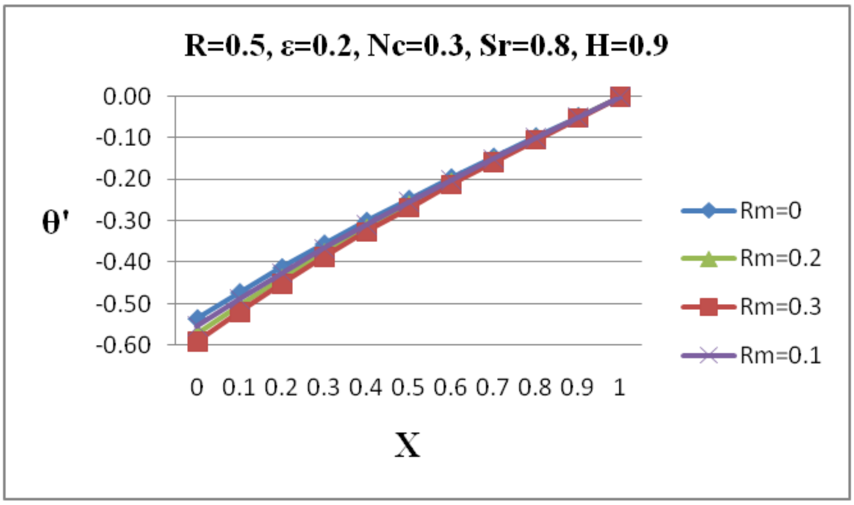


Fig.1 Plot for θ and θ' for different value of R_m

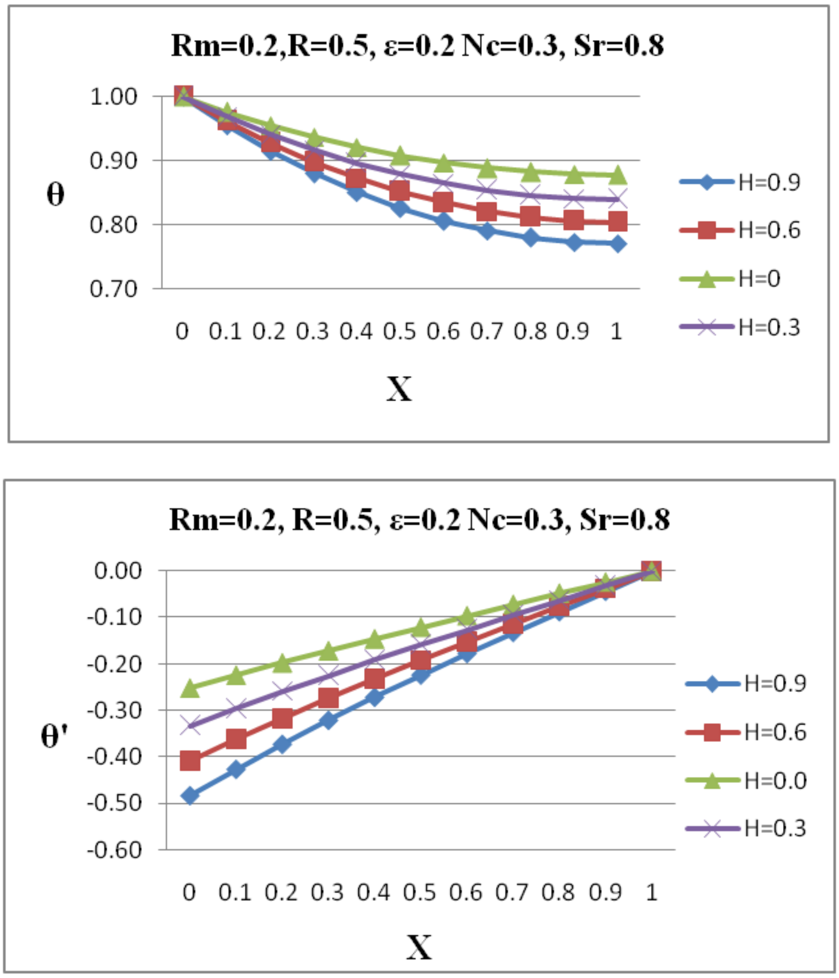
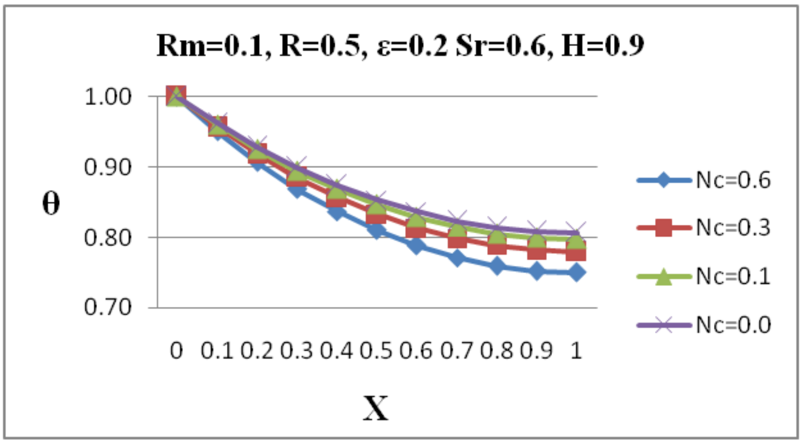


Fig.2 Plot for θ and θ' for different value of H



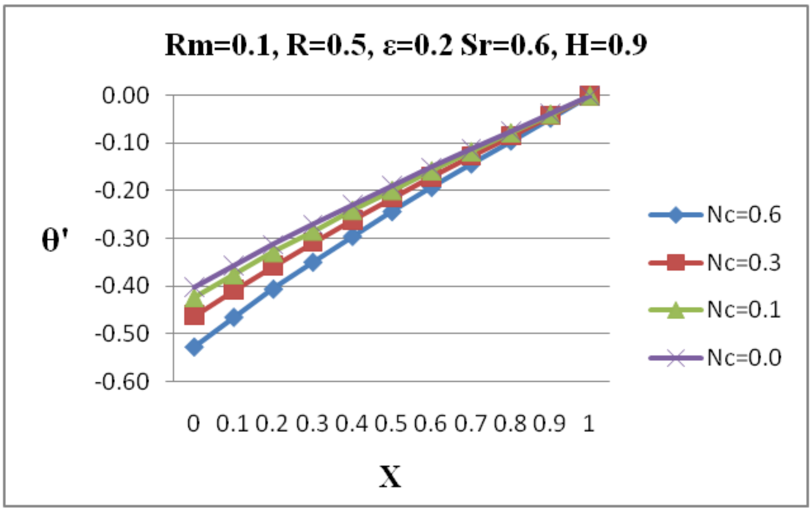
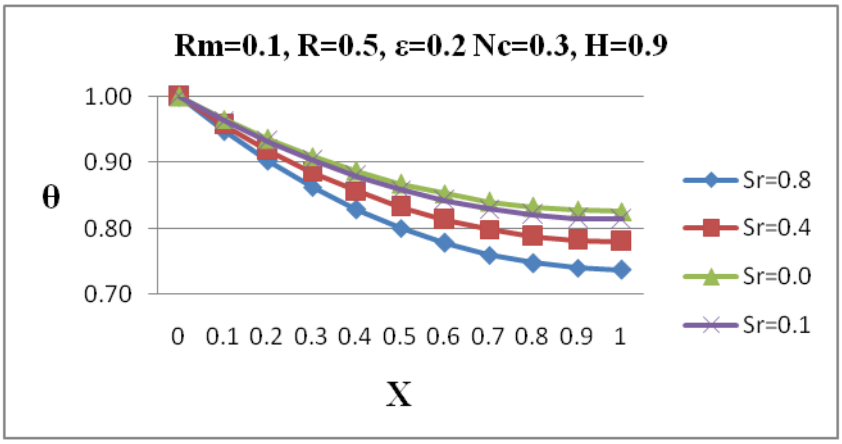


Fig.3 Plot for θ and θ' for different value of Nc



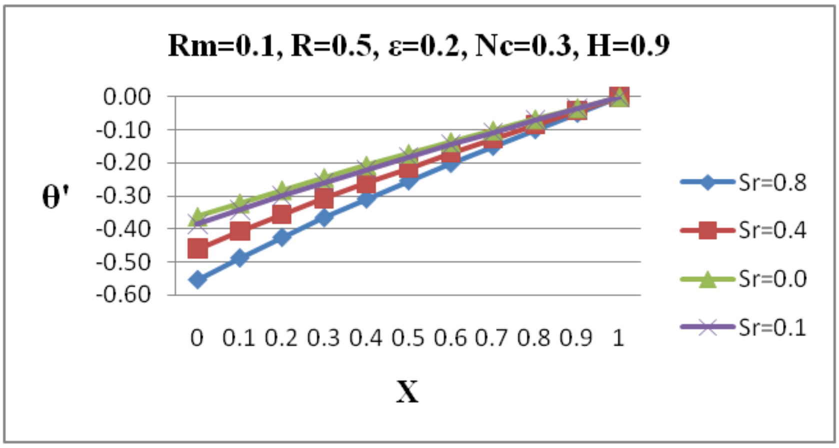


Fig.4 Plot for θ and θ' for different value of Sr

Table 1-The results of MSLM, LSM and Numerical methods for θ(X) for
R= 0.1, R_m = 0.5,ε = 0.2,N_c= 0.4 ,S_r =0.3 and H = 0.9.

X	$\theta(X)$		
	MLSM	LSM (Hoshyar et al, 2016)	Numerical method (Hoshyar et al, 2016)
0.0	1.00000000000	1.000000000	1.000000000
0.1	0.95698781941	0.956987665	0.956988020
0.2	0.91913158249	0.919132060	0.919132513
0.3	0.88621788672	0.886217828	0.886217964
0.4	0.85805676249	0.858057970	0.858057690
0.5	0.83449103175	0.834492969	0.834492509
0.6	0.81538766673	0.815389906	0.815389668
0.7	0.80063914861	0.800641589	0.800641806
0.8	0.79016282623	0.790165668	0.790166187
0.9	0.78390027477	0.78390376	0.783904154
0.10	0.78181665447	0.781820569	0.781820729

Table 2- The results of MSLM, LSM and Numerical methods for θ'(X) for
R= 0.1, R_m = 0.5,ε = 0.2,N_c= 0.4 ,S_r =0.3 and H = 0.9.

X	$\theta'(X)$		
	MLSM	LSM (Hoshyar et al, 2016)	Numerical method (Hoshyar et al, 2016)
0.0	0.4566527307380000	0.456697507	0.456697420
0.1	0.4039532353102840	0.403956702	0.403953575
0.2	0.3535270670991870	0.353514184	0.353515512
0.3	0.3050966906963400	0.305081978	0.305086466
0.4	0.2583909837781150	0.258380892	0.258384515
0.5	0.2131452371056250	0.213140520	0.213140387
0.6	0.1691011545247230	0.169099238	0.169095471
0.7	0.1260068529660040	0.126004208	0.125999796
0.8	0.0836168624448032	0.083611375	0.083610171
0.9	0.0416921260611962	0.041685468	0.041688474
0.10	0.0000000	0.000000000	0.000000000

Table 1& 2 is showing comparative study of result obtain by MLSM and LSM. Result shows temperature distribution can be studied more accurately using MLSM.

V. CONCLUSION

In this research temperature of porous fin which is under uniform magnetic field has been analyzed using modified least square method and compared with the result of least square method. Result reveals that both MLSM and LSM give good result over approximate numerical result. It can also be concluded that both methods have wide verities of capability to solve nonlinear problems very conveniently.

NOMENCLATURE	
A cross-sectional or profile area, d diameter of fin, w Width of fin h convection heat transfer coefficient	T temperature, x height coordinate, t fin thickness, m mass flow rate

h_o convection heat transfer coefficient at the base k thermal conductivity μ permeability of the porous fin L fin length, p fin perimeter, q_i ideal fin heat transfer rate, q_f fin heat transfer rate, J total current intensity I conduction current intensity H magnetic field intensity C_p specific heat R_m modified Rayleigh number R radiation–conduction parameter V_a average velocity of the fluid passing through the fin at any point N_c convection parameter S_r surface radiation k_r thermal conductivity ratio,	α thermal diffusivity β_R Roseland extinction coefficient φ porosity Ɛ emissivity A emissivity of fin at the radiation sink temperature η fin efficiency ρ density of the fluid ρ_ε electrical density θ dimensionless temperature θ_b dimensionless radiation temperature a ambient conditions b base of fin pp porous properties f fluid properties s solid properties u axial velocity H_n Hartman number MLSM modified Least square method LSM Least square Method

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