

Effect of MHD on Pivoted Curved Slider Bearing Lubricated with Non-Newtonian Fluid

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Abstract: This paper presents a theoretical study of effect of applied magnetic field on the performance of pivoted curved slider bearing lubricated with couple stress fluid. Based on the Stokes couple stress theory, the modified Reynolds equation is derived which governs the fluid pressure in the flow region. The closed form expressions are obtained for pressure, load carrying capacity, frictional force and coefficient of friction to study the effect of magnetic field and couple stress parameter on these characteristics. The results predict that, the application of external magnetic field and the use of couple stress fluid increases the pressure, load carrying capacity and frictional force whereas decreases the coefficient of friction.

Key words: MHD, Couple Stress Fluid, Pivoted Slider Bearing,.

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I. INTRODUCTION

Magneto-hydrodynamic(MHD) is the study of the magnetic properties and behaviour electrically conducting fluids. Examples of such magneto fluids include plasmas, liquid metals, salt water and electrolytes. Bearing ensure high efficiency and reliability depending on their size, type of functioning, Materials used in Manufacturing and use of fluids for lubrication. Many researcher studied effect of Magnetic field on the performance of bearing by Hughes [1] presentedmagnetohydrodynamiclubrication flow between parallel rotating disks, The Magneto-hydrodynamic finite journal bearing by Kuzma [2], Analysis of finite magneto-hydrodynamic journal bearings by Malik and Singh [3], Lin [4] analysed the magneto-hydrodynamic lubrication of finite slider bearings. They realized that the application of Magnetic field which increases the load carrying capacity and decreases the frictional forces. Kamiyama [5] studied Magneto-hydrodynamic journal bearing (report-I). Snyder [6] presented magneto-hydrodynamic slider bearing and it is observed that load carrying capacity of the bearing depends on the electromagnetic boundary condition passing through the conductivity of the surface. All these studies are belongs to classical hydrodynamic lubrication in which lubricant assume to behave as the Newtonian fluid. This is not much satisfactory prediction for practical application in engineering fields.

The researchers found the desirable lubricant by adding some polymers to Newtonian fluid known as non-Newtonian fluid and they started to study the characteristics of bearings lubricated with non-Newtonian fluids. Lin [7] studied the effects on MHD steady and dynamic characteristics wide tapered-land slider bearings, author reported that the MHD bearing provides higher values of load carrying capacity.Lin and Lu [8] presented MHD study and dynamic characteristics wide tapered land slider bearing, they found that the presence of applied magnetic fields signifies an enhancement of film pressure. In this result the use of applied magnetic field effects of the characterized by the Hartmann number gives increase in the values of load carrying capacity. Stochastic Reynolds equation for diverse shaped slider bearing when lubricated with couple stress fluid and by applying MHDstudied by SyedaTasneem Fatima et al [9] are found that the load carrying capacity of Parabolic slider is more significant and which improves the normal functioning of the bearings.

When we mix additives in the oil or fluid, the forces which are present in the fluid opposes the forces of additives. This opposition make a couple forces and hence couple stress is induced in the fluid. This type fluid is known as couple stress fluid. The simplest generalization classical theory proposed by Stokes [10] which allow polar effects such as the presence of body couples and couple stresses. Many researchers used this couple

stress fluid theory to study the effect of couple stress on the bearing performance. Lin [11] presented the squeeze film characteristic of long partial journal bearing lubricated with couple stress fluids. It is observed that effect of couple stress provide an enhancement in the load carrying capacity and delay the squeeze film time. Ramanaiah and Sarkar[12] studied the slider bearings lubricated by fluids with couple stress, and concluded that with increase of couple stress parameter the load capacity and frictional force increases but the frictional coefficient decreases. Effect of couple stresses on the lubrications of finite journal bearings studied by Lin [13] it is reported that the effect of couple stresses improves the lubrication characteristics of the journal bearing system. On the performance of dynamically loaded journal bearings lubricated with couple stress fluids is analysed by Xiao-Li Wang et al [14] it is shown that the friction force and friction coefficient vary considerably with time by the effect of couple stress. Hydrodynamic lubrication of rough slider bearings with couple stress fluids is analysed by Naduvanamani et al [15], The effect of Magnetohydrodynamics couple stress dynamic characteristics of exponential slider bearing studied by Naduvanamani et al [16] concluded that the use of magnetohydrodynamic couple stress lubrication gives higher steady load carrying capacity, dynamic stiffness, damping coefficient.

Effect of magneto-hydrodynamics and couple stress steady and dynamic characteristics of plane slider bearing was studied by Hanumagouda [17] it is found that the application of MHD and couple stress fluid lubricant improves the steady state and dynamic stiffness and damping characteristics of the plane slider bearings. Surface roughness effects on the pivoted slider bearings with couple stress fluid was studied by Naduvanamani and Biradar[18].

Fatima et al [19] studied the effect of MHD and couple stress fluid on the performance characteristics of wide slider bearing with an exponential and secant film profile a comparative study. It is observed that exponential slider bearing has more significant load carrying capacity and friction as compared to secant slider. The Magnetohydrodynamic lubrication of curved circular plates with couple stress fluid was studied by Hanumagouda et al [20], author reported that the influence of magnetic field and couple stress parameters between the curved circular plates is to increase load carrying capacity, pressure and squeeze film time. Hanumagouda and Biradar[21] studied the MHD effects on composite slider bearing lubricated with couple stress fluids, it account that with increasing the strength of magnetic field is to increase the film pressure, load carrying capacity, frictional force and coefficient of friction. Pivoted slider bearing with convex pad surface in micro polar fluid was studied by Datta [22]. It is found that the load carrying capacity more as compare to Newtonian fluid.

Analysis of MHD effect on pivoted curved slider bearing lubricated with non-Newtonian fluid has not been studied so far. In this paper the Analysis of MHD effect on pivoted curved slider bearing lubricated with non-Newtonian fluid is analysed.

II. MATHEMATICAL ANALYSIS

The physical configuration of the pivoted curved slider bearing of length L lubricated with incompressible electrically conducting couple stress fluid in the presence of applied magnetic field is shown in Fig. 1. The lower plate of the bearing slides with velocity U in x - direction and an external transverse magnetic field B_0 is applied in y – direction. It is assumed that couple and body forces are absent and the effect of internal forces is negligible compare to viscous terms. Under these condition, the governing basic equations for the flow of couple stress fluid in the presence of applied magnetic field reduces to the form,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\mu \frac{\partial^2 u}{\partial y^2} - \eta \frac{\partial^4 u}{\partial y^4} - \sigma B_0^2 u = \frac{\partial p}{\partial x} + \sigma E_z B_0 \tag{2}$$

$$\frac{\partial p}{\partial y} = 0 \tag{3}$$

where u, v denote the velocity components in x and y directions respectively, p is the pressure, μ is the fluid viscosity, η is a new material constant responsible for the couple stress fluid. M_0 represents the Hartmann

number denoted by $M_0 = B_0 h_0 \left(\frac{\sigma}{\mu} \right)^{1/2}$.

For the pivoted curved slider bearing, the mathematical expression for film thickness is

$$h = H_c \left\{ 4 \left(\frac{x}{L} - \frac{1}{2} \right)^2 - 1 \right\} + h_0 \left\{ a - \frac{a}{L} x + \frac{x}{L} \right\}$$

where $a = h_1/h_0$ with h_0 denoting the steady minimum thickness at the exit.

The relevant boundary conditions are:

(i) At the upper surfaces ($y = h$)

$$u=0, \quad \frac{\partial^2 u}{\partial y^2} = 0, \quad v = 0 \tag{4}$$

(ii) At the lower surfaces ($y = U$)

$$u=0, \quad \frac{\partial^2 u}{\partial y^2} = 0 \quad v = 0 \tag{5}$$

On solving equation (1) and (2) using the boundary condition (4) and (5) we get

$$u = -\frac{U}{2} \xi_1 - \frac{h_0^2 h}{2l \mu M_0^2} \frac{\partial p}{\partial x} \xi_2 \tag{6}$$

where

$$\xi_1 = \xi_{11} - \xi_{12}, \quad \xi_2 = \xi_{13} - \xi_{14} \text{ for } 4M_0^2 l^2 / h_0^2 < 1 \tag{7a}$$

$$\xi_1 = \xi_{21} - \xi_{22}, \quad \xi_2 = \xi_{23} - \xi_{24} \text{ for } 4M_0^2 l^2 / h_0^2 = 1 \tag{7b}$$

$$\xi_1 = \xi_{31} - \xi_{32}, \quad \xi_2 = \xi_{33} - \xi_{34} \text{ for } 4M_0^2 l^2 / h_0^2 > 1 \tag{7c}$$

$$M_0 = B_0 h_0 (\sigma/\mu)^{1/2} \text{ and } \eta/\mu = l^2$$

The associated relations in equations (7a), (7b) and (7c) are given in Appendix A.

Integration of the continuity equation (1) over the film thickness and the use of boundary conditions (4) and (5) give the modified Reynolds equation in the form

$$\frac{\partial}{\partial x} \left\{ f(h, l, M_0) \frac{\partial p}{\partial x} \right\} = 6U \frac{dh}{dx} \tag{8}$$

$$\text{where } f(h, l, M_0) = \begin{cases} \frac{6h_0^2 h^2}{\mu l M_0^2} \left\{ \frac{A^2 - B^2}{\frac{A^2}{B} \tanh \frac{Bh}{2l} - \frac{B^2}{A} \tanh \frac{Ah}{2l}} - \frac{2l}{h} \right\} & \text{for } 4M_0^2 l^2 / h_0^2 < 1 \\ \frac{6h_0^2 h^2}{\mu l M_0^2} \left\{ \frac{2(\text{Cosh}(h/\sqrt{2}l) + 1)}{3\sqrt{2}\text{Sinh}(h/\sqrt{2}l) - h/l} - \frac{2l}{h} \right\} & \text{for } 4M_0^2 l^2 / h_0^2 = 1 \\ \frac{6h_0^2 h^2}{\mu l M_0^2} \left\{ \frac{M_0 (\text{Cos}B_1 h + \text{Cosh}A_1 h)}{h_2 (A_2 \text{Sin}B_1 h + B_2 \text{Sinh}A_1 h)} - \frac{2l}{h} \right\} & \text{for } 4M_0^2 l^2 / h_0^2 > 1 \end{cases} \tag{9}$$

$$A_2 = (B_1 - A_1 \text{Cot}\theta) \quad B_2 = (A_1 + B_1 \text{Cot}\theta)$$

Introducing non-dimensional quantities

$$x^* = \frac{x}{L}, \quad P^* = \frac{p^* h_0^2}{\mu U L}, \quad l^* = \frac{2l}{h_0}, \quad h^* = \frac{h}{h_0}, \quad \beta = \frac{H_c}{h_0}, \quad M_0 = B_0 h_0 \left(\frac{\sigma}{\mu} \right)^{1/2}$$

$$h^* = h_1^* + (1 - h_1^* - 4\beta) x^* + 4\beta x^{*2}$$

Substituting above values in (7) we get

$$\frac{\partial}{\partial x^*} \left\{ f(h^*, l^*, M_0) \frac{\partial P^*}{\partial x^*} \right\} = 6 \frac{dh^*}{dx^*} \tag{10}$$

where

$$f^*(h^*, l^*, M_0) = \begin{cases} \frac{12h^{*2}}{l^* M_0^2} \left\{ \frac{(A^{*2} - B^{*2})}{A^{*2} \tanh \frac{B^* h^*}{l^*} - \frac{B^{*2}}{A^*} \tanh \frac{A^* h^*}{l^*}} - \frac{l^*}{h^*} \right\} & \text{for } M_0^2 l^{*2} < 1 \\ \frac{12h^{*2}}{l^* M_0^2} \left\{ \frac{1 + \text{Cosh}(\sqrt{2}h^*/l^*)}{(3/\sqrt{2}) \text{ Sinh}(\sqrt{2}h^*/l^*) - (h^*/l^*)} - \frac{l^*}{h^*} \right\} & \text{for } M_0^2 l^{*2} = 1 \\ \frac{12h^{*2}}{l^* M_0^2} \left\{ \frac{M_0 (\text{Cos}B_1^* h^* + \text{Cosh}A_1^* h^*)}{A_2^* \text{Sin}B_1^* h^* + B_2^* \text{Sinh}A_1^* h^*} - \frac{l^*}{h^*} \right\} & \text{for } M_0^2 l^{*2} > 1 \end{cases} \quad (11)$$

$$A^* = \left\{ \frac{1 + (1 - l^{*2} M_0^2)^{1/2}}{2} \right\}^{1/2} \quad B^* = \left\{ \frac{1 - (1 - l^{*2} M_0^2)^{1/2}}{2} \right\}^{1/2}$$

$$A_1^* = \sqrt{2M_0/l^*} \text{Cos}(\theta^*/2) \quad B_1^* = \sqrt{2M_0/l^*} \text{Sin}(\theta^*/2) \quad \theta^* = \tan^{-1}(\sqrt{l^{*2} M_0^2 - 1})$$

$$A_2^* = (B_1^* - A_1^* \text{Cot}\theta^*) \quad B_2^* = (A_1^* + B_1^* \text{Cot}\theta^*)$$

The pressure boundary conditions are given by

$$p^* = 0 \text{ at } x^* = 0, 1 \quad (12)$$

Integrating equation (10) using equation (12) we obtain

$$p^* = 6 \int_{x^*=0}^{x^*} \frac{h^*}{f^*(h^*, l^*, M_0)} dx^* + C_1 \int_{x^*=0}^{x^*} \frac{1}{f^*(h^*, l^*, M_0)} dx^* \quad (13)$$

where

$$C_1 = - \frac{6 \int_{x^*=0}^1 \frac{h^*}{f^*(h^*, l^*, M_0)} dx^*}{\int_{x^*=0}^1 \frac{1}{f^*(h^*, l^*, M_0)} dx^*}$$

The load carrying capacity per unit width is given by

$$w = \int_0^L p dx \quad (14)$$

The non-dimensional load carrying capacity is given by

$$W^* = 6 \int_0^1 \int_{x^*=0}^{x^*} \frac{h^*}{f^*(h^*, l^*, M_0)} dx^* dx^* - \frac{6 \int_{x^*=0}^1 \frac{h^*}{f^*(h^*, l^*, M_0)} dx^*}{\int_{x^*=0}^1 \frac{1}{f^*(h^*, l^*, M_0)} dx^*} \int_0^1 \int_{x^*=0}^{x^*} \frac{1}{f^*(h^*, l^*, M_0)} dx^* dx^* \quad (15)$$

The components of stress tensor required for calculating frictional force is

$$\tau_{yx} = \mu \frac{\partial u}{\partial y} - \eta \frac{\partial^3 u}{\partial y^3}$$

$$\tau_{yx} |_{y=0} = -G(h, l, M_0) - \frac{h}{2} \frac{\partial p}{\partial x}$$

where

$$G(h, l, M_0) = \begin{cases} \frac{\mu U l M_0^2}{2h_0^2(A^2 - B^2)} \left\{ \frac{A^2}{B} \coth\left(\frac{Bh}{2l}\right) - \frac{B^2}{A} \coth\left(\frac{Ah}{2l}\right) \right\} & \text{for } 4M_0^2 l^2 / h_0^2 < 1 \\ \frac{\mu U}{16l^2} \left\{ \frac{h + 3\sqrt{2}l \operatorname{Sinh}(h/\sqrt{2}l)}{\operatorname{Cosh}(h/\sqrt{2}l) - 1} \right\} & \text{for } 4M_0^2 l^2 / h_0^2 = 1 \\ \frac{\mu U}{2} \left\{ \frac{(K_1 - K_2 \cot\theta) \operatorname{Sinh} A_1 h + (K_1 \cot\theta + K_2) \operatorname{Sin} B_1 h}{\operatorname{Cosh} A_1 h - \operatorname{Cos} B_1 h} \right\} & \text{for } 4M_0^2 l^2 / h_0^2 > 1 \end{cases}$$

$$K_1 = \sqrt{M_0 / l h_0} \operatorname{Cos}(\theta/2) \left[1 - (l M_0 / h_0) \{ 1 - 4 \operatorname{Sin}^2(\theta/2) \} \right]$$

$$K_2 = \sqrt{M_0 / l h_0} \operatorname{Sin}(\theta/2) \left[1 + (l M_0 / h_0) \{ 1 - 4 \operatorname{Cos}^2(\theta/2) \} \right]$$

The frictional force at $y = 0$ is given by

$$F = \int_0^l (t_{21})_{y=0} dx = \int_0^l \left[-G(h, l, M_0) - \frac{h}{2} \frac{\partial p}{\partial x} \right] dx \quad (16)$$

The non dimensional frictional force is

$$F^* = \int_0^1 G(h^*, l^*, M_0) dx^* + 3 \int_0^1 \left\{ \frac{h^*}{\xi(h^*, l^*, M_0)} \right\} dx^* - 3 \left\{ \frac{\int_{x^*=0}^1 \frac{h^*}{f^*(h^*, l^*, M_0)} dx^*}{\int_{x^*=0}^1 \frac{1}{f^*(h^*, l^*, M_0)} dx^*} \right\} \int_0^1 \left(\frac{1}{\xi(h^*, l^*, M_0)} \right) dx^* \quad (17)$$

Where

$$G^*(h^*, l^*, M_0) = \begin{cases} \frac{l^* M_0^2}{24(A^{*2} - B^{*2})} \left(\frac{A^{*2}}{B^*} \coth \frac{B^* h^*}{l^*} - \frac{B^{*2}}{A^*} \coth \frac{A^* h^*}{l^*} \right) & \text{for } M_0^2 l^{*2} < 1 \\ \frac{1}{48l^{*2}} \left\{ \frac{2h^* + 3\sqrt{2}l^* \operatorname{Sinh}(\sqrt{2}h^*/l^*)}{\operatorname{Cosh}(\sqrt{2}H/l^*) - 1} \right\} & \text{for } M_0^2 l^{*2} = 1 \\ \frac{(K_1^* - K_2^* \cot\theta^*) \operatorname{Sinh} A_1^* h^* + (K_1^* \cot\theta^* + K_2^*) \operatorname{Sin} B_1^* h^*}{12(\operatorname{Cosh} A_1^* h^* - \operatorname{Cos} B_1^* h^*)} & \text{for } M_0^2 l^{*2} > 1 \end{cases}$$

$$K_1^* = \sqrt{2M_0 / l^*} \operatorname{Cos}(\theta^*/2) \left[1 - (l^* M_0 / 2) \{ 1 - 4 \operatorname{Sin}^2(\theta^*/2) \} \right]$$

$$K_2^* = \sqrt{2M_0 / l^*} \operatorname{Sin}(\theta^*/2) \left[1 + (l^* M_0 / 2) \{ 1 - 4 \operatorname{Cos}^2(\theta^*/2) \} \right]$$

$$\xi(h^*, l^*, M_0) = \begin{cases} \frac{12h^*}{l^* M_0^2} \left\{ \frac{(A^{*2} - B^{*2})}{A^{*2} \tanh \frac{B^* h^*}{l^*} - \frac{B^{*2}}{A^*} \tanh \frac{A^* h^*}{l^*}} - \frac{l^*}{h^*} \right\} & \text{for } M_0^2 l^{*2} < 1 \\ \frac{12h^*}{l^* M_0^2} \left\{ \frac{1 + \text{Cosh}(\sqrt{2}h^*/l^*)}{(3/\sqrt{2}) \text{Sinh}(\sqrt{2}h^*/l^*) - (h^*/l^*)} - \frac{l^*}{h^*} \right\} & \text{for } M_0^2 l^{*2} = 1 \\ \frac{12h^*}{l^* M_0^2} \left\{ \frac{M_0 (\text{Cos}B_1^* h^* + \text{Cosh}A_1^* h^*)}{A_2^* \text{Sin}B_1^* h^* + B_2^* \text{Sinh}A_1^* h^*} - \frac{l^*}{h^*} \right\} & \text{for } M_0^2 l^{*2} > 1 \end{cases}$$

The coefficient of friction is given by

$$C = \frac{F^*}{W^*} \tag{18}$$

III. RESULTS AND DISCUSSIONS

The effect of non-dimensional couple stress parameter l^* on magneto hydrodynamic curved pivoted slider bearing is presented in this paper. The parameter M_0 is the Hartmann number l^* ($= 2l/h$) where l ($= (\eta/h)^{1/2}$) arises due the presence of small polar additives in the lubricant. The dimension of (η/μ) is of length square and this length may be regarded as chain length of the polar additives in the lubricant. Hence in the present analysis the following range of values of non-dimensional parameters are used to obtain the bearing characteristics.

- a) Hartmann number $M_0 = 0 - 6$
- b) Couple stress parameter $l^* = 0 - 0.4$.
- c) Curvature parameter $\beta = 0 - 0.6$.

3.1 Pressure

The Figure.2 and Figure.3 Shows the variation of non-dimensional pressure P^* with x^* for different values of the Hartmann number M_0 and couple stress parameter l^* . It is observed that the non-dimensional pressure P^* increases with increasing values of M_0 (Fig.2) and l^* (Fig.3), which represents respectively Newtonian and Non-Newtonian case. The variation of non-dimensional pressure P^* with x^* for different values of curvature parameter β is presented in the Figure.4. It is observed that the non-dimensional pressure P^* increases with increasing values of β .

3.2 Load carrying capacity

Figure.5 and Figure.6 shows the variation of non-dimensional load carrying capacity W^* with h_1^* for different values M_0 and l^* . It is observed that the load carrying capacity W^* increases with increasing values of Hartmann number M_0 (Fig.5) and couple stress l^* (Fig.6) as compared to the Newtonian and non-Newtonian case. The variation of non-dimensional load carrying capacity W^* with h_1^* for different values of curvature parameter β is depicted in Figure.7. It is observed that the load carrying capacity W^* increases with increasing values of β .

3.3 Frictional force

The variation of non-dimensional frictional force F^* with h_1^* for different values of curvature parameter is depicted in Figure.8 it is observed that the non-dimensional frictional force F^* increases with increasing values of β . Figure.9 and Figure.10 shows the variation of non-dimensional frictional force F^* with h_1^* for different values of l^* and M_0 . It is observed that the effect of frictional force F^* increases with increasing values of l^* (Fig.9) and M_0 (Fig.10) as compared to the Newtonian and non-Newtonian case.

3.4 Coefficient of friction

The variation of non-dimensional coefficient of friction C with h_1^* for different values of M_0 and l^* . It is observed that, the non-dimensional coefficient of friction C increases with increasing values of M_0 (Fig.11) and decreases with increasing values couple stress l^* (Fig.12) as compared to the Newtonian and non-Newtonian case. Figure.13 shows the variation of non-dimensional coefficient of friction C with h_1^* for different values of curvature parameter β . It is observed that the coefficient of friction decreases with increasing values of β .

IV. CONCLUSIONS

The combined discussion of Analysis of MHD effect on Pivoted Curved Slider Bearings lubricated with Non-Newtonian Fluids by using basis of Stokes (1966) couple stress fluid theory and the modified Darcy's equations for the flow of conducting couple stress fluid.

The following conclusions are drawn from above results obtained.

- 1) Due to magnetic field we found enhancement in the pressure, load-supporting capacity, frictional force, and decreases the coefficient of friction.
- 2) Due to Couple Stress characteristics we observed that the pressure, load carrying capacity and frictional force increases, and coefficient of friction decreases.
- 3) Due to curvature parameter β we found that the pressure increases at certain point then it decreases, and increases the load carrying capacity, frictional force, decreases the coefficient of friction.

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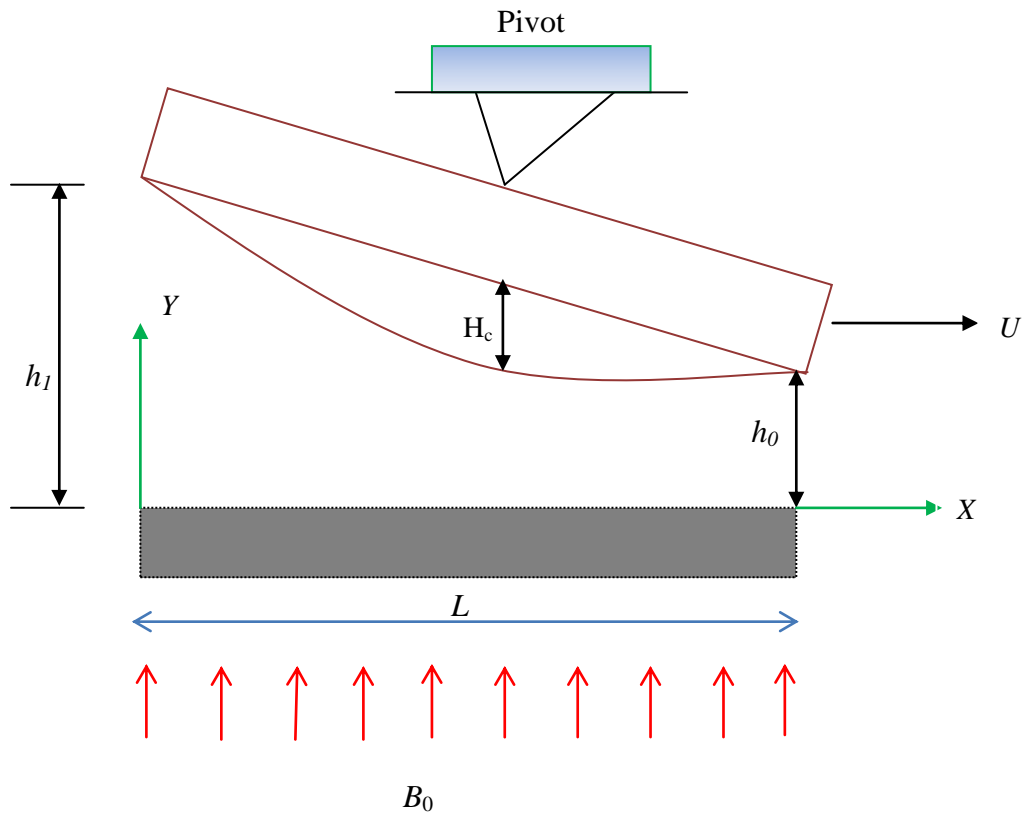


Figure 1: Physical geometry of Pivoted Slider Bearing with convex pad surface

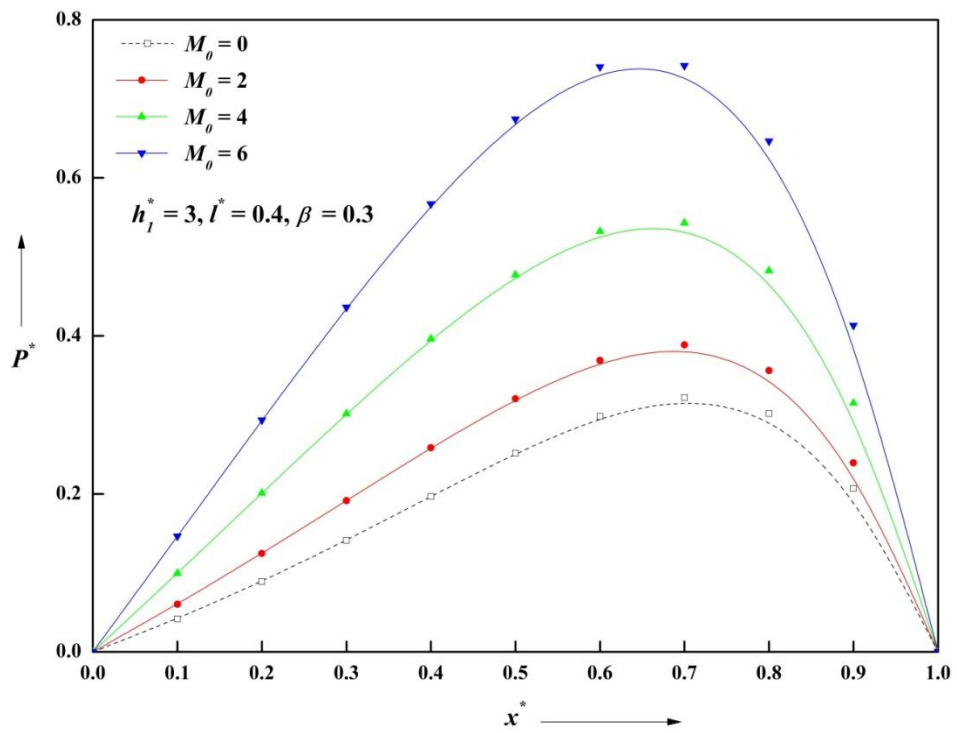


Figure 2: Variation of non-dimensional pressure P^* with x^* for different values of M_0 with $h_l^* = 3, l^* = 0.4$ and $\beta = 0.3$.

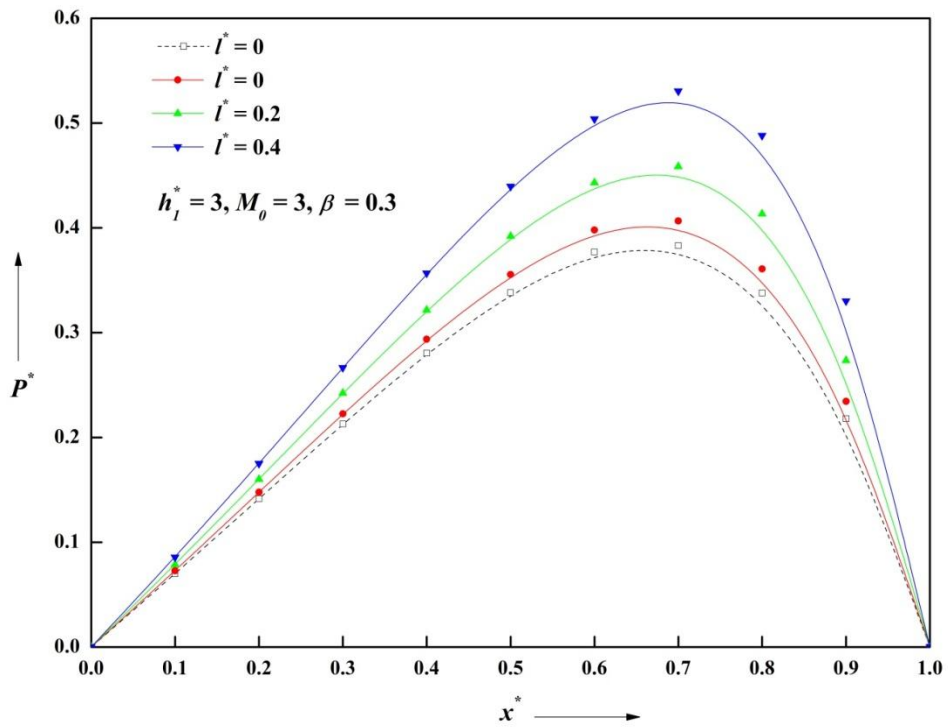


Figure 3: Variation of non-dimensional pressure P^* with x^* for different values of l^* with $h_1^* = 3$, $M_0 = 3$ and $\beta = 0.3$.

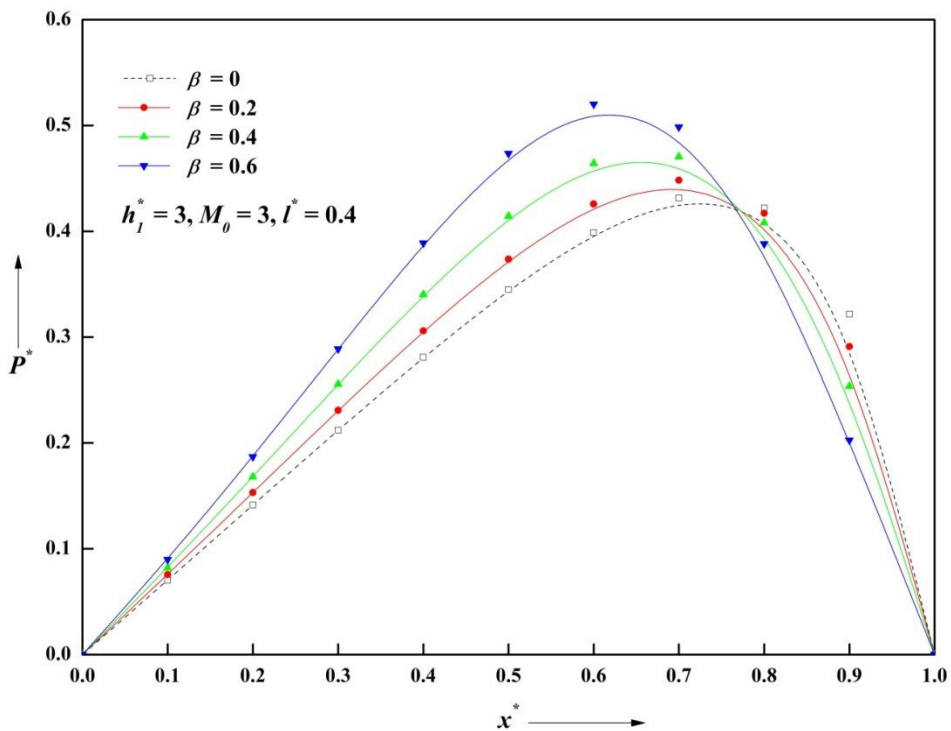


Figure 4: variation of non-dimensional pressure P^* with x^* for different values of β with

$$h_l^* = 3, M_0 = 3 \text{ and } l^* = 0.4..$$

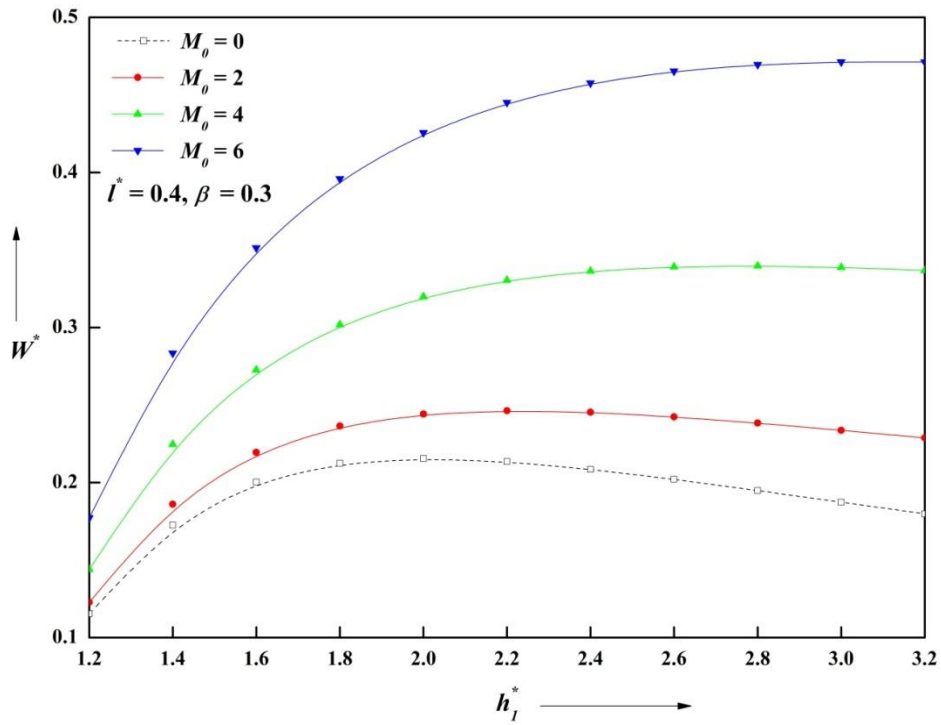


Figure 5: Variation of non-dimensional load carrying capacity W^* with h_l^* for different values of M_0 with $l^*=0.4$ and $\beta = 0.3$.

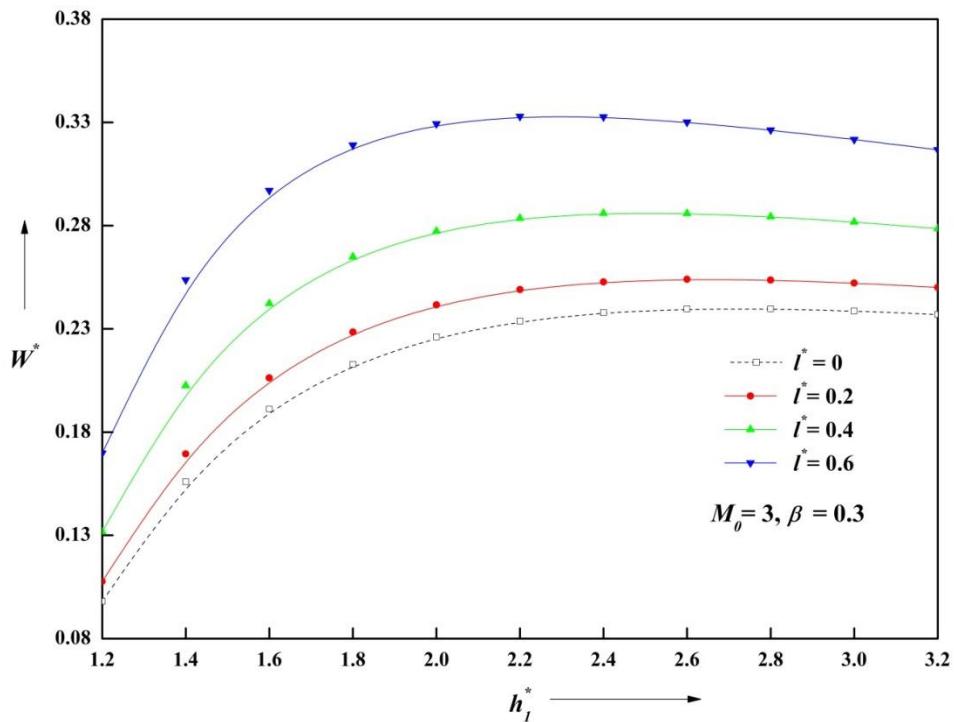


Figure 6: Variation of non-dimensional load carrying capacity W^* with h_l^* for different values of l^* with $M_0 = 3$ and $\beta = 0.3$.

values of l^* with $M_0 = 3$ and $\beta = 0.3$.

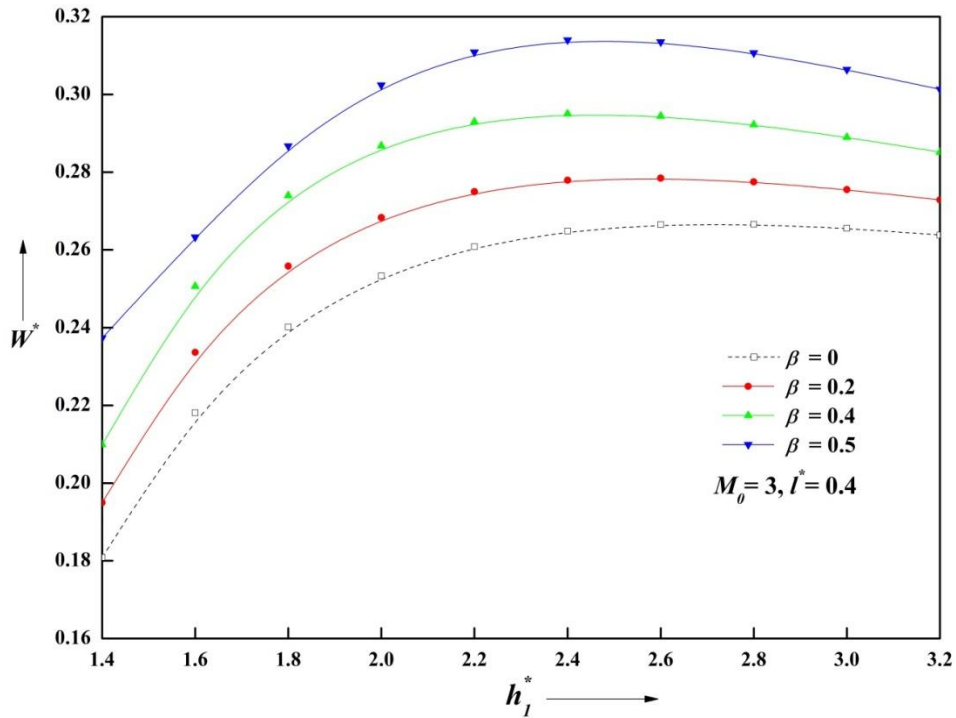


Figure 7: Variation of non-dimensional load carrying capacity W^* with h_1^* for different values of β with $M_0=3$, $l^* = 0.4$.

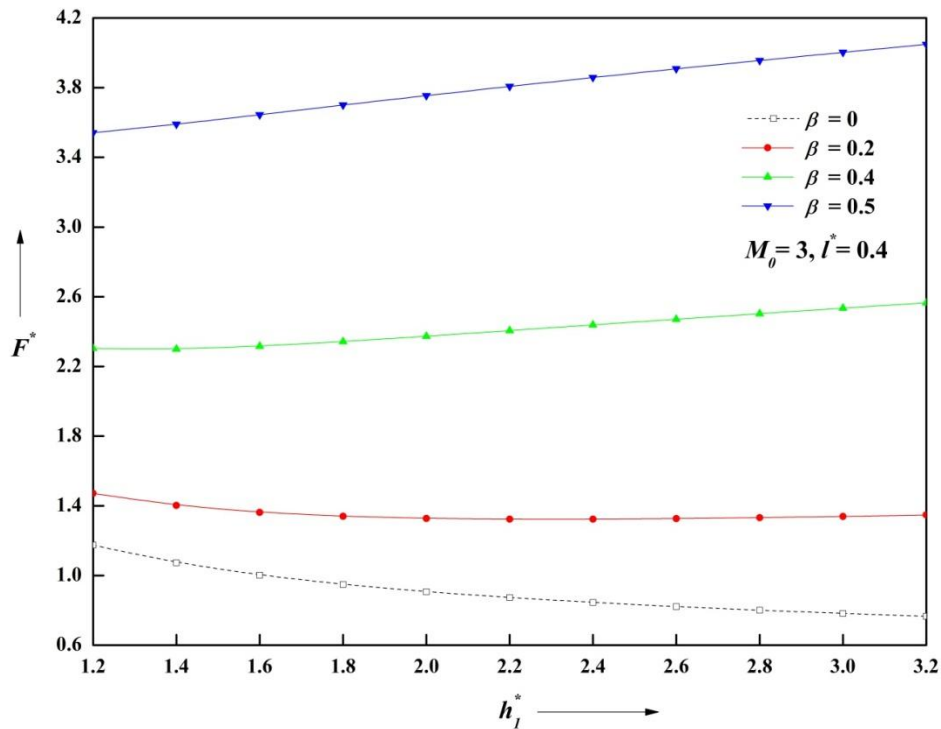


Figure 8: Variation of non-dimensional frictional force F^* with h_1^* for different values of β with $M_0=3$ and $l^*=0.4$.

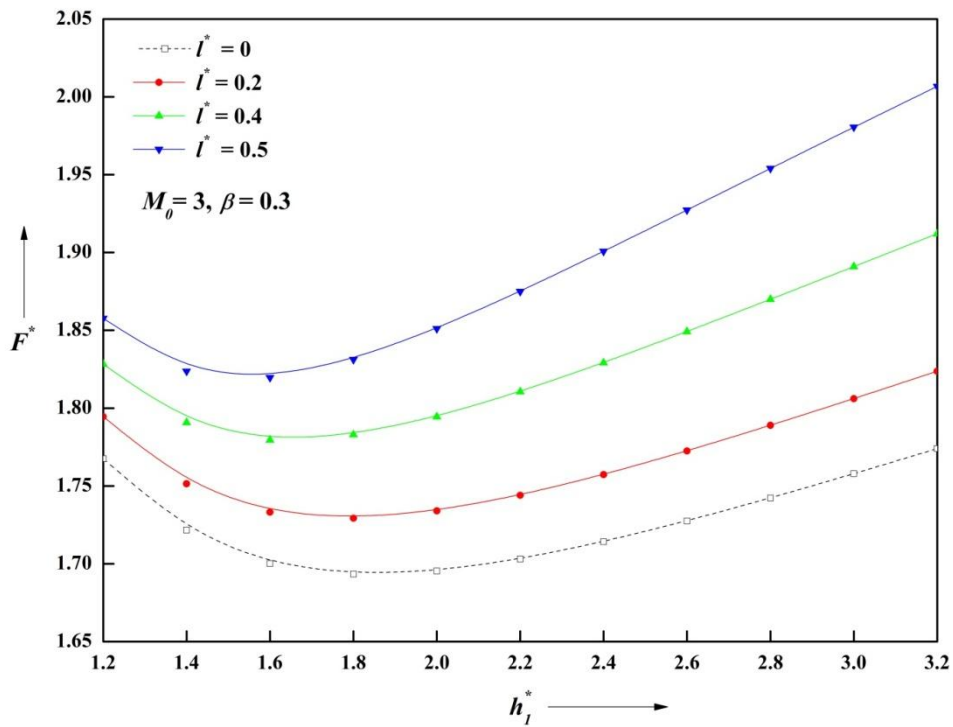


Figure 9: Variation of non-dimensional frictional force F^* with h_1^* for different values of l^* with $M_0=3$ and $\beta=0.3$.

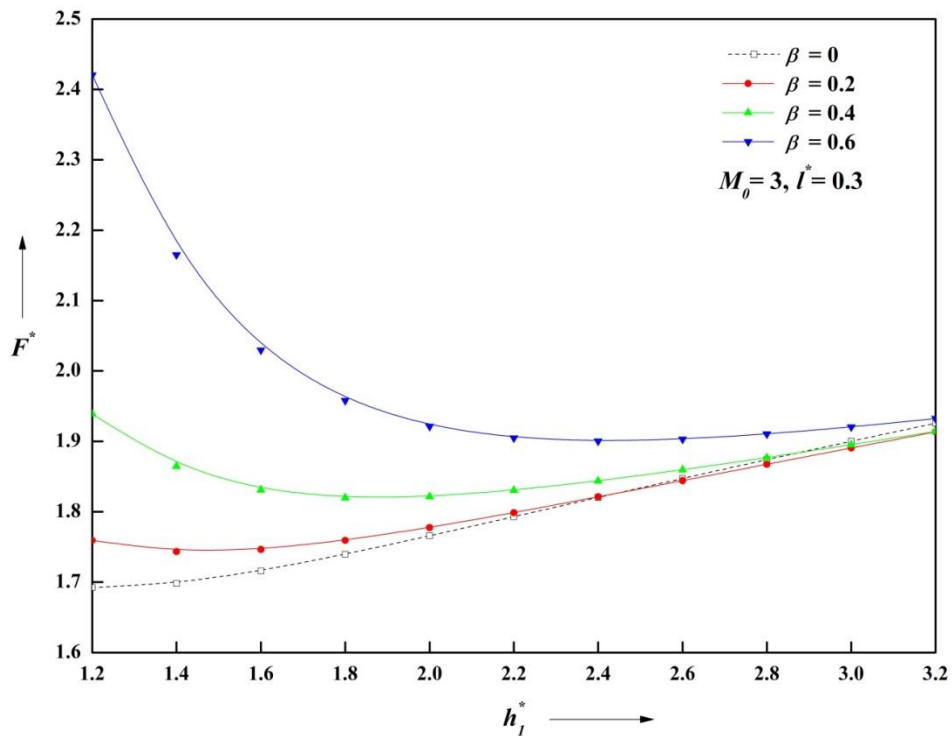


Figure 10: Variation of non-dimensional frictional force F^* with h_1^* for different values of β With $M_0=3$ and $l^*=0.3$.

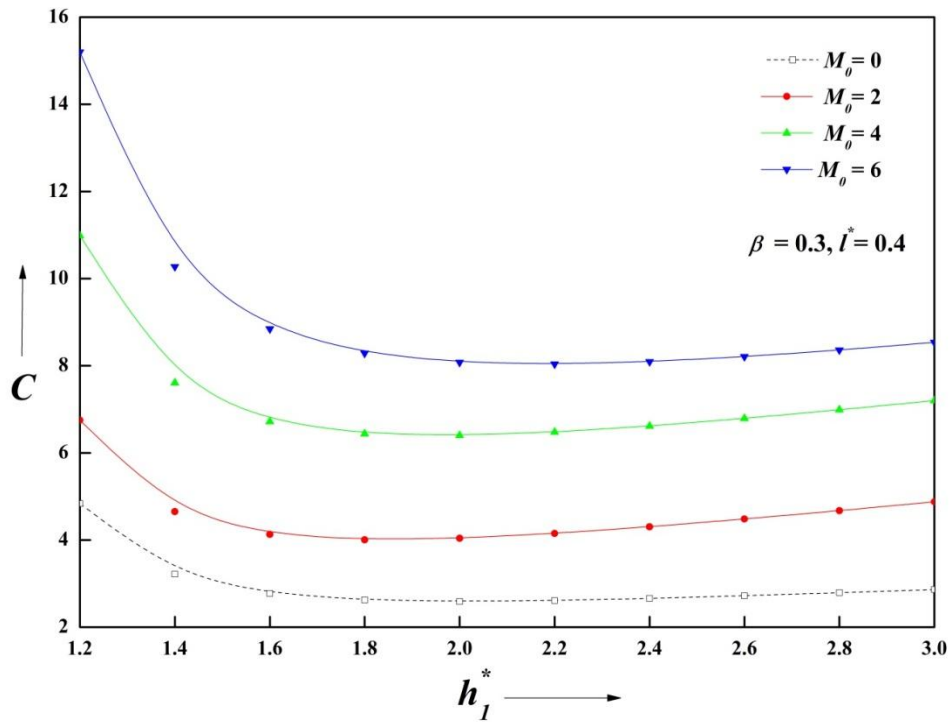


Figure 11: Variation of non-dimensional co-efficient of friction C with h_1^* for different values of M_0 with $\beta=3$ and $l^*=0.4$.

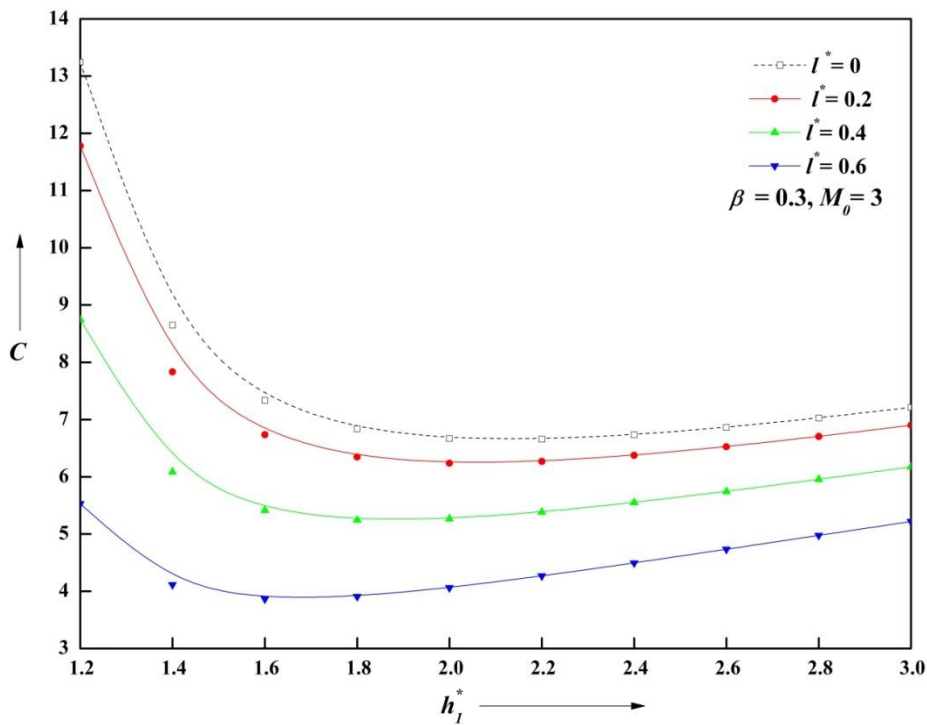


Figure 12: Variation of non-dimensional coefficient of friction C with h_1^* for different values of l^* with $\beta=0.3$ and $M_0=3$.

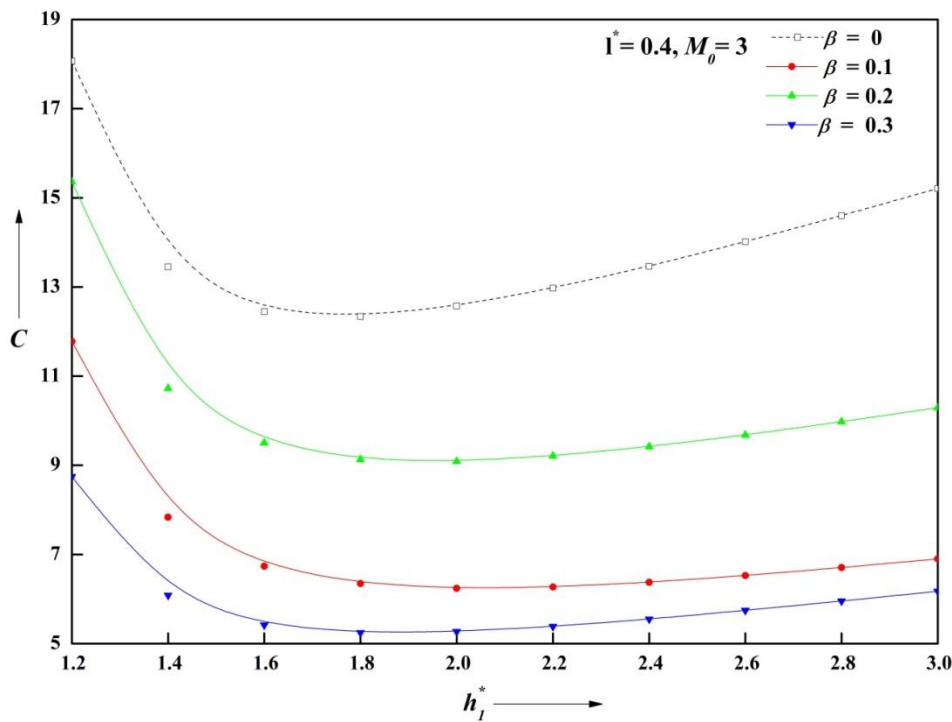


Figure 13: Variation of non-dimensional co-efficient of friction C with h_1^* for different values Of β with $l^* = 0.4$ and $M_0=3$.

Nomenclature

- B_0 applied magnetic field
- C coefficient of friction
- F frictional force
- F^* non-dimensional frictional force $(= -Fh_0 / \mu UL)$
- h film thickness
- h_0 inlet film thickness
- h_0^* non-dimensional inlet film thickness
- h_1 outlet film thickness
- H non-dimensional film thickness $(= h/h_0)$
- l couple stress parameter $(\eta/\mu)^{1/2}$
- l^* non-dimensional couple stress parameter $(2l/h_0)$
- L Bearing length
- M_0 Hartmann number $(= B_0 h_0 (\sigma/\mu)^{1/2})$
- p pressure in the film region
- P^* non-dimensional pressure $(= ph_0^2 / \mu UL)$
- x, y rectangular coordinates
- x^* non-dimensional rectangular coordinates $(x^* = x/L)$
- u, v velocity components in film region
- w load carrying capacity

- W^* non-dimensional load carrying capacity ($= -wh_0^2 / \mu UL^2$)
 η material constant characterizing couple stress
 μ viscosity coefficient
 σ electrical conductivity

Appendix A

$$\xi_{11} = \frac{B^2}{(A^2 - B^2)} \left\{ \frac{\text{Sinh}(Ah/l) - \text{Sinh}(Ay/l) - \text{Sinh } A(h-y)/l}{\text{Sinh}(Ah/l)} \right\} \quad (\text{A1a})$$

$$\xi_{12} = \frac{A^2}{(A^2 - B^2)} \left\{ \frac{\text{Sinh}(Bh/l) - \text{Sinh}(By/l) - \text{Sinh } B(h-y)/l}{\text{Sinh}(Bh/l)} \right\} \quad (\text{A1b})$$

$$\xi_{13} = \frac{B^2 \{ \text{Sinh}(Ah/l) - \text{Sinh}(Ay/l) + \text{Sinh } A(h-y)/l \}}{\text{Sinh}(Ah/l) \{ (B^2/A) \tanh(Ah/2l) - (A^2/B) \tanh(Bh/2l) \}} \quad (\text{A1c})$$

$$\xi_{14} = \frac{A^2 \{ \text{Sinh}(Bh/l) - \text{Sinh}(By/l) + \text{Sinh } B(h-y)/l \}}{\text{Sinh}(Bh/l) \{ (B^2/A) \tanh(Ah/2l) - (A^2/B) \tanh(Bh/2l) \}} \quad (\text{A1d})$$

$$A = \left[\frac{1 + \{ 1 - (4l^2 M_0^2 / h_0^2) \}^{1/2}}{2} \right]^{1/2} \quad (\text{A1e})$$

$$B = \left[\frac{1 - \{ 1 - (4l^2 M_0^2 / h_0^2) \}^{1/2}}{2} \right]^{1/2} \quad (\text{A1f})$$

$$\xi_{21} = \frac{\text{Sinh}\{(y-h)/\sqrt{2l}\} + \text{Sinh}(y/\sqrt{2l}) - \text{Sinh}(h/\sqrt{2l})}{\text{Sinh}(h/\sqrt{2l})} \quad (\text{A2a})$$

$$\xi_{22} = \frac{y \text{Cosh}\{(y-h)/\sqrt{2l}\} + y \text{Cosh}(y/\sqrt{2l}) - h \text{Cosh}(h/\sqrt{2l}) - h}{2\sqrt{2l} \text{Sinh}(h/\sqrt{2l})} \quad (\text{A2b})$$

$$\xi_{23} = \frac{y \text{Sinh}\{(y-h)/\sqrt{2l}\} + y \text{Sinh}(y/\sqrt{2l}) - h \text{Sinh}(y/\sqrt{2l})}{6l \text{Sinh}(h/\sqrt{2l}) - \sqrt{2}h} \quad (\text{A2c})$$

$$\xi_{24} = \frac{2 \text{Cosh}\{(y-h)/\sqrt{2l}\} + 2 \text{Cosh}(y/\sqrt{2l}) - 2 \text{Cosh}(h/\sqrt{2l}) - 2}{3\sqrt{2} \text{Sinh}(h/\sqrt{2l}) - (h/l)} \quad (\text{A2d})$$

$$\xi_{31} = \frac{\text{Cosh}A_1 y \text{Cos}B_1 (y-h) - \text{Cos}B_1 y \text{Cosh}A_1 (y-h)}{(\text{Cosh}A_1 h - \text{Cos}B_1 h)} \quad (\text{A3a})$$

$$\xi_{32} = \frac{\text{Cot}\theta \{ \text{Sinh}A_1 y \text{Sin}B_1 (y-h) - \text{Sin}B_1 y \text{Sinh}A_1 (y-h) \} + (\text{Cosh}A_1 h - \text{Cos}B_1 h)}{(\text{Cosh}A_1 h - \text{Cos}B_1 h)} \quad (\text{A3b})$$

$$\xi_{33} = \frac{\text{Cot}\theta \{ \text{Sin}B_1 y \text{Sinh}A_1 (y-h) + \text{Sinh}A_1 y \text{Sin}B_1 (y-h) \} + (\text{Cos}B_1 h + \text{Cosh}A_1 h)}{(B_1 - A_1 \text{Cot}\theta) \text{Sin}B_1 h + (A_1 + B_1 \text{Cot}\theta) \text{Sinh}A_1 h} \quad (\text{A3c})$$

$$\xi_{34} = \frac{\text{Cos}B_1 y \text{Cosh}A_1 (y-h) + \text{Cosh}A_1 y \text{Cos}B_1 (y-h)}{(B_1 - A_1 \text{Cot}\theta) \text{Sin}B_1 h + (A_1 + B_1 \text{Cot}\theta) \text{Sinh}A_1 h} \quad (\text{A3d})$$

$$A_1 = \sqrt{M_0/lh_0} \cos(\theta/2) \text{ (A3e)}$$

$$B_1 = \sqrt{M_0/lh_0} \sin(\theta/2) \text{ (A3f)}$$

$$\theta = \tan^{-1} \left(\sqrt{4l^2 M_0^2 / h_0^2 - 1} \right) \text{ (A3g)}$$

Ayyappa GH .“Effect of MHD on Pivoted Curved Slider Bearing Lubricated with Non-Newtonian Fluid.” IOSR Journal of Engineering (IOSRJEN), vol. 09, no. 03, 2019, pp. 01-16.