

On Quasi Intuitionistic A-Open Maps

T.A.Albinaa* And Gnanambalilango*

Government Artscollege, CBE
 Corresponding Author: T.A.Albinaa

Abstract: We introduce the concept of quasi intuitionistic α -open function and quasi intuitionistic α -preopen functions. Further α -normal and α -quasi normal spaces are introduced in intuitionistic topological spaces.

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I. INTRODUCTION

In 2011 [6] introduced Quasi α -Closed, Strongly α -Closed and Weakly α -Irresolute Functions in topological spaces. In the present paper we define quasi intuitionistic α -open function, quasi intuitionistic α -preopen function, intuitionistic α -normal and intuitionistic α -quasi normal spaces and obtain their basic properties.

II. PRELIMINARIES

Definition 2.1. [4] An intuitionistic set A is an object having the form $\langle X, A_1, A_2 \rangle$ where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \emptyset$. The set A_1 is called the set of members of A , while A_2 is called the set of nonmembers of A . Furthermore, let $\{A_i : i \in I\}$ be an arbitrary family of intuitionistic sets in X , where $A_i = \langle X, A_i^1, A_i^2 \rangle$ then

- $\varphi \sim = \langle X, \varphi, X \rangle$, $X \sim = \langle X, X, \varphi \rangle$
- A if $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$
- $\bar{A} = \langle X, A_2, A_1 \rangle$
- $A - B = A \cap \bar{B}$
- $[] A = \langle X, A, A_1^c \rangle$
- $\langle \rangle A = \langle X, A_2^c, A_2 \rangle$
- $\cap A_i = \langle X, \cap A_i^1, \cup A_i^2 \rangle$ and $\cup A_i = \langle X, \cup A_i^1, \cap A_i^2 \rangle$.

Definition 2.2. [4] An intuitionistic topological space on a nonempty set X is a family τ of intuitionistic sets in X satisfying the following axioms:

- $\varphi \sim, X \sim \in \tau$
- $G_1 \cap G_2 \in \tau$ for $G_1, G_2 \in \tau$
- $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$

In this case the pair (X, τ) is called intuitionistic topological space and any intuitionistic set in τ is known as an intuitionistic open set in X , and the complement of intuitionistic open sets is known as intuitionistic closed set in X .

Definition 2.3. [10] Let (X, τ) be an intuitionistic topological space. An intuitionistic set A of X is said to be

- Intuitionistic semiopen if $A \subseteq I \text{ cl}(I \text{ int}(A))$
- Intuitionistic preopen if $A \subseteq I \text{ int}(I \text{ cl}(A))$
- Intuitionistic regular open if $A = I \text{ int}(I \text{ cl}(A))$.

The family of all intuitionistic preopen and intuitionistic regular open sets of (X, τ) are denoted by $IPOS(X)$ and $IROS(X)$ respectively.

Definition 2.4. [4] Let (X, τ) be an intuitionistic topological space on X and $A = \langle X, A_1, A_2 \rangle$ be an intuitionistic set in X . Then the several topologies generated by (X, τ) are

- $\tau_{0,1} = \{ [] A : A \in \tau \}$

- $\tau_{0,2} = \{ \langle A : A \in \tau \rangle \}$
- $\tau_1 = \{ A_1 : \langle X, A_1, A_2 \rangle \in \tau \}$
- $\tau_2 = \{ (A_2)^c : \langle X, A_1, A_2 \rangle \in \tau \}$

Definition 2.5.[4]

- If $B = \langle Y, B_1, B_2 \rangle$ is an intuitionistic set in Y , then the preimage of B under f , denoted by $f^{-1}(B)$, is the intuitionistic set in X defined by $f^{-1}(B) = \langle X, f^{-1}(B_1), f^{-1}(B_2) \rangle$
- If $A = \langle X, f(A_1), f(A_2) \rangle$ is an intuitionistic set in X , then the image of A under f , denoted by $f(A)$, is the intuitionistic set in Y defined by $f(A) = \langle Y, f_+(A_1), f_-(A_2) \rangle$ where $f_-(A_2) = Y - (f(X - A_2))$.
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III. ON QUASI INTUITIONISTIC A-OPEN MAPS

Definition 3.1. An intuitionistic map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be quasi intuitionistic α -open if the image of every intuitionistic α -open set in X is intuitionistic open in Y .

Theorem 3.2. Every quasi intuitionistic α -open map is intuitionistic open

Proof. Let U be intuitionistic open in X . Since every intuitionistic open set is intuitionistic α -open, U is intuitionistic α -open in X . Then $f(U)$ is intuitionistic open in Y . Hence f is intuitionistic open.

Theorem 3.3. An intuitionistic map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be quasi intuitionistic α -open iff for every subset U of X , $f(I_\alpha \text{int}(U)) \subset I \text{int}(f(U))$

Proof. Let f be quasi intuitionistic α -open function. For an intuitionistic set U of X , $I \text{int}(U) \subset U$ and $I_\alpha \text{int}(U)$ is an intuitionistic α -open set. Hence, $f(I_\alpha \text{int}(U)) \subset f(U)$. As $f(I_\alpha \text{int}(U))$ is intuitionistic open, $f(I_\alpha \text{int}(U)) \subset I \text{int}(f(U))$.

Conversely, assume $f(I_\alpha \text{int}(U)) \subset I \text{int}(f(U))$. If U is intuitionistic α -open in X then $f(U) = f(I_\alpha \text{int}(U)) \subset I \text{int}(f(U))$. But we know that $I \text{int}(f(U)) \subset f(U)$.

Therefore $f(U) = I \text{int}(f(U))$ and so f is quasi intuitionistic α -open.

Lemma 3.4. An intuitionistic function $f : (X, \tau) \rightarrow (Y, \sigma)$ is quasi intuitionistic α -open then $I_\alpha \text{int} f^{-1}(B) \subset f^{-1}(I \text{int}(B))$ for every set B of Y .

Proof. Let B be any intuitionistic set in Y and $I_\alpha \text{int} f^{-1}(B)$ is an intuitionistic α -open set in X . As f is quasi intuitionistic α -open, $f(I_\alpha \text{int} f^{-1}(B)) \subset I \text{int} f(f^{-1}(B)) \subset I \text{int}(B)$. Thus $I_\alpha \text{int} f^{-1}(B) \subset f^{-1}(I \text{int}(B))$.

Definition 3.5. An intuitionistic subset S is called intuitionistic α -neighbourhood of a point x_\sim in X if there exists an intuitionistic α -open set U such that $x_\sim \in U \subset S$

Theorem 3.6. Let (X, τ) and (Y, σ) be intuitionistic topological space and $f : (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic function. Then the following are equivalent.

- (i) f is quasi intuitionistic α -open.
- (ii) For each subset U of X , $f(I_\alpha \text{int}(U)) \subset I \text{int}(f(U))$.
- (iii) For each $x_\sim \in X$ and each intuitionistic α -neighbourhood U of $x_\sim \in X$ there exists intuitionistic neighbourhood V of $f(x_\sim)$ in Y such that $V \subset f(U)$.

Proof. (i) \Rightarrow (ii) By Theorem

(ii) \Rightarrow (iii) Assume (ii) holds. Let $x_\sim \in X$ and U be an arbitrary intuitionistic α -neighbourhood of $x_\sim \in X$. Then there exists an intuitionistic α -open set V in X such that $x_\sim \in V \subset U$. Then by (ii) $f(V) = f(I(V)) \subset I \text{int}(f(V))$ and hence $f(V) = I \text{int}(f(V))$. Therefore it follows $f(V)$ is intuitionistic open in Y such that $f(x_\sim) \in f(V) \subset f(U)$. Hence (iii) holds.

(iii) \Rightarrow (i) Assume (iii) holds, let U be an arbitrary intuitionistic α -open set in X . Then for each $y_\sim \in f(U)$, by (iii) there exists intuitionistic neighbourhood V of y_\sim in Y such that $V \subset f(U)$. As V is an intuitionistic neighbourhood of y_\sim there exists intuitionistic open set W in Y such that $y_\sim \in W \subset V$. Thus, $f(U) = \cup W / y_\sim \in f(U)$ which is intuitionistic open in Y . this implies f is intuitionistic α -open.

Theorem 3.7. An intuitionistic function $f : (X, \tau) \rightarrow (Y, \sigma)$ is quasi intuitionistic α -open iff for any subset B of Y and for any intuitionistic α -closed set F of X containing $f^{-1}(B)$ there exists intuitionistic closed set G of Y containing B such that $f^{-1}(G) \subset F$.

Proof. Let f be quasi intuitionistic α -open function. Let $B \subset Y$ and F be intuitionistic α -closed set of X containing $f^{-1}(B)$. Put $G = Y - f(X - F)$. As $f^{-1}(B) \subset F$ we have $B \subset G$. As f is quasi intuitionistic α -open, G is intuitionistic closed in Y . Then $f^{-1}(G) \subset F$. Conversely, let U be intuitionistic α -open in X . Put $B = Y - f(U)$ then $X - U$ is intuitionistic α -closed

closed in X containing $f^{-1}(B)$. By hypothesis, there exists intuitionistic closed set G of Y such that $B \subset G$ and $f^{-1}(G) \subset X - U$. Hence, $f(U) \subset Y - G$. Since $B \subset G$, $Y - G \subset Y - B = f(U)$ we obtain $f(U) = Y - G$ which is intuitionistic open. Therefore it follows f is quasi intuitionistic α -open function.

Theorem 3.8. Let (X, τ) , (Y, σ) and (Z, η) be intuitionistic topological space and $f: (X, \tau) \rightarrow (Y, \sigma)$, $g: (Y, \sigma) \rightarrow (Z, \eta)$ be two intuitionistic functions and $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ be quasi intuitionistic α -open. Then

(i). If f is intuitionistic α -open surjective, then g is quasi intuitionistic α -open.

(ii). If g is intuitionistic continuous and injective, then f is quasi intuitionistic α -open.

Proof. (i). Let U be intuitionistic open in X , then $f(U)$ is intuitionistic α -open in Y . As $g \circ f$ is quasi intuitionistic α -open, $g(f(U)) = g \circ f [f(U)]$ is intuitionistic open in Z . Hence g is quasi intuitionistic α -open.

(ii). Let V be intuitionistic α -open in X . As $g \circ f$ is quasi intuitionistic α -open, $g \circ f(V)$ is intuitionistic open in Z . Since g is intuitionistic continuous, $f(V) = g^{-1}(g \circ f(V))$ is intuitionistic open in Y .

IV. ON QUASI INTUITIONISTIC α -PREOPEN FUNCTION

Definition 4.1. Let (X, τ) be an intuitionistic topological space and $A = \langle X, A_1, A_2 \rangle$ be an intuitionistic set in X . Then intuitionistic pre-interior of A , denoted by $Ipint(A)$ is defined as the union of all intuitionistic preopen sets contained in A .

Definition 4.2. An intuitionistic map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be quasi intuitionistic α -preopen if the image of every intuitionistic α -open set in X is intuitionistic preopen in Y .

Theorem 4.3. An intuitionistic map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be quasi intuitionistic α -preopen iff for every subset U of X , $f(Iaint(U)) \subset Ipint(f(U))$

Proof. Let f be quasi intuitionistic α -preopen function. In general we know that $Ipint(U) \subset U$ and $Iaint(U)$ is an intuitionistic α -open set. Hence, $f(Iaint(U)) \subset f(U)$. As $f(Iaint(U))$ is intuitionistic preopen then $f(Iaint(U)) \subset Ipint(f(U))$. Conversely, assume $f(Iaint(U)) \subset Ipint(f(U))$. If U is intuitionistic α -open in X then $f(U) = f(Iaint(U)) \subset Ipint(f(U))$. But we know that $Ipint(f(U)) \subset f(U)$. Therefore $f(U) = Ipint(f(U))$. Therefore f is quasi intuitionistic α -preopen.

Lemma 4.4. An intuitionistic function $f: (X, \tau) \rightarrow (Y, \sigma)$ is quasi intuitionistic α -preopen then $Iaint f^{-1}(G) \subset f^{-1}(Ipint(G))$ for every set G of Y .

Proof. Let G be an intuitionistic set of Y . Then $Iaint f^{-1}(G)$ is an intuitionistic α -open in X . As f is quasi intuitionistic α -preopen, $f(Iaint f^{-1}(G)) \subset Ipint(f(f^{-1}(G))) \subset Ipint(G)$. Therefore $Iaint f^{-1}(G) \subset f^{-1}(Ipint(G))$

Definition 4.5. An intuitionistic set S is called intuitionistic pre-neighbourhood of a point x in X if there exists an intuitionistic pre-open set U such that $x \in U \subset S$

Theorem 4.6. Let (X, τ) and (Y, σ) be intuitionistic topological space and $f: (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic function. Then the following are equivalent.

- (i) f is quasi intuitionistic α -preopen.
- (ii) For each intuitionistic set U of X , $f(Iaint(U)) \subset Ipint(f(U))$.
- (iii) For each $x \in X$ and each intuitionistic α -neighbourhood U of $x \in X$ there exists intuitionistic pre neighbourhood V of $f(x)$ in Y such that $V \subset f(U)$.

Proof. (i) \Rightarrow (ii) By previous theorem

(ii) \Rightarrow (iii) Assume for an intuitionistic subset U of X , $f(Iaint(U)) \subset Ipint(f(U))$.

Let $x \in X$ and U be an arbitrary intuitionistic α -neighbourhood of $x \in X$. Then there exists an intuitionistic α -open set V in X such that $x \in V \subset U$. Then, $f(V) = f(Iaint(V)) \subset Ipint(f(V))$ and hence $f(V) = Ipint(f(V))$. Therefore $f(V) = W$ is intuitionistic preopen in Y such that $f(x) \in W \subset f(U)$. Hence (iii) holds.

(iii) \Rightarrow (i) Assume (iii) holds, let U be intuitionistic α -open set in X . Then for every $y \in f(U)$, by (iii) there exists intuitionistic pre-neighbourhood V of y in Y such that $V \subset f(U)$. As V is an intuitionistic pre-neighbourhood of y , there exists intuitionistic preopen set in Y . This implies f is quasi intuitionistic α -preopen.

Theorem 4.7. An intuitionistic function $f: (X, \tau) \rightarrow (Y, \sigma)$ is quasi intuitionistic α -preopen iff for any subset B of Y and for any intuitionistic α -closed set F of X containing $f^{-1}(B)$ there exists intuitionistic pre-closed set G of Y containing B such that $f^{-1}(G) \subset F$

Proof. Let f be quasi intuitionistic α -preopen function. Let B of Y and F be intuitionistic α -closed set

of X containing $f^{-1}(B)$. Put $G = Y - f(X - F)$. As $f^{-1}(B) \subset F$ we have $B \subset G$. Since f is quasi intuitionistic α -preopen, G is intuitionistic preclosed in Y . Then $f^{-1}(G) \subset F$. Conversely, let U be intuitionistic α -open in X . Put $B = Y - f(U)$ then $X - U$ is intuitionistic α -closed in X containing $f^{-1}(B)$. By hypothesis, there exists intuitionistic preclosed set G of Y such that $B \subset G$ and $f^{-1}(G) \subset X - U$. Hence, $f(U) \subset Y - G$. On the other hand $B \subset G$, $Y - G \subset Y - B = f(U)$. Thus $f(U) = Y - G$ which is intuitionistic preopen. Therefore f is quasi intuitionistic α -preopen function.

Lemma 4.8. Let $(X, \tau), (Y, \sigma)$ and (Z, η) be intuitionistic topological spaces and $f: (X, \tau) \rightarrow (Y, \sigma), g: (Y, \sigma) \rightarrow (Z, \eta)$ be two intuitionistic functions and $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ be quasi intuitionistic α -preopen. Then

- If f is intuitionistic α -continuous, preopen then g is quasi intuitionistic α -preopen.
- If f is intuitionistic pre-irresolute, f is quasi intuitionistic α -preopen.

Proof. Let U be intuitionistic α -open in Y . Then since f is intuitionistic α -continuous, $f^{-1}(U)$ is intuitionistic α -open in X . Since g is quasi intuitionistic α -preopen, $g(U) = g \circ f[f^{-1}(U)]$ is intuitionistic preopen in Z . Hence g is quasi intuitionistic α -preopen.

Let V be intuitionistic α -open in X then $g(f(V))$ is intuitionistic preopen in Z . As g is intuitionistic pre-irresolute, $f(V) = g^{-1}(g \circ f(V))$ is intuitionistic preopen in Y . Hence f is quasi intuitionistic α -preopen.

Theorem 4.9. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be two intuitionistic functions. Then the following statements are valid:

- If $f: (X, \tau) \rightarrow (Y, \sigma)$ is intuitionistic α -open and $g: (Y, \sigma) \rightarrow (Z, \eta)$ is I -pre- α -open, then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is intuitionistic α -open.
- If $f: (X, \tau) \rightarrow (Y, \sigma)$ is I -pre- α -open and $g: (Y, \sigma) \rightarrow (Z, \eta)$ is quasi intuitionistic α -open.
- If $f: (X, \tau) \rightarrow (Y, \sigma)$ is quasi intuitionistic α -open and $g: (Y, \sigma) \rightarrow (Z, \eta)$ is intuitionistic preopen then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is quasi intuitionistic α -preopen.
- If $f: (X, \tau) \rightarrow (Y, \sigma)$ is intuitionistic α -open and $g: (Y, \sigma) \rightarrow (Z, \eta)$ is quasi intuitionistic α -open then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is intuitionistic open.

5. Intuitionistic α -Normal And α -Quasi Normal

Definition 5.1. An intuitionistic topological space (X, τ) is intuitionistic normal iff for every pair L, M of disjoint intuitionistic closed sets of X there exists intuitionistic open sets G and H such that $L \subset G, M \subset H$ and $G \subseteq H^c$.

Definition 5.2. An intuitionistic topological space (X, τ) is intuitionistic α -normal if for every pair F_1, F_2 of disjoint intuitionistic closed sets of X there exists disjoint intuitionistic α -open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$ and $U \subseteq V^c$.

Definition 5.3. An intuitionistic topological space (X, τ) is intuitionistic α -quasi normal iff for every pair of disjoint intuitionistic closed set A and intuitionistic α -closed set B of X there exists intuitionistic α -open sets U and V such that $A \subseteq U, B \subseteq V$ and $U \subseteq V^c$.

Theorem 5.4. If (X, τ) is intuitionistic α -normal then for every pair of intuitionistic open sets U and V whose union is X , there exists intuitionistic α -closed sets A and B such that $A \subset U, B \subset V$ and $A \cup B = X$.

Proof. Let U and V be a pair of intuitionistic open sets in intuitionistic α -normal space (X, τ) such that $U \cup V = X$. Then U^c and V^c are intuitionistic closed sets. Since (X, τ) is intuitionistic α -normal there exist intuitionistic α -open sets U_1 and V_1 such that $U^c \subset U_1, V^c \subset V_1$ and $U_1 \subset V^c$. Let $A = U^c$ and $B = V^c$. Then A and B are intuitionistic α -closed sets such that $A \subset U, B \subset V$ and $A \cup B = X$.

Theorem 5.5. An intuitionistic topological space (X, τ) is intuitionistic α -quasi normal iff for every intuitionistic closed set A and intuitionistic α -open set V containing A there is intuitionistic α -open set U such that $A \subseteq U \subseteq \text{Iacl}(U) \subseteq V$.

Proof. Suppose (X, τ) is intuitionistic α -quasi-normal in which A is intuitionistic closed and V is intuitionistic α -open containing A . Then V^c is intuitionistic α -closed such that $A \cap V^c = \emptyset$. Since (X, τ) is intuitionistic α -quasi-normal there exists intuitionistic α -open sets U and W such that $A \subseteq U, V^c \subseteq W$ and $U \subset W^c$, i.e. $A \subseteq U \subseteq W^c \subseteq V$. Since W^c is intuitionistic α -closed set U containing intuitionistic α -open set $U, U \subseteq \text{Iacl}(U) \subseteq W^c$. Therefore $A \subseteq U \subseteq \text{Iacl}(U) \subseteq V$. Conversely, let A and B be disjoint pair of intuitionistic closed set and intuitionistic α -closed set respectively. Then $A \cap B = \emptyset \Rightarrow A \subseteq B^c$ and B^c is intuitionistic α -open. Then by hypothesis, there exists intuitionistic α -open

set U such that $A \subseteq U \subseteq \text{Iacl}(U) \subseteq B^c$. This implies $A \subseteq U$, $B \subseteq [\text{Iacl}(U)]^c$ and $U \subseteq \text{Iacl}(U)$. Hence (X, τ) is intuitionistic α -quasinormal.

Theorem 5.6. *Intuitionistic α -quasi normality is a topological property*

Proof. Let (X, τ) be intuitionistic α quasi-normal and let (Y, σ) be intuitionistic α -homeomorphic image of (X, τ) under intuitionistic α -homeomorphism f . To show that (Y, σ) is intuitionistic α quasi-normal, let A and B be pair of disjoint intuitionistic closed and intuitionistic α -closed sets in Y . Since f is intuitionistic α -continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are intuitionistic α -closed in X . Also $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = f^{-1}(\emptyset) = \emptyset$. So, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint intuitionistic α -closed of X . Since X is intuitionistic α -quasinormal, there exists intuitionistic α -open sets U and V such that $f^{-1}(A) \subseteq U$, $f^{-1}(B) \subseteq V$ and $U \cap V = \emptyset$. But $f^{-1}(A) \subseteq U \Rightarrow f(f^{-1}(A)) \subseteq f(U) \Rightarrow A \subseteq f(U)$. Similarly $B \subseteq f(V)$. Also since f is bijective intuitionistic α -open, $f(U)$ and $f(V)$ are intuitionistic α -open set of Y such that $f(U) \cap f(V) \subseteq f(U \cap V) = \emptyset \Rightarrow f(U) \subseteq (f(V))^c$. Hence (Y, σ) is intuitionistic α quasinormal. So, intuitionistic α -quasinormal is a topological property.

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